Stochastic Bi-level Private Household PV System Investment Optimization

Seminar Paper

Author: Niklas Bockholt (7380604), Janosch Hagenbeck (7431426), Conrad Reintjes (7430921), Christian Zwiessler (7431669)

Supervisor: Dr. Saber Talari

Co-Supervisor: Lennard Sund

Department of Information Systems for Sustainable Society

Faculty of Management, Economics and Social Sciences

University of Cologne

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Hagenbeck J., Reintjes, C., Bockholt, N., Zwiessler C.

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Abstract

The transition to sustainable energy systems is essential for mitigating climate change, with photovoltaic (PV) systems playing a key role in decentralizing electricity generation. However, private households face significant challenges when determining the economic viability and optimal size of a PV installation due to uncertainties in energy prices, weather conditions, and consumption patterns. This research introduces a stochastic bi-level optimization model to support informed investment decisions.

The upper-level of the model addresses long-term investment decisions by minimizing total costs, including installation, operational expenses, and the social cost of carbon emissions. The lower-level optimizes electricity dispatch under uncertain conditions using scenario-based stochastic modeling. To transform the bi-level problem into a single-level optimization framework, Karush-Kuhn-Tucker (KKT) conditions and Big-M linearization techniques are applied, allowing for computational efficiency in solving the mixed-integer linear programming (MILP) formulation.

The results demonstrate that the optimal PV system size is 59.25 m^2 with total annual cost of €3499 covering €1514 investment, €1771 operational and €214 SCC expenses. A detailed energy distribution analysis reveals that only 26.5% of the generated solar energy is directly used for household consumption, while the remaining portion is exported to the grid, underscoring the dependency on grid interactions. Sensitivity analysis highlights that increasing SCC and rising grid electricity prices drive larger PV installations, whereas low feed-in tariffs constrain expansion. Notably, if the feed-in tariff surpasses €0.10/kWh, the system transitions into a profitable setup, encouraging significantly larger PV investments exceeding 5000 m^2.

Future research should explore dynamic energy pricing, variations in consumption patterns, and the integration of energy storage systems to enhance decision-making frameworks for sustainable residential energy solutions.

Contents

List of Figures

Team Assignment Contribution

In preparing this assignment, we initially divided the tasks among team members to ensure efficiency and clear responsibilities. However, throughout the process, everyone actively supported each other, stepped in where needed, and contributed to different parts of the work.

Conrad Reintjes

- Task 1: Developed a Minimum Viable Product (MVP) for solving the Bi-Level problem using the `BiLevelJuMP` library and initially formulated the Bi-Level problem.
- Task 2: Researched and wrote the introduction section 1 of the report.
- Task 3: Conducted an investigation on real-world values for the model constants and authored Section 2.3.

Janosch Hagenbeck

- Task 1: Coded the 03_MonteCarloSimulationConsumption Notebook including the data research.
- Task 2: Coded the 04_MergeAndCopulas Notebook together with Christian.
- Task 3: Wrote the section 4.

Niklas Bockholt

- Task 1: Implemented the model in Julia.
- Task 2: Created plots for model evaluation and write Section 3 Results.
- Task 3: Reformulated as a Single-Level Model.

Christian Zwiessler

- Task 1: Coded the 02 and 03_MonteCarloSimulation Notebook including the data research.
- Task 2: Coded the 04_MergeAndCopulas Notebook together with Janosch.
- Task 3: Reformulated as a Single-Level Model and mainly authored Section 2.

1 Introduction

1.1 Motivation

Climate change has a profound impact on natural and social systems, for example the increase in extreme weather events (IPCC, 2021), rising sea levels (Nicholls & Cazenave, 2018) and water scarcity (Thornton et al., 2014).

A key driver of climate change is the energy sector, which accounts for around 40% of greenhouse gas emissions, with electricity and heat generation being the largest sources of emissions (Foster & Bedrosyan, 2014).

The energy transition offers a solution to reduce emissions. A key aspect of this transformation is the expansion of decentralized energy systems, in particular PV systems in private households (Breyer et al., 2017).

However, households are faced with the challenge of deciding whether a PV system is economically viable for them and, if so, what size (Lazdins et al., 2021). This decision is complicated by numerous uncertainties, including weather conditions and their own energy consumption (Komendantova et al., 2011).

To support households in this decision, it is necessary to develop a model that takes into account both investment and operational aspects of a PV system. Our goal is to formulate a stochastic bi-level optimization model that minimizes investment costs, operating costs and social costs related to CO_2 emissions depending on uncertain influencing factors. This raises the central research question:

> *How can a stochastic bi-level optimization model be designed to support private households in determining the economically optimal size of a PV system under uncertainties in weather conditions and consumption patterns?*

1.2 Background

1.2.1 Bi-level Optimization

The *Bi-level Optimization* is a hierarchical optimization method in which a superordinate (leader) and a subordinate (follower) decision level interact (Dempe & Zemkoho, 2020). The upper-level optimizes its objective function taking into account the optimal response of the lower-level:

$$\min_{x \in X} F(x, y^*(x)) \quad \text{with} \quad y^*(x) = \arg \min_{y \in Y(x)} f(x, y). \tag{1}$$

This type of model is used in energy optimization, e.g. for tariff design or investment planning for PV systems (Aghamohamadi et al., 2021). Typical solution methods include the KKT reformulation, branch-and-bound or heuristic approaches (Colson et al., 2007).

1.2.2 Stochastic Optimization

Stochastic optimization enables the consideration of uncertainties in optimization problems that are modeled by random variables (Morales et al., 2013). These variables can be discrete or continuous and are described by probability distributions (Fouskakis & Draper, 2002). In contrast to deterministic optimization, stochastic optimization uses probabilities to make robust decisions. A classic model is two-stage optimization, in which initial decisions are made under uncertainty and later adjusted (Morales et al., 2013). A central solution method is the Monte Carlo simulation, which approximates random influences through random sampling (Shapiro, 2001).

1.3 Related Work

Previous studies have investigated various aspects of investment and optimization of PV systems. Cervilla et al. (2015) developed a bi-level optimization model for electricity tariffs and decentralized PV investments without considering uncertainties. Schram et al. (2018) analyzed PV battery systems for self-consumption maximization and load peak reduction, but without stochastic modeling. Bianchi et al. (2014) focused on off-grid PV battery systems without including external price or grid effects. Aghamohamadi et al. (2021) developed a robust two-stage optimization that includes uncertainties, but was primarily optimized for battery systems.

2 Methodology

2.1 Modeling: Problem Formulation

We develop a stochastic bi-level optimization model that integrates long-term investment decisions with short-term operational optimization. The upper-level determines optimal capacity investment, while the lower-level optimizes generation dispatch under uncertainties (see Figure 1). The detailed formulation follows in the next sections 2.1.1 and 2.1.2.

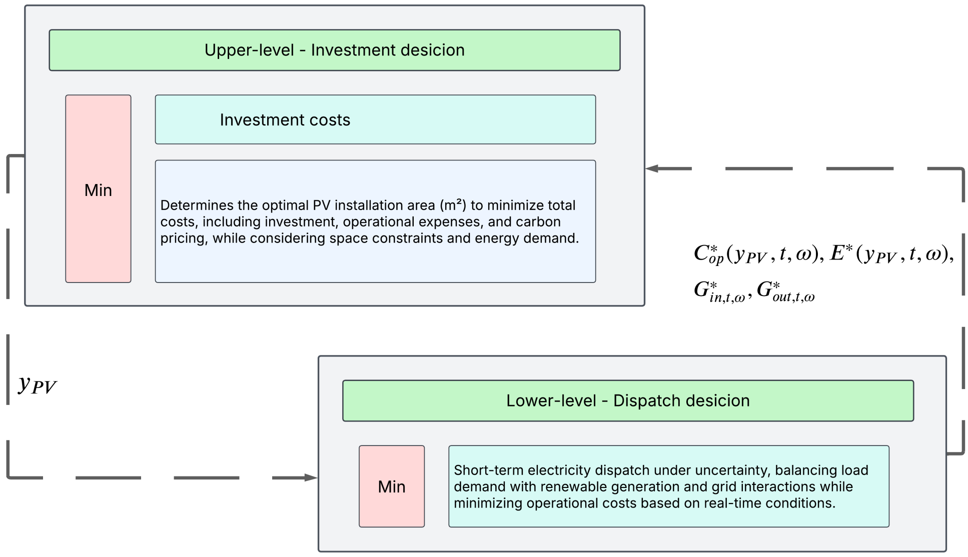

Figure 1: Bi-level optimization framework for PV investment and dispatch: The upper-level determines the optimal PV installation area, while the lower-level optimizes electricity dispatch under uncertainty.

2.1.1 Upper-Level (Investment Decision)

The upper-level models the long-term investment decision of a central planner, who determines the optimal PV installation area in square meters. The objective is to minimize the expected total cost, which consists of investment costs for PV panels, operational costs, and potential carbon pricing, while considering space constraints and energy demand:

$$\left[\min_{y_{PV}} \quad C_{\text{inv}}(y_{PV}) + \sum_{t=1}^{T} \sum_{\omega_t \in \Omega_t} \Phi_{\omega_t} C^*_{op}(y_{PV}, t, \omega_t) + \text{SCC} \cdot \sum_{t=1}^{T} \sum_{\omega_t \in \Omega_t} \Phi_{\omega_t} E^*(y_{PV}, t, \omega_t) \right], \tag{2}$$

where:

- $C_{\text{inv}}(y_{\text{PV}})$ are the (annually allocated) investment costs for the PV system,
- Φ_ω are the probabilities of scenario occurrences ω,
- $C^*_{\text{op}}(y_{\text{PV}}, t, \omega_t)$ is the minimum operating cost in scenario ω after *optimal* dispatch (see lower-level),

- SCC is the (politically determined) *Social Cost of Carbon* per ton of CO_2,
- $E^*(y_{PV}, t, \omega_t)$ is the resulting CO_2 emissions in optimal operation.

2.1.2 Lower-Level (Scenario-Based Operational Optimization)

The lower-level represents the short-term operational problem, which optimizes electricity dispatch for each scenario while considering uncertain parameters such as load demand and renewable generation.

Decision Variables:

$$G_{in,t,\omega} \geq 0 \quad \text{(Grid Purchase)}, \quad G_{out,t,\omega} \geq 0 \quad \text{(Grid Feed-in)}.$$

Model:

$$\begin{aligned} \min_{\{G_{in,t,\omega}, G_{out,t,\omega}\}} \quad & c_{grid}^{buy} G_{in,t,\omega} - c_{grid}^{feed} G_{out,t,\omega} \\ \text{s.t.} \quad & L_{t,\omega} = y_{PV} P_{PV}(t,\omega) + G_{in,t,\omega} - G_{out,t,\omega}, \\ & G_{in,t,\omega} \geq 0, \quad G_{out,t,\omega} \geq 0. \end{aligned} \tag{3}$$

The PV power production at time step t under scenario ω is modeled as:

$$P_{\text{PV}}(t,\omega) = \frac{S_{t,\omega}}{1000} \cdot \eta_0 \cdot \left(1 - \beta \left(T_{t,\omega} - T_{\text{ref}}\right)\right). \tag{4}$$

where:

- $P_{\text{PV}}(t,\omega)$: PV power output at time t under scenario ω (in kWh/m^2).
- $S_{t,\omega}$: Solar irradiance at time t under scenario ω (in W/m^2).
- η_0: Nominal efficiency of the PV panel at the reference temperature T_{ref} (in %).
- $T_{t,\omega}$: Temperature at time t under scenario ω (in $^\circ C$).
- T_{ref}: Reference temperature (in $^\circ C$).
- β: Temperature coefficient of the PV panel.
- c_{grid}^{buy}: Cost of buying electricity from the grid (in Euro per kWh).
- c_{grid}^{feed}: Feed-in tariff (revenue per kWh fed into the grid).
- $L_{t,\omega}$: Load demand at time t under scenario ω (in kWh).

The optimal dispatch values $G^*_{\text{in},t,\omega}(y_{\text{PV}})$ and $G^*_{\text{out},t,\omega}(y_{\text{PV}})$ are dependent on the investment decision y_{PV}. These results further determine the scenario-specific emissions:

$$E(y_{\text{PV}}, \omega) \ = \ e_{\text{grid}} \ \cdot \ G^*_{\text{in},t,\omega,y_{\text{PV}}}, \tag{5}$$

where e_{grid} represents the grid's emission intensity (tCO_2/kWh) in scenario ω.

Furthermore, the annualized investment cost is defined by the function based on Bergner and Quaschning (2019)[1]:

$$C_{\text{inv}}(y_{\text{PV}}) = I_0 \cdot (0.2 \cdot y_{\text{PV}}) \cdot (1 + \text{VAT}) \cdot \frac{r(1+r)^n}{(1+r)^n - 1}, \tag{6}$$

where I_0 represents the base investment cost per kWp, the factor 0.2 converts the y_{PV} from square meter to kWp (GASAG, 2022), VAT is the value-added tax, r is the discount rate, and n is the lifetime.

2.2 Solution Strategy: Reformulation as a Single-Level Model

Since the problem is inherently a bi-level optimization problem, we transform it into a Single-Level MILP problem. This is achieved through a KKT reformulation of the lower-level problem, which allows embedding its optimality conditions directly into the upper-level formulation. Additionally, Big-M linearization is applied to handle complementarity constraints, while Monte Carlo simulations in combination with a Gaussian copula are used to generate realistic scenarios for wind and load profiles.

2.2.1 Reformulation Using KKT Conditions

The lower-level problem is replaced by its optimality conditions, characterized by the Karush-Kuhn-Tucker (KKT) conditions. These include primal feasibility, dual feasibility, stationarity, and complementary slackness.

Primal feasibility ensures the power balance at each time step:

$$L_{t,\omega} = y_{PV} P_{PV}(t, \omega) + G_{in,t,\omega} - G_{out,t,\omega}. \tag{7}$$

The dual variables $\lambda_{t,\omega}$ correspond to the power balance constraint and act as shadow prices, while $\mu^{in}_{t,\omega}$ and $\mu^{out}_{t,\omega}$ are the dual multipliers for the non-negativity con-

[1] We removed the exponent p from the orginal formula, as otherwise the solver would not have been able to solve the problem due to non-linarity.

straints of grid import and export:

$$\lambda_{t,\omega} = c_{grid}^{buy} - \mu_{t,\omega}^{in}, \quad \lambda_{t,\omega} = c_{grid}^{feed} + \mu_{t,\omega}^{out}. \tag{8}$$

Dual feasibility requires that $\mu_{t,\omega}^{in} \geq 0$ and $\mu_{t,\omega}^{out} \geq 0$, while complementary slackness ensures that either grid exchange or its corresponding dual multiplier must be zero:

$$\mu_{t,\omega}^{in} G_{in,t,\omega} = 0, \quad \mu_{t,\omega}^{out} G_{out,t,\omega} = 0. \tag{9}$$

2.2.2 Big-M Linearization

Since complementarity constraints introduce non-linearity, they are reformulated using the Big-M method. Binary variables $z_{t,\omega}^{in}, z_{t,\omega}^{out} \in \{0,1\}$ are introduced to enforce the conditions $\mu_{t,\omega}^{in} G_{in,t,\omega} = 0$ and $\mu_{t,\omega}^{out} G_{out,t,\omega} = 0$. The constraints are then rewritten as:

$$G_{in,t,\omega} \leq M\, z_{t,\omega}^{in}, \qquad \mu_{t,\omega}^{in} \leq M\,(1 - z_{t,\omega}^{in}), \tag{10}$$

$$G_{out,t,\omega} \leq M\, z_{t,\omega}^{out}, \qquad \mu_{t,\omega}^{out} \leq M\,(1 - z_{t,\omega}^{out}). \tag{11}$$

Here, M is a sufficiently large parameter ensuring proper enforcement of complementarity conditions. This transformation allows the problem to be formulated as an MILP, which can be solved efficiently using standard solvers, such as CPLEX.

2.2.3 Integrated Single-Level Model

The complete formulation as a single-level model is then given as follows:

$$\begin{aligned} \min_{\substack{y_{PV}, \{G_{in,t,\omega}, G_{out,t,\omega}, \lambda_{t,\omega}, \\ \mu_{t,\omega}^{in}, \mu_{t,\omega}^{out}, z_{t,\omega}^{in}, z_{t,\omega}^{out}\}}} \quad & C_{\text{inv}}(y_{PV}) \\ & + \sum_{t=1}^{T} \sum_{\omega_t \in \Omega_t} \Phi_{\omega_t} \left(c_{grid}^{buy} G_{in,t,\omega_t} - c_{grid}^{feed} G_{out,t,\omega_t} \right) \\ & + \text{SCC} \cdot \sum_{t=1}^{T} \sum_{\omega_t \in \Omega_t} \Phi_{\omega_t} (e_{grid} \cdot\ G_{in,t,\omega_t}), \end{aligned} \tag{12}$$

subject to the following constraints:

$$
\begin{aligned}
&L_{t,\omega} = y_{PV} P_{PV}(t,\omega) + G_{in,t,\omega} - G_{out,t,\omega}, && \forall t,\omega, \\
&\lambda_{t,\omega} = c_{grid}^{buy} - \mu_{t,\omega}^{in} = c_{grid}^{feed} + \mu_{t,\omega}^{out}, && \forall t,\omega, \\
&\mu_{t,\omega}^{in}, \mu_{t,\omega}^{out}, G_{in,t,\omega}, G_{out,t,\omega}, y_{PV} \geq 0, && \forall t,\omega, \\
&G_{in,t,\omega} \leq M z_{t,\omega}^{in}, \quad G_{out,t,\omega} \leq M z_{t,\omega}^{out}, && \forall t,\omega, \\
&\mu_{t,\omega}^{in} \leq M(1 - z_{t,\omega}^{in}), \quad \mu_{t,\omega}^{out} \leq M(1 - z_{t,\omega}^{out}), && \forall t,\omega, \\
&z_{t,\omega}^{in}, z_{t,\omega}^{out} \in \{0,1\}, && \forall t,\omega.
\end{aligned}
\tag{13}
$$

2.2.4 Copula-Based Modeling and Monte-Carlo Simulation

To account for uncertainties in irradiation, temperature, and load profiles, we employ Monte-Carlo simulations in combination with copula-based modeling. This enables realistic dependency modeling, crucial for risk assessments in energy market optimization.

The process starts with an exploratory analysis, where the distributions of the individual stochastic variables, power consumption, solar irradiance, and temperature, are determined and their correlations examined. These empirical distributions are then transformed into uniform distributions via inverse transform sampling. To reconstruct the joint dependencies while maintaining the original correlation structure, the Gaussian copula function is fitted to the data.

Once the copula model is established, synthetic data points are generated to create realistic correlated scenarios. This approach ensures that simulated scenarios reflect the statistical properties of real-world conditions.

2.3 Data Sources and Model Assumptions

The parameter values in the investment formula follow the empirical analysis according to Bergner and Quaschning (2019) with $I_0 = 1923$ €/kWp, $p = 0.16$ and VAT of 19%. The annual investment costs are calculated using the annuity formula with an interest rate of 2.75% (EZB, 2025) and a lifespan of 25 years (Kost et al., 2021).

The costs for grid electricity and feed-in are 47 ct/kWh and -8.03 ct/kWh respectively according to Wirth (2025). The social cost of carbon (SCC) is set at 100 €/tCO_2 (Mauleón, 2017; Pindyck, 2019; Tol, 2011). The average emissions intensity of the

German electricity mix is 0.00038 tCO_2/kWh (Umweltbundesamt, 2024).

The PV power results as a function of the radiation $S_{t,\omega}$ and temperature $T_{t,\omega}$ (see Equation 4) with $\eta_0 = 0.18$, $\beta = 0.004$ and $T_{\text{ref}} = 25°C$ (IEC61215, 2021).

The PV system settings were derived from the EU's Photovoltaic Geographical Information System (PVGIS) (European Commission, Joint Research Centre, 2023), using optimal parameters for Cologne, Germany ($50.936N$, $6.954E$) from the PVGIS-SARAH3 database. The module slope was set to 39 and the azimuth angle to 3. The dataset covers the period 2005-2023 and includes temperatures and solar radiation. In addition, the energy consumption were extracted from a one household electric power consumption dataset, created by University of California Irvine (2016). This dataset contains measurements recorded between December 2006 and November 2010.

Representative days were simulated for each month of the year (12 records) by creating representative hours for each day (24 records). As a result the dataset contains 288 records (12·24) with 5 scenarios each.

3 Results

The optimization model provides insights into the most cost-effective configuration of solar panels for energy generation and consumption. When running the model with the predefined constants (see section 2.3), the following are the key results:

- Optimal area of solar panels: 59.25m^2
- Yearly total cost: 3498.8€
- With investment cost of: 1514.2€
- With operation cost of: 1770.6€
- With social cost of carbon (SCC) of: 214.1€

The initial cost for setting up the system includes purchasing and installing the solar panels, while the operational cost includes recurring expenses such as grid imports.

3.1 Energy Distribution and Usage

The optimization model also provides insights into how the generated solar energy is utilized and how the load is served.

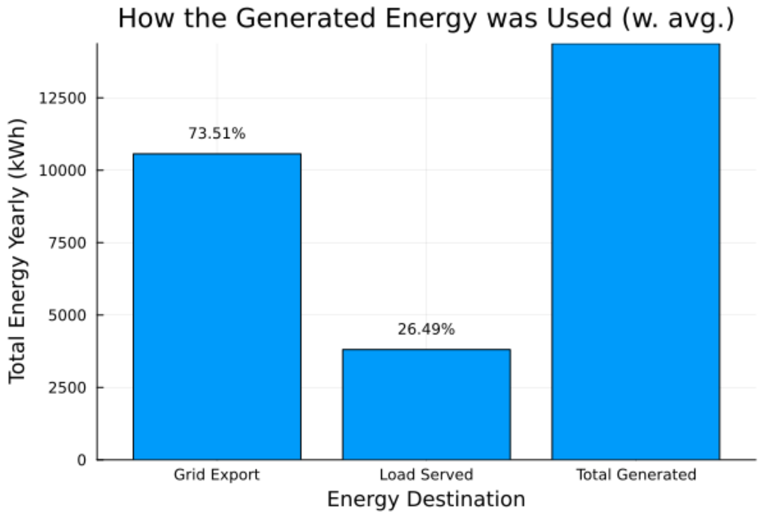

Figure 2: Distribution of total generated power by PV, showing the share exported to the grid versus the share used to serve the load, calculated as the weighted average (w. avg.) across all scenarios.

From the results, we observe that only 26.49% of the energy generated by the solar panels is used to serve the load, while the remaining energy is exported to the grid. This distribution highlights the importance of grid exports in the system's overall functioning. The relatively small percentage of energy used directly for the load can be attributed to several factors, including limited solar irradiance during certain periods, which reduces the amount of energy available to meet the load demand.

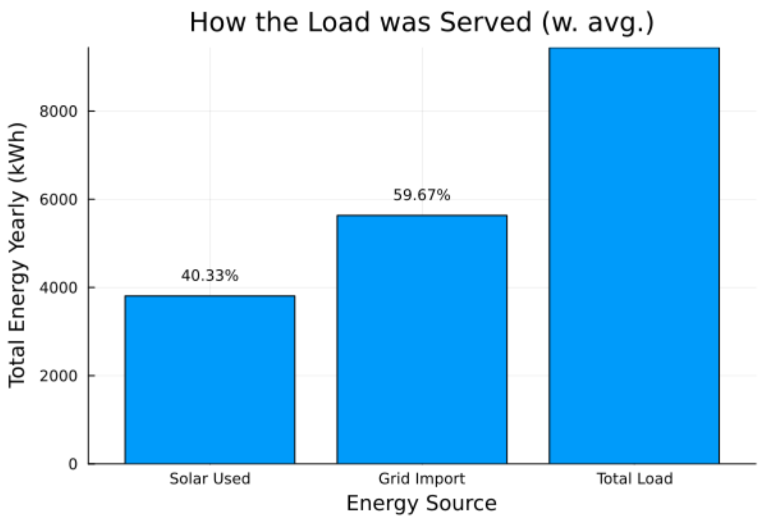

Figure 3: Proportion of total energy demand met by PV production versus grid imports, calculated as the weighted average (w. avg.) across all scenarios.

Figure 3 illustrates how the total energy demand (load) is met in the most probable scenario. While grid imports predominantly serve the load, solar energy contributes a smaller portion. This limitation is not due to the inefficiency of solar power but rather the inherent constraints of solar generation, such as the daily and seasonal variability of solar irradiance.

These variabilities are further emphasized in Figure 4, which shows the typical energy management behavior with PV production. Grid imports increase in the evenings and colder seasons, while energy export occurs mostly during midday and warmer periods. This highlights the necessity of grid imports or other energy sources, especially during times of low solar generation.

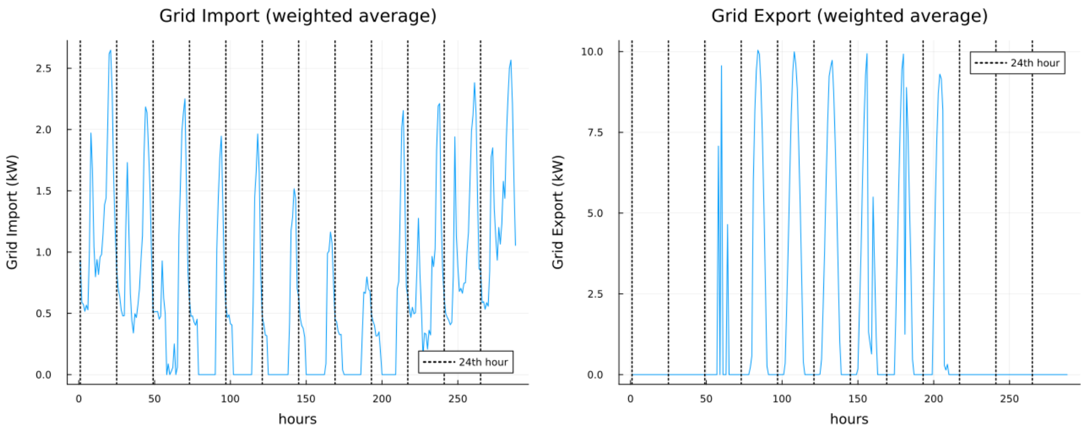

Figure 4: Time series of energy imports from and exports to the grid, averaged across weighted scenarios.

3.2 Sensitivity Analysis

After evaluating the optimized model, a sensitivity analysis was conducted to explore how changes in the key parameters SCC, cost of energy imports, and price for energy exports affect the model's decisions.

The optimal PV size and associated costs are shown to vary with changes in these parameters (see Figure 5 Appendix A).

For the SCC, at 0, the optimal PV area is around 56m^2. As SCC increases, both the optimal area and costs rise linearly, as grid imports become less attractive. A similar linear trend is observed with the cost of energy imports, where higher import costs lead to a larger PV system. This aligns with expectations, as higher import costs push the model to maximize solar generation and minimize external grid dependence.

However, the response to export revenue differs. When the export price surpasses €0.10 per kWh, the optimal PV size increases sharply, exceeding 5000m^2. At these higher export prices, the total cost actually drops below zero, meaning that the system starts generating a profit from energy exports. This surge reflects the strong financial incentive to install a larger system that not only covers the demand but also generates revenue.

4 Discussion

4.1 Interpretation

As the results show, the optimal PV size for a private household is 59.25m^2. We can interpret that the optimization of the size can provide value and seems plausible under consideration of the cost, mentioned in Chapter 3. The relative use of the energy of 26.49% is typical for PV according to the research of Klingler (2019).

The near zero energy export during each hour between November and February (see Figure 4) shows that nearly all produced energy in this months is consumed by the household. However, the energy import from the grid over time (see Figure 4) reveals that during this time, roughly 0.5 kW is drawn from the grid every hour. Since this represents the characteristics of an optimally sized PV system, we can conclude that increasing its size to reduce grid imports would not offset the higher investment costs. This is likely because the excess energy generated during the remaining eight months would primarily be sold at a low retail price, offering only a marginal reduction in the import of more expensive grid energy.

The hypothesis that the optimal size is limited by the retail price is supported by the graph with retailed prices in our sensitivity analysis in Figure 5 which shows that if the feed-in tariff surpasses €0.10/kWh, the system transitions into a profitable setup for significantly larger PV investments exceeding 5000m^2. Surprisingly, even a sharp increase in the SCC by over 100% results in only a modest increase of just over 10% in the optimal PV size. This suggests that self-sufficiency in energy production is not necessarily the most cost-effective solution.

4.2 Shortcomings

Despite the valuable contributions of our work, the following limitations should be acknowledged.

Starting with methodological limitations we decided to restrict the number of clusters to five to reduce the computational effort. Moreover, the selected granularity of hourly-rounded energy consumption and production can influence the outcomes, as well as the assumed energy costs. High demand at specific moments may necessitate electricity purchases, however, averaging this demand over an hour smooths out such peaks. This approach could reduce the need for additional purchases which leads to an increased artificial financial profitability of the solar plant.

Moreover, the following aspects result in scope limitations, reducing the transferability to other households. The first point is the solar irradiance which can vary between regions. Since we utilized the solar irradiance for Cologne, our results can be transferred to other regions to certain extend. However, this parameter can be switched easily. Besides the varying production, the energy consumption could also fluctuate between households. In particular, the consumption pattern, as well as the general overall consumption can differ and fundamentally effect the optimal size.

In connection with the scope limitations, we also acknowledge data limitations. Key aspects are the quality and time span of the data, used for the simulations.

4.3 Future Work

Future research should aim to overcome our limitations by developing multiple models or constructing a single, more complex model. This includes the use of more detailed energy prices such as dynamic energy prices as well as forecasts of energy prices in the coming years. Besides that, these models could incorporate data from different regions within Germany to make the results transferable to a broader audience. In order to make the result more accurate, a higher number of clusters and samples could be used as well. In addition, future studies could add a energy storage as part of the investment and determine it's impact on the return.

A Appendix

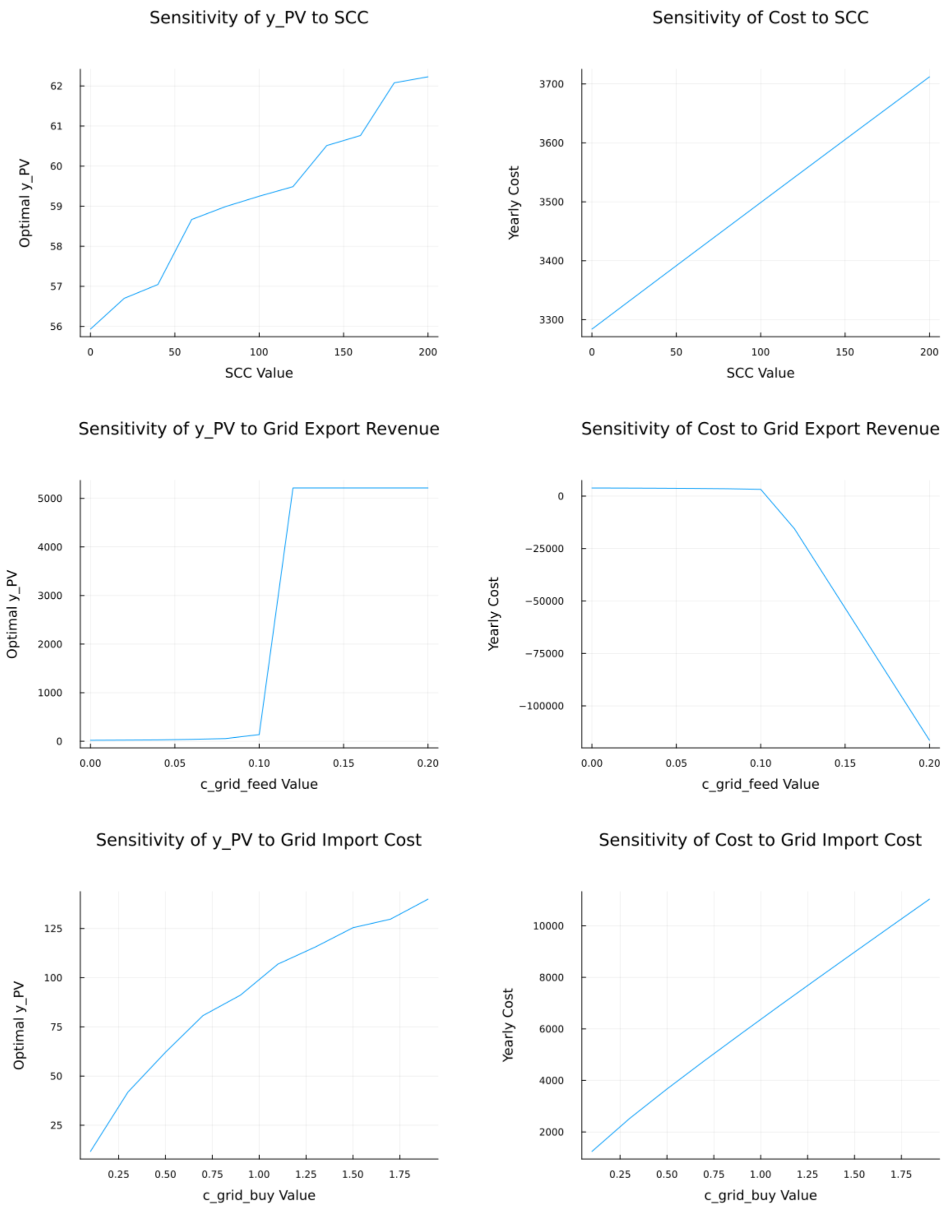

Figure 5: Sensitivity analysis of the optimal PV area (y_PV) and total cost in response to changes in individual parameters, considered in isolation.

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