Noname manuscript No.
(will be inserted by the editor)

Comparison of Mixed Integer Programming Formulations for the Shared Multicast Tree Problem

Tightening the LP bounds

Dag Haugland · Marika Ivanova

Received: date / Accepted: date

Abstract Insert your abstract here. Include keywords, PACS and mathematical subject classification numbers as needed.

Keywords First keyword · Second keyword · More

1 Introduction

The purpose of a multicast communication in a wireless ad-hoc network is to route information from a sending device to a set of receiving devices. Given a set of wireless devices and distances between them, the task is to assign power to each device, so that the demands of the communication are met and the energy consumption is as low as possible, assuming their locations are fixed. Power efficiency is an important measure in designing ad-hoc wireless networks since the devices typically use batteries as power supply and are therefore heavily energy-constrained. Individual devices work as transceivers, which means that they have the ability to both transmit and receive a signal. Moreover, the power level of a device can be dynamically adjusted during a multicast session.

Unlike wired networks, nodes in ad-hoc wireless networks use omnidirectional antennas, and hence a message reaches all nodes within the communication range of the sender. This range is determined by the power assigned to the sender, which is the maximum rather than the sum of the powers necessary

F. Author
first address
Tel.: +123-45-678910
Fax: +123-45-678910
E-mail: fauthor@example.com

S. Author
second address

to reach all intended receivers. This feature is often referred to as the wireless multicast advantage [**?**].

1.1 Related work

1.2 Definitions and notation

An ad-hoc wireless network is modelled by a complete graph $G = (V, E)$

2 MIP Formulations

In this section, we state the known MIP formulation of the shared multicast tree problem and relations between them. Several strengthening valid inequalities are also presented.

2.1 Original SMT Model

The first model we consider is slightly improved SMT model introduced in **??**, which contains a weaker version of constrainte (**??**). This model extends the SBT formulation from **??** by the Steiner nodes in order to formulate the multicast version of the problem. The model uses three sets of variables defined as follows:

$$z_{ij} = \begin{cases} 1 & \text{if edge } \{i,j\} \in E \text{ is in the solution,} \\ 0 & \text{otherwise,} \end{cases}$$

$$x_{ij}^{s} = \begin{cases} 1 & \text{if arc } (i,j) \in A \text{ is used to transmit a message from } s \in D, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{ij}^{s} = \begin{cases} 1 & \text{if node } i \in V \text{ uses power } p_{ij} \text{ to transmit a message from } s \in D, \\ 0 & \text{otherwise.} \end{cases}$$

$$\min \sum_{(i,j)\in A} \sum_{s\in D} p_{ij} y^s_{ij} \tag{1a}$$

s.t.

$$\sum_{\{i,j\}\in E} z_{ij} \le N-1 \tag{1b}$$

$$\sum_{j\in V_i} x^s_{ji} = 1 \qquad i,s\in D, i\ne s \tag{1c}$$

$$\sum_{j\in V_i} x^s_{ji} \le 1 \qquad i\in V\setminus D, s\in D \tag{1d}$$

$$x^s_{ik} \le \sum_{j\in V_i\setminus\{k\}} x^s_{ji} \qquad i\in V\setminus D, (i,k)\in A, s\in D \tag{1e}$$

$$\sum_{j\in V_i} x^s_{ji} \le \sum_{j\in V_i} x^s_{ij} \qquad i\in V\setminus D, s\in D \tag{1f}$$

$$x^s_{ij} + x^s_{ji} = z_{ij} \qquad \{i,j\}\in E, s\in D \tag{1g}$$

$$x^i_{ji} = 0 \qquad i\in D, (j,i)\in A \tag{1h}$$

$$x^s_{ij} \le \sum_{k\in V: p_{ik}\ge p_{ij}} y^s_{ik} \qquad s\in D, (i,j)\in A \tag{1i}$$

$$\mathbf{z}\in\{0,1\}^E, \mathbf{x},\mathbf{y}\in\{0,1\}^{A\times D} \tag{1j}$$

Let $(\mathbf{x},\mathbf{y},\mathbf{z})$ be an optimal solution to the SMT model above. Then, the vector $\mathbf{x^s}\in\{0,1\}^A$ encapsulates broadcast Steiner arborescences for a source $s\in D$. From $\mathbf{z}\in\{0,1\}^E$ we obtain the resulting (undirected) broadcast Steiner tree. Finally, $\mathbf{y^s}\in\{0,1\}^A$ describes the links determining the power levels. The graph induced by $\mathbf{y}$ is a subgraph of the tree induced by $\mathbf{x}$, and is not necessarily connected.

The number of edges in the resulting Steiner tree is constrained by (1b). Imposing a lower bound $M-1$ on the size of the spanning tree would neither reduce the space of feasible solutions nor increase the strength of the model. If the tree does not contain any Steiner nodes, its size is the lower bound, while if all nodes are used (either as Steiner nodes, or $D=V$), its size equals the upper bound. Constraints (1c) ensures that a message from source s reaches a destination i from exactly one neighbour $j\in V_i$. Analogously, (1d) covers the case when $j\in V\setminus D$: For every source s, there is at most one inbound arc to a non-destination i.

If a non-destination i forwards a message from s towards k, the message must come from exactly one neighbour j different from k, because there is no point in sending the signal backwards. This is ensured by constraint (1e). Note that assuming there is no outgoing arc from a non-destination j, (1e) does not prevent j from being a leaf in $G_{\mathbf{x^s}}$. We make such undesired solutions impossible by adding constraint (1f) reducing the set of feasible solutions. However, (1f) is not necessary, because a solution, where a non-destination that does not relay any message is assigned a non-zero power, would be filtered out by optimality. The expression (1g) enforces that an edge $\{i,j\}$ is a part of a solution if and only if for every $s \in D$, either (i,j) or (i,j) is an arc used for sending a message from s. The next constraint (1h) expresses that a transmission initiated by $s \in D$ cannot reach s again, which implies non-existence of a directed cycle containing s. Finally, by (1i), we define a relation between x-variables and y-variables. When arc (i,j) is used for transmission of a message from $s \in D$, vertex i relaying the message must be assigned power at least p_{ij}.

2.2 s,t-Flow Extension

Consider a network flow problem where one unit of flow must be sent between every pair (s,t) of destinations. In order to model this requirements, we introduce a variable f_{ij}^{st} as follows:

$$f_{ij}^{st} = \begin{cases} 1 & \text{if arc } \{i,j\} \in A \text{ carries 1 unit of flow from } s \in D \text{ to } t \in D, \\ 0 & \text{otherwise.} \end{cases}$$

The original SMT formulation can be extended and strengthened by flow constraints for each (s,t)-pair.

$$\min \sum_{(i,j)\in A} \sum_{s\in D} p_{ij} y_{ij}^s \tag{2a}$$

$$\text{s.t.}$$

$$(2a), \ldots, (1i)$$

$$\sum_{j\in V_i} f_{ij}^{st} - \sum_{j\in V_i} f_{ji}^{st} = 0 \qquad i \in V \setminus \{s,t\}, s,t \in D, s \neq t \tag{2b}$$

$$\sum_{j\in V_t} f_{tj}^{st} - \sum_{j\in V_t} f_{jt}^{st} = -1 \qquad s,t \in D, s \neq t \tag{2c}$$

$$f_{ij}^{st} \leq x_{ij}^s, \qquad (i,j) \in A, s,t \in D, s \neq t \tag{2d}$$

$$f_{ij}^{st} = f_{ji}^{ts}, \qquad (i,j) \in A, s,t \in D, s \neq t \tag{2e}$$

$$\mathbf{z} \in \{0,1\}^E, \mathbf{x}, \mathbf{y} \in \{0,1\}^{A\times D}, \mathbf{f} \in \{0,1\}^{A\times D\times D}. \tag{2f}$$

The flow conservation constraints (2b)-(2c) guarantee that for each $s,t \in D$ there is a flow of one unit from s to t. Next constraint (2d) expresses that if an arc (i,j) carries an s,t-flow , then this arc is used for sending a message initiated in s. The flow symmetry (2e) states that arc (i,j) carries a flow from s to t if and only if arc (j,i) carries a flow from t to s.

This model can be further extended by valid inequalities strengthening LP bounds

$$x^s_{ij} \leq \sum_{t \in D \setminus \{s\}} f^{st}_{ij}, \qquad (i,j) \in A, s \in D \tag{3a}$$

$$\sum_{j \in V \setminus \{s\}} y^s_{sj} = 1, \qquad s \in D \tag{3b}$$

$$f^{st_1}_{ij} - f^{st_2}_{ij} + f^{t_1 t_2}_{ij} \geq 0, \qquad i,j \in V, s, t_1, t_2 \in D, \; i \neq j, s \neq t_1, s \neq t_2, t_1 \neq t_2 \tag{3c}$$

$$\sum_{i \in V \setminus \{j\}} y^s_{ji} \geq \sum_{i \in V \setminus \{j\}} x^s_{ij}, \qquad j \in V \setminus D, s \in D \tag{3d}$$

$$\sum_{i \in V \setminus \{j\}, p_{ji} \geq p_{jk}} f^{st}_{ji} \leq \sum_{i \in V \setminus \{j\}, p_{ji} \geq p_{jk}} y^s_{ji}, j \in V, s, t \in D, s \neq t, k \in V \tag{3e}$$

2.3 SMT based on Polzin's Minimum Steiner Tree formulation

There are many formulations for the Steiner minimum tree problem, that can be a basis for our SMT problem. We consider the formulation P_{F^2}, the strongest model studied in [**?**]. The model contains variables

$$\check{f}^{st}_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \in A \text{ carries a flow from the source to both } s,t \in D_1, \\ 0 & \text{otherwise.} \end{cases}$$

$$f^{t}_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \in A \text{ carries a flow from the source to } t \in D_1, \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if arc } (i,j) \in A \text{ is in the solution,} \\ 0 & \text{otherwise.} \end{cases}$$

The x-variables encapsulating the resulting tree correspond to arcs, while analogous z-variables in s,t-flow model correspond to edges. Hence, a solution obtained by s,t-flow is an undirected tree, while P_{F^2} produces an arborescence.

2.4 Relation between SMT a PF2

3 Experimental Evaluation

The practical part of this work focuses on comparison of the models presented in the previous section. As the main focus of this study is to determine tighter bounds, the conducted experiments are designed for this purpose.

Table 1 Please write your table caption here

first	second	third
number	number	number
number	number	number

3.1 Constraint generation

The stronger SMT models are too large and are therefore not very practical as computation of rather smaller instances takes prohibitively long time. That is also the case of the model SMT-MF. The main idea of how to make this model more useful in practice is to dynamically add only those flow constraints that are violated. First, we relax the flow constraints which means that we solve only the LP relaxation of the original SMT model. This gives the vector $\mathbf{x}$, which, by constraint (**??**), acts as a capacity vector, and determines the maximum possible value of a flow through certain arc. Next, we go through all possible $s-t$ pairs of destinations and check whether the flow constraints are fulfilled for the particular s and t. New flow constraints for some (possibly all) $s-t$ pairs that violate flow constraints are added to the model and the whole process is repeated until there are no violated flow constraints for any $s-t$ pair. The algorithm **??** describes this process formally.

There are various strategies how to determine which violated flow constraints will be added to the model.

4 Conclusion and Future Work

as required. Don't forget to give each section and subsection a unique label (see Sect. 2).

Paragraph headings Use paragraph headings as needed.

$$a^2 + b^2 = c^2 \tag{4}$$

References

1. Author, Article title, Journal, Volume, page numbers (year)
2. Author, Book title, page numbers. Publisher, place (year)

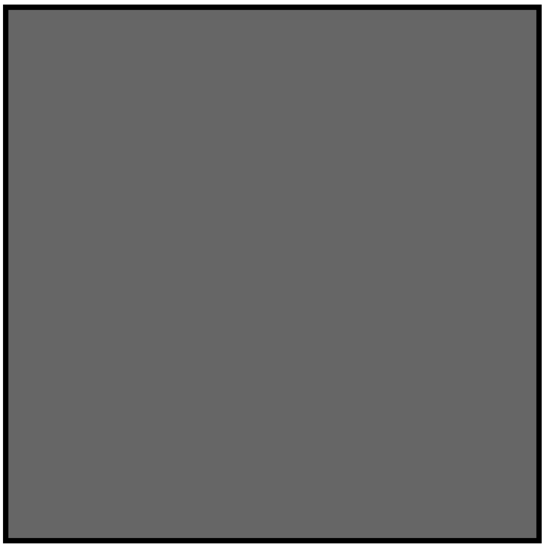

Fig. 1 Please write your figure caption here

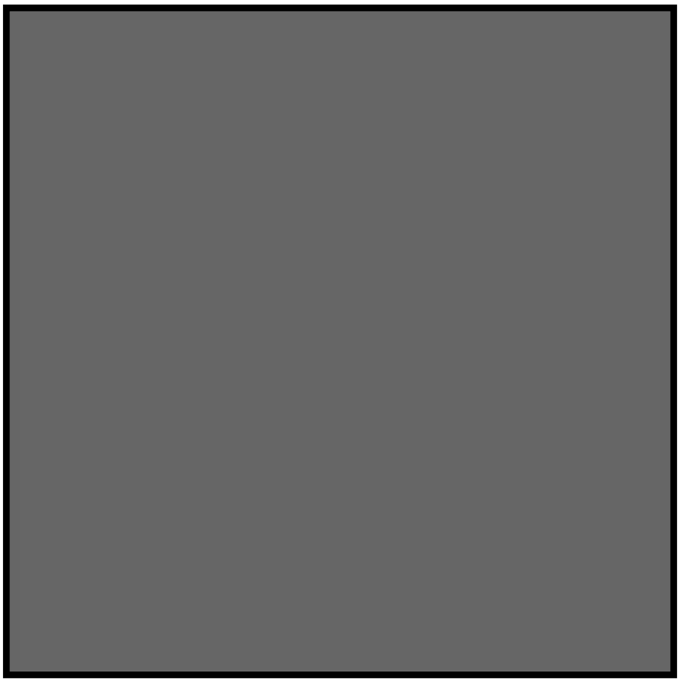

Fig. 2 Please write your figure caption here