The Shared Broadcast Tree Problem and MST

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Abstract

The shared broadcast tree (SBT) problem in Euclidean graphs resembles the minimum spanning tree (MST) problem, but differs from MST in the definition of the objective function. The SBT problem is known to be NP-hard. In the current work, we analyse how closely the MST-solution approximates the SBT-solution, and we prove in particular that the approximation ratio is at least 6. Further, we conduct numerical experiments comparing the MST-solution and the optimum. The results show that the cost of the MST-solution is around 20% higher than the optimal cost.

Keywords: shared broadcast tree, MST, approximation algorithm

1 Introduction

The purpose of a broadcast communication in a wireless ad-hoc network is to route information from one source node to all other nodes. Given a set of devices and distances between them, the task is to assign a power to each node, so that the communication demands are met and the energy consumption is minimized, assuming their locations are fixed. The devices are able to both transmit and receive a signal, as well as dynamically adjust their power level.

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Omnidirectional antennas are used, and hence a message reaches all nodes within the communication range given by a power assigned to the sender, i.e. the maximum of the powers necessary to reach all intended recipients.

Minimum Energy Broadcast [3] (MEB) is the problem of constructing an optimal arborescence for broadcasting from a given source to all remaining nodes, such that the total power consumption is minimized. A separate tree has to be stored for each source. The idea of SBT [2][4] is to construct a common source-independent tree, instead of a set of individual arborescences. The power levels then depend merely on the immediate neighbour from which a message is received. This idea is based on the observation that a forwarded signal does not have to reach the neighbour from which it originally came.

The decentralized nature of wireless ad-hoc networks implies its suitability for applications, where it is not possible to rely on central nodes, or where network infrastructure does not exist. This is typical for various short-term events like conferences or fixtures. Simple maintenance makes them useful in emergency situations, military conflicts, and home networking.

We model a wireless network as a complete graph $G = (V, E)$, where V corresponds to the network nodes (points in $\mathbb{R}^2$), and the edges E correspond to the potential links between them. The energy requirement for transmission from i to j is denoted by $p_{ij} = \kappa d_{ij}^{\alpha}$, where d_{ij} is the Euclidean distance between i and j, α is an environment-dependent parameter (typically valued between 2 and 4) and κ is a constant. In this work, we use $\alpha = 2$ and $\kappa = 1$. Let $T = (V, E_T)$, $E_T \subseteq E$ be a spanning tree of G. Then $T_{i/j}$ denotes the subtree of T consisting of all vertices k such that the path from k to j visits i, as introduced in [4]. For a non-leaf node i in T, i_1 and i_2 denote the first and the second most distant neighbour of i in T, respectively. If i is a leaf, i_2 is not defined, and we let $p_{ii_2} = 0$. If a message is generated at a node k in $T_{i_1/i}$ then i needs power p_{ii_2} to relay the message to i_2 and other neighbours in $T \setminus T_{i_1/i}$. Power p_{ii_1} is needed to relay messages initiated in $T \setminus T_{i_1/i}$. Assuming that all nodes initiate messages equally frequently, the SBT problem is to construct a spanning tree T minimizing the objective function

$$P(T) = \sum_{i \in V} |T_{i_1/i}| p_{ii_2} + |T \setminus T_{i_1/i}| p_{ii_1}. \tag{1}$$

2 MST as an approximate solution to the SBT problem

Since the SBT problem is NP-hard, inexact solutions are often considered. Because any spanning tree is a feasible solution, the MST-solution yields one such approximation. This approach is also valid for MEB, where MST approximates the optimum with factor 6 [1]. We define the *MST approximation ratio* ρ as the supremum, taken over all SBT instances, of the ratio between the power consumptions in the MST solution and an optimal SBT.

Theorem 2.1 *The MST approximation ratio for SBT is at least 6.*

Proof. For an integer $k \geq 2$, let G_k be a complete Euclidean graph with a node o located in the center of a unit circle, nodes $t_1, \ldots, t_6$ evenly distributed on the circumference, and nodes $s_{i1}, \ldots, s_{ik}$, $(i = 1, \ldots, 6)$, evenly distributed on the radial line $[o, s_{ik}] \subset [o, t_i]$, where s_{ik} is located $1/k$ units from o. Thus, since arc costs p_{uv} are the square of arc lengths d_{uv}, we have $p_{uv} = 1/k^4$ for $u = s_{ij}$, $v = s_{i,j+1}$, whereas $p_{uv} = (1 - 1/k)^2$ for $u = s_{ik}$, $v = t_i$. A possible MST (denoted T_k) of G_k consists of the 6 paths $(o, s_{i1}, \ldots, s_{ik}, t_i)$. For this tree, the objective function (1) evaluates to

$$P(T_k) = \underbrace{6\left(1 - k^{-1}\right)^2}_{t_i} + \underbrace{6\left[(6k+6)\left(1 - k^{-1}\right)^2 + k^{-4}\right]}_{s_{ik}} + \underbrace{(6k-5)(6k+7)k^{-4}}_{o,s_{i1},\ldots,s_{i,k-1}}.$$

Another spanning tree of G_k is the star T_k^* centered at node o. For this solution, (1) evaluates to

$$P(T_k^*) = \underbrace{6}_{t_i} + \underbrace{6\sum_{i=1}^{k}\left(i\frac{1}{k^2}\right)^2}_{s_{i1},\ldots,s_{ik}} + \underbrace{6k+7}_{o}.$$

Thus, the MST-approximation ratio satisfies $\rho \geq \frac{P(T_k)}{P(T_k^*)}$. Since $\lim_{k\to\infty} \frac{P(T_k)}{P(T_k^*)} = 6$, the claim follows. $\square$

3 Numerical Experiments

The SBT problem can be modelled as a MILP [2][4], and moderately sized instances can be solved. We have generated instances of a specific number of nodes with random coordinates distributed uniformly on a square, and compared the MST-solution to the optimal one. The MILP solver CPLEX is used to compute the optimal solution. Each number of nodes is tested in 100 instances. Although the theoretical approximation ratio suggests that MST is not very suitable for SBT, the experimental results summarized in Tab. 1 reveal that in practice, MST represents a feasible solution with objective value approximately 1.2 times the optimum. This factor does not seem to change much with growing number of nodes. However, calculation of the optimum

Table 1
Average SBT costs of MST and optimal solutions for various instance sizes.

Number of nodes	10	12	14	16	18	20
P(OPT)	46268	56060	66747	69727	84250	94039
P(MST)	9432	68833	80195	84262	101816	119679
P(MST)/P(OPT)	1.198	1.232	1.206	1.210	1.209	1.271

for larger instances takes prohibitively long time, so we have access only to limited data. The largest ratio observed in the experiments is 1.59.

4 Conclusion and Future Work

This paper studies the relation between MST and the optimal solution to SBT in terms of the objective value. It has been shown that the MST approximation ratio is at least 6. Numerical experiments suggest that even though there are instances where MST is nearly 60% above the optimum, it represents a good solution in the vast majority of cases. The current research leads to several interesting questions that merit further investigation. A prominent question is whether there exists a constant upper bound on the MST-approximation ratio. For the related MEB problem, approximation algorithms with constant performance guarantee are well studied. Adapting these methods and the corresponding analysis to SBT is a research question to be pursued.

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