Applied Statistics,
Probability theory and mathematical statistics

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INDEPENDENCE OF EVENTS

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Exercise 6.1 (M1)

A coin is tossed ten times. Let A denote the event that there are both heads and tails among the results, while B denotes the event that there is at most one tail among them. Are A and B independent?

$A - there\ is\ at\ least\ a\ head\ and\ at\ least\ a\ tail$
$B - there\ is\ at\ most\ one\ tail$

$$P(AB) \stackrel{?}{=} P(A)P(B)$$

$$P(AB) = \frac{10}{2^{10}}$$

$$P(A) = 1 - \frac{2}{2^{10}} = \frac{2^9 - 1}{2^9}$$

$$P(B) = \frac{1 + 10}{2^{10}} = \frac{11}{2^{10}}$$

$$\frac{10}{2^{10}} \neq \frac{11}{2^{10}} \times \frac{2^9 - 1}{2^9}$$

Exercise 6.1 (M1) MATLAB solution

```
N = 10^7;
A = 0;
B = 0;
AB = 0;
for i=1:N
    sim = randi(2,1,10)-1;
    s = sum(sim);
    if s>0 & s<10
        A = A+1;
    end
    if s<=1
        B = B+1;
    end
    if s==1
        AB = AB+1;
    end
End
(A/N) * (B/N)
AB/N
```

Exercise 6.3

In a box we have 8 cards numbered from 1 to 8. A card is chosen randomly. Let events A, B and C denote the following:

A: the chosen number is even;

B: the chosen number is not greater than 4;

C: the chosen number is either 2 or greater than 5.

Show that $P(ABC) = P(A)P(B)P(C)$ and the three events are not mutually independent.

$$P(A) = \frac{1}{2}$$

$$P(B) = \frac{1}{2}$$

$$P(C) = \frac{1}{2}$$

$$P(ABC) = \frac{1}{8}$$

$$P(AC) \neq P(A)P(C)$$

Exercise 6.5 (M2)

Two lottery coupons (5 from 90) are filled independently of each other. What is the probability of winning, i.e., of hitting at least two winning numbers?

$$q - The\ probability, that\ one\ coupon\ wins$$

$$q = \frac{\binom{5}{2}\binom{85}{3} + \binom{5}{3}\binom{85}{2} + \binom{5}{4}\binom{85}{1} + \binom{5}{5}\binom{85}{0}}{\binom{90}{5}}$$

$$1 - q \quad - The\ probability, that\ one\ coupon\ loses$$

$$(1 - q)^2 \quad - The\ probability, that\ both\ coupons\ lose$$

$$1 - (1 - q)^2 \quad - The\ probability, that\ at\ least\ one\ coupon\ wins$$

Exercise 6.5 (M2) MATLAB solution

```
winner = sort(datasample(1:90,5,'Replace',false));
n = 1000;
c = 0;
w = [];
for i=1:n
    x = datasample(1:90,5,'Replace',false);
    y = datasample(1:90,5,'Replace',false);
    if length(intersect(winner,x))>1 |
length(intersect(winner,y))>1
        c = c+1;
        w = [w; sort(x); sort(y)];
    end
end
c/n
1-((nchoosek(85,5)+5*nchoosek(85,4))/nchoosek(90,5))^2
winner
i = 1;
w([i*2-1,i*2],:)
```

Exercise 6.7 (M3)

Two soldiers shoot on a target in turn until the first hit. The probability that the starter hits it is 0.2, while the second hits it with probability 0.3. What is the probability that the first successful shot belongs to the soldier who started the shooting?

$p - the\ probability, that\ the\ first\ soldier\ hits\ the\ target\ first$

$P_1(H_i) - the\ probability, that\ the\ first\ soldier\ hits\ the\ target\ in\ round\ i$

$P_2(H_i) - the\ probability, that\ the\ second\ soldier\ hits\ the\ target\ in\ round\ i$

$$P_1(H_1) = \frac{2}{10}$$

$$P_1(H_2) = \frac{8}{10} \times \frac{7}{10} \times \frac{2}{10}$$

$$p = P_1(H_1) + P_1(H_2) + \cdots = \sum_{k=0}^{\infty} 0.2 \times 0.8^k \times 0.7^k = 0.2 \times \sum_{k=0}^{\infty} 0.8^k \times 0.7^k = 0.2 \times \frac{1}{1 - 0.56}$$

Exercise 6.7 (M3) MATLAB solution

```
f = 0;
s = 0;
n = 1000;

for i=1:n
    while true
        if rand()<.2
            f = f+1;
            break;
        elseif rand()<.3
            s = s+1;
            break;
        end
    end
end

f/n

0.2 / 0.44
```

Exercise 6.9

The ten digits are written on ten separate cards. A card is chosen randomly, the digit on it is noted and the card is replaced. How many cards should be chosen to have an even number among them with probability greater than 0.9?

$P(A) - the\ probability\ that\ at\ least\ one\ digit\ is\ even$
$P(A) > 0.9$

$$P(n\ odd\ digits, from\ n\ samples) = \left(1 - \frac{1}{2}\right)^n$$

$$1 - \left(1 - \frac{1}{2}\right)^n > 0.9$$

$$0.1 > \frac{1}{2^n}$$

$2^n > 10$
$n > 3$

Literature

- Ágnes Baran: Mathematics for engineers 1. (Laboratory slides)
- Sándor Baran: Probability theory and statistics
- Fazekas István: Valószínűségszámítás, Debreceni Egyetemi Kiadó, 2009
- Fazekas István: Bevezetés a matematikai statisztikába, Debreceni Egyetemi Kiadó, 2009
- Matlab examples: https://www.mathworks.com/help/examples.html