APPLIED STATISTICS, PROBABILITY THEORY AND MATHEMATICAL STATISTICS

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Classical probability space

Discription to this presentation

After solving each exercise with „pen and paper”, solve it in Matlab as well.

Exercise 3.1

Two fair dice are thrown. Find the probability that the sum of the numbers obtained is 8. Illustrate the sample space and the set of favorable events.

$$P(A) = \frac{5}{36}$$

Hint:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

	1	2	3	4	5	6

Exercise 3.3

A fair die is thrown twice. Find the probability that the result of the first throw is greater than the result of the second.

Hint:

$$P(A) = \frac{15}{36}$$

Result of the first throw

Sum of the two throws

6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
	1	2	3	4	5	6

Result of the second throw

Exercise 3.7

Ten people, 5 women and 5 men are sitting around a round table. Find the probability, that neither two women nor two men are sitting next to each other.

$$P(A) = \frac{4!\,5!}{9!} = \frac{1}{126}$$

Exercise 3.9

From a deck of cards three cards are dealt. Find the probability that there isn't any spade among them.

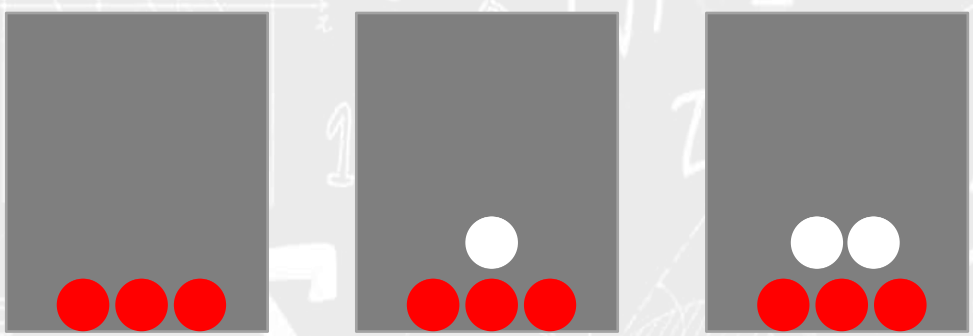
$$P(A) = \frac{C_{39}^{3}}{C_{52}^{3}}$$

Exercise 3.11

In an urn we have three red balls. Find the minimal number of white balls to be added to have the probability of choosing a white ball be greater than 0.9.

At least 28 balls

Hint:



$P(white) = 0$ $\quad P(white) = \frac{1}{4}$ $\quad P(white) = \frac{2}{5}$

Exercise 3.14

In an urn we have 20 red and 30 white balls. 10 balls are chosen without replacement. Find the probability that

a) all the chosen balls are red. $$P(A) = \frac{C_{20}^{10}}{C_{50}^{10}}$$

b) 4 red, 6 white. $$P(A) = \frac{C_{20}^{4} C_{30}^{6}}{C_{50}^{10}}$$

c) at least one red. $$P(A) = 1 - \frac{C_{30}^{10}}{C_{50}^{10}}$$

Solve the previous exercise (question b)under the assumption that the balls are chosen with replacement.

$$P(A) = C_{10}^{4} \cdot \frac{20^4 \cdot 30^6}{50^{10}}$$

Exercise 3.17

Find the probability that on the lottery 5 from 90 we hit at least three winning numbers.

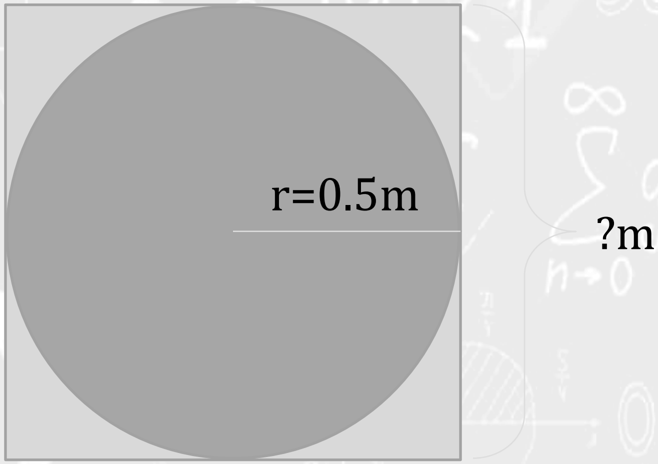
$$P(A) = \frac{C_5^3 C_{85}^2 + C_5^4 C_{85}^1 + C_5^5 C_{85}^0}{C_{90}^5}$$

Exercise 4.1

On a rectangular target with sides of 1-meter lengths each a circle is drawn with a radius of 0.5 meter. Find the probability that a random shot (given it hits the target) hits the target outside the circle.

$$P(A) = 1 - \frac{\pi}{4}$$

Hint:



Exercise 4.2

A stick of length one meter is randomly broken into two parts. What is the probability that from the obtained parts and from a new stick of half a meter length a triangle can be formed?

$$P(A) = \frac{1}{2}$$

Exercise 4.8

In 24 hours, time two ships arrive independently into the harbour of Chewbakka Bay, denoted by A and B, respectively. Ship A can be unloaded in an hour, while ship B in two hours. Workers start to unload a ship immediately after it's arrival and if the other ship arrives before they finish it has to wait. What is the probability that none of the ships has to wait?

$$P(A) = \frac{22^2 + 23^2}{2 \cdot 24^2}$$

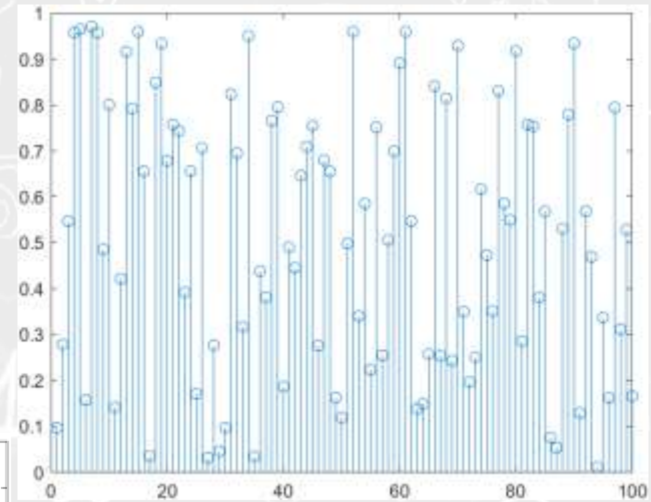
Exercise 4.10

Two points are chosen randomly from the interval (0, a). What is the probability that the sum of thei squares is greater than a^2?

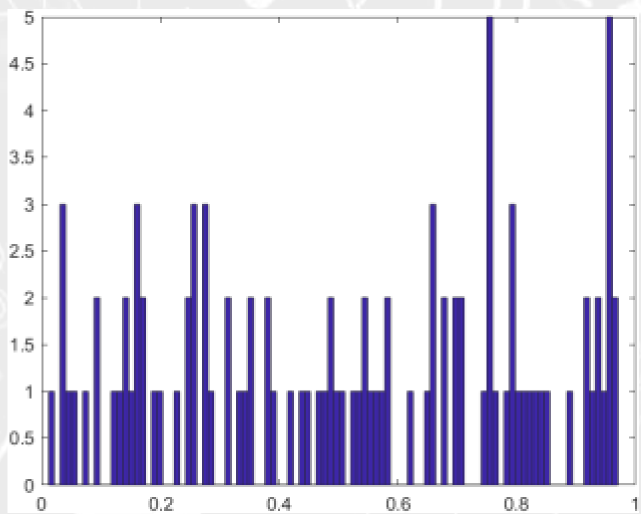
$$P(A) = 1 - \frac{\pi}{4}$$

Random Numbers

```
x=rand(100,1);
stem(x);
```




```
hist(x,100)
```



Coin Tosses

Simulate the outcomes of 100 fair coin tosses

```
x=rand(100,1);
p=sum(x<0.5)/100
p =
    0.5400
```

Simulate the outcomes of 1000 fair coin tosses

```
x=rand(1000,1);
p=sum(x<0.5)/1000
p =
    0.5110
```

Coin Tosses

Simulate the outcomes of 1000 biased coin tosses with p[Head]=0.4

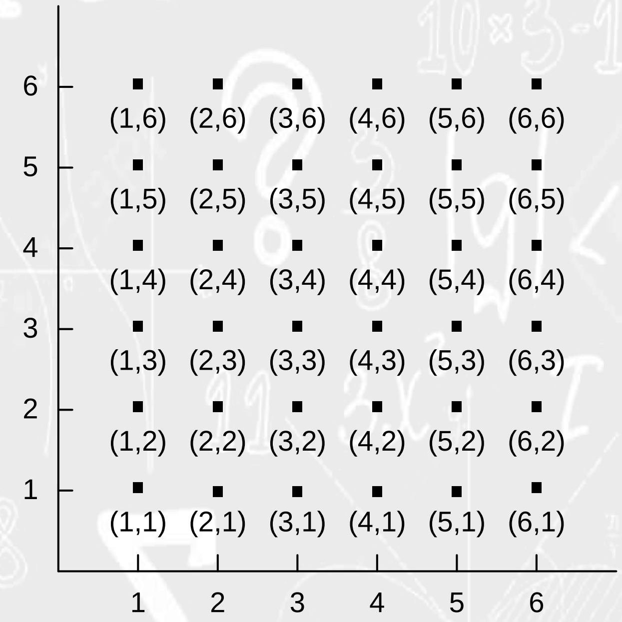
```
x=rand(1000,1);
p=sum(x<0.4)/1000

p =

    0.4160
```

Sum of Two Dice

Simulate 10000 observations of the sum of two fair dice



Sum of Two Dice

Simulate 10000 observations of the sum of two fair dice

```
x1=floor(6*rand(10000,1)+1);
x2=floor(6*rand(10000,1)+1);
y=x1+x2;
sum(y==2)/10000      ans = 0.0275        p[2]=0.0278
sum(y==3)/10000      ans = 0.0554        p[3]=0.0556
sum(y==4)/10000      ans = 0.0841        p[4]=0.0833
sum(y==5)/10000      ans = 0.1082        p[5]=0.1111
sum(y==6)/10000      ans = 0.1397        p[6]=0.1389
sum(y==7)/10000      ans = 0.1705        p[7]=0.1667
sum(y==8)/10000      ans = 0.1407        p[8]=0.1389
sum(y==9)/10000      ans = 0.1095        p[9]=0.1111
sum(y==10)/10000     ans = 0.0794        p[10]=0.0833
sum(y==11)/10000     ans = 0.0585        p[11]=0.0556
sum(y==12)/10000     ans = 0.0265        p[12]=0.0278
```

Sum of Two Dice

Simulate 10000 observations of the sum of two fair dice

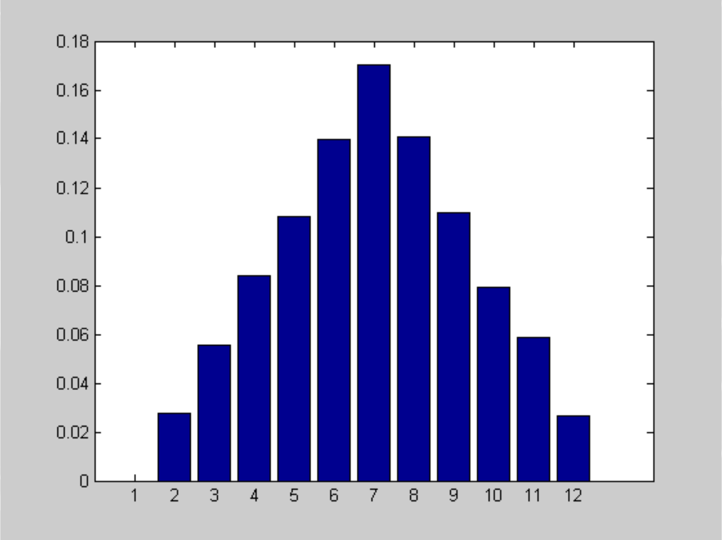
```
clf;

ISM=10000;
d1=randi(6,1,ISM);d2=randi(6,1,ISM);

[count, bin]=hist(d1+d2,2:12);
bar(bin, count/ISM)
```

Sum of Two Dice

```
for i=2:12
        z(i)=sum(y==i)/10000
end
bar(z)
```



Literature

- Ágnes Baran: Mathematics for engineers 1. (Laboratory slides)
- Sándor Baran: Probability theory and statistics
- Fazekas István: Valószínűségszámítás, Debreceni Egyetemi Kiadó, 2009
- Fazekas István: Bevezetés a matematikai statisztikába, Debreceni Egyetemi Kiadó, 2009
- Matlab examples: https://www.mathworks.com/help/examples.html