Engineering Thermodynamics homework(4)

March 2024

1 第四章热力学一般关系

(4-1): 在定容条件下 dv = 0, 有:

$$ds = (\frac{\partial s}{\partial T})_v dT + (\frac{\partial s}{\partial v})_T dv = \frac{c_V}{T} dT$$

可得定容线斜率

$$(\frac{\partial T}{\partial s})_p = \frac{T}{c_V} > 0$$

在定压条件下 dp = 0, 有:

$$ds = (\frac{\partial s}{\partial T})_v dT + (\frac{\partial s}{\partial p})_T dp = \frac{c_p}{T} dT$$

可得定压线斜率

$$(\frac{\partial T}{\partial s})_p = \frac{T}{c_p} > 0$$

因为 $c_p - c_v = Tv\frac{\alpha_V^2}{\kappa_T} > 0 \Rightarrow c_p > c_v \Rightarrow \frac{T}{c_V} > \frac{T}{c_p}$, 可知经过同一状态点时定容线更陡峭.

(4-5) 证明:$(\frac{\partial h}{\partial p})_T = -\mu_j c_p, (\frac{\partial T}{\partial v})_s = -\frac{T\alpha_V}{c_V \kappa_T}$

$$\begin{aligned}
(\frac{\partial h}{\partial p})_T &= T(\frac{\partial s}{\partial p})_T + v = -T(\frac{\partial v}{\partial T})_p + v \\
&= -Tv\alpha_V + v = v(1 - T\alpha_V) = -v\frac{\mu_j c_p}{v} = -\mu_j c_p \\
(\frac{\partial T}{\partial v})_s &= -(\frac{\partial p}{\partial s})_v = -(\frac{\partial p}{\partial T})_v(\frac{\partial T}{\partial s})_v \\
&= \frac{-(\frac{\partial p}{\partial T})_v}{(\frac{\partial s}{\partial T})_v} = \frac{-(\frac{\partial p}{\partial T})_v}{\frac{C_v}{T}} = -\frac{T\alpha_V}{c_V \kappa_T}
\end{aligned}$$

(4-13): 范德瓦尔状态方程:

$$p = \frac{R_g T}{v - b} - \frac{a}{v^2}$$

(1):

$$\begin{aligned}
(\frac{\partial v}{\partial T})_p &= \frac{\frac{R_g}{v-b}}{\frac{R_g T}{(v-b)^2} - \frac{2a}{v^3}} \\
(\frac{\partial p}{\partial T})_v &= \frac{R_g}{v - b} \\
c_p - c_v =& T(\frac{\partial v}{\partial T})_p(\frac{\partial p}{\partial T})_v = \frac{R_g}{1 - \frac{2a(v-b)^2}{R_g T v^3}}
\end{aligned}$$

(2):

$$\begin{aligned}
\text{已知:}(\frac{\partial c_V}{\partial v})_T &= T(\frac{\partial^2 p}{\partial T^2})_v \\
(\frac{\partial^2 p}{\partial T^2})_v &= \frac{\partial}{\partial T}(\frac{R_g}{v - b}) = 0 \\
\text{可得:}(\frac{\partial c_V}{\partial v})_T &= 0
\end{aligned}$$

(3):

$$\mu_j = (\frac{\partial T}{\partial p})_h = \frac{1}{c_p}[T(\frac{\partial v}{\partial T})_p - v] = \frac{v}{c_p}[\frac{1}{\frac{v}{v-b} - \frac{2a(v-b)}{R_g T v^2}} - 1]$$

(4-14):

(1):

$$\begin{aligned}
(h_2 - h_1)_T = \int_1^2 dh = \int_1^2 du + \int_1^2 d(pv) \\
&= (p_2 v_2 - p_1 v_1) + \int_1^2 c_v dT + [T(\frac{\partial p}{\partial T})_v - p]dv \\
&= \int_1^2 (T\frac{R}{v-b} - p)dv + (p_2 v_2 - p_1 v_1) \\
&= (p_2 v_2 - p_1 v_1) + a(\frac{1}{v_1} - \frac{1}{v_2})
\end{aligned}$$

(2):

$$ds = \frac{c_p}{T}dT + (\frac{\partial p}{\partial T})_v dv = (\frac{\partial p}{\partial T})_v dv = \frac{R}{v-b}d(v-b)$$

$$(s_2 - s_1)_T = R\int_1^2 dln(v-b) = Rln(\frac{v_2 - b}{v_1 - b})$$