Exercise 2

Sort with Swap(0,*)

https://pintia.cn/problem-sets/16/problems/678

Slides adapted from material by
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问题描述

Given any permutation of the numbers {0, 1, 2,…, N−1}, it is easy to sort them in increasing order. But what if Swap(0, *) is the ONLY operation that is allowed to use?

For example, to sort {4, 0, 2, 1, 3} we may apply the swap operations in the following way:

Swap(0, 1) => {4, 1, 2, 0, 3}

Swap(0, 3) => {4, 1, 2, 3, 0}

Swap(0, 4) => {0, 1, 2, 3, 4}

Now you are asked to find the minimum number of swaps need to sort the given permutation of the first *N* nonnegative integers.

输入输出格式

- Input Specification:
- Each input file contains one test case, which gives a positive N($\leq 10^5$) followed by a permutation sequence of {0, 1, …, N−1}. All the numbers in a line are separated by a space.

- Output Specification:
- For each case, simply print in a line the minimum number of swaps need to sort the given permutation.

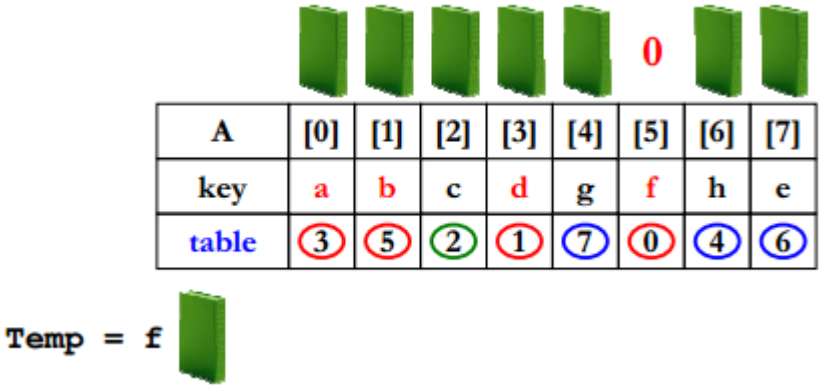
输入输出示例

Sample Input:
10
3 5 7 2 6 4 9 0 8 1

Sample Output:
9

分析

- 给定N个数字的排列，如何仅利用与0交换达到排序目的?
  - N个数字的排列由若干个独立的环组成



环的分类

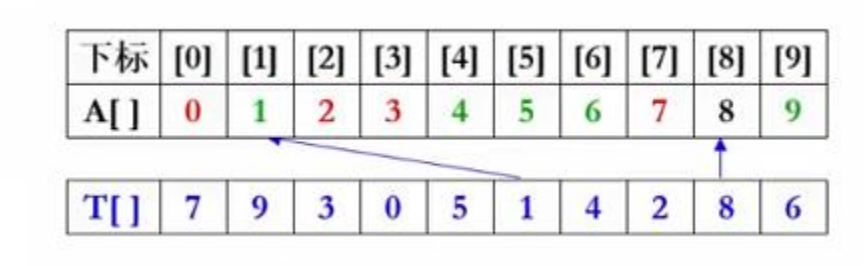
- 环分3种
  - ① 只有1个元素：不需要交换
  - ② 环里n_0个元素，包括0：需要n_0–1次交换
  - ③ 第i个环里有n_i个元素，不包括0：先把0换到环里，再进行(n_i+1)–1次交换 —— 一共是n_i+1次交换

- 若N个元素的序列中包含S个单元环、 K个多元环，则交换次数为：

$$n_0 - 1 + \sum_{i=1}^{K-1}(n_i + 1)$$

$$= \sum_{i=0}^{K-1} n_i + K - 2 = N - S + K - 2$$

算法示例



下标	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
A[ ]	0	1	2	3	4	5	6	7	8	9
T[ ]	7	9	3	0	5	1	4	2	8	6

3

5

T[A[i]] = i; 元素i在A[T[i]]中存放

4-1=3次　　5+1=6次

$$N - S + K - 2 = 10 - 1 + 2 - 2 = 9$$

完整代码

完整代码实现

Data Structures

To be continued…