A comparison of two interaction based random graph models

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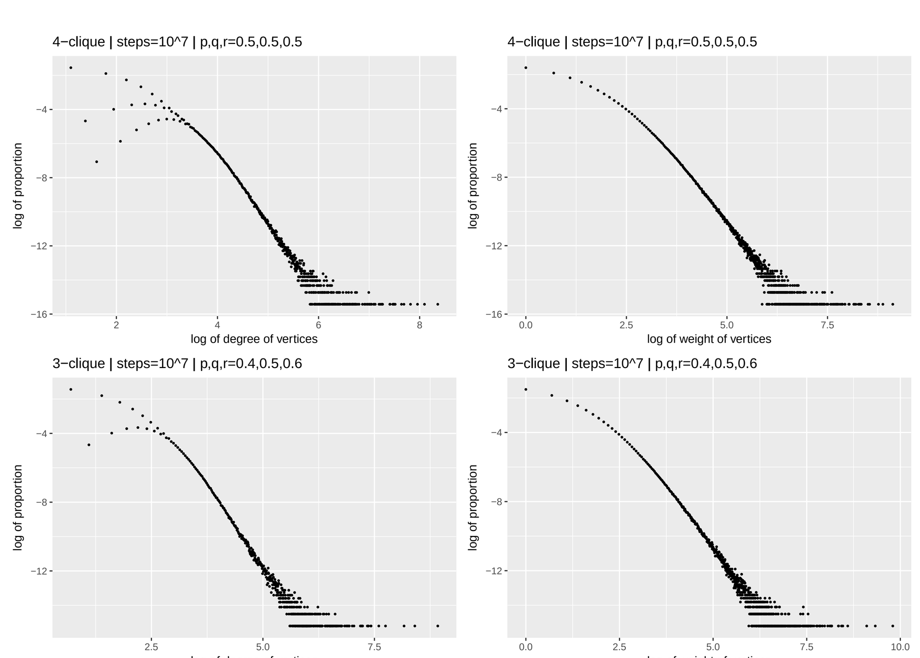
The N-clique model

Backhausz and Móri in [BM14] introduced a new class of random graphs based on interactions of three vertices. They proved almost sure results exhibiting the scale free property. The model and the results were further generalized by Fazekas and Porvázsnyik in [FP16]. Here is a short recipe of the generation: Start with an N-clique, with weigths one all of its subcliques. At each step N vertices interact. W.p. (i.e. with probabilty) p add a new vertex v to the graph and select an $N-1$ element set K from the old vertices: w.p. r from the $N-1$ cliques proportional to their weights or w.p. $1-r$ from the existing vertices uniformly, then set $K = K \cup v$. W.p. $1-p$ select an N element set K from the old vertices: w.p. q select from the N cliques proportional to their weights or w.p. $1-q$ from the existing vertices uniformly. In both cases add the N-clique generated by K to the graph and increase all of its subcliques weights by one.

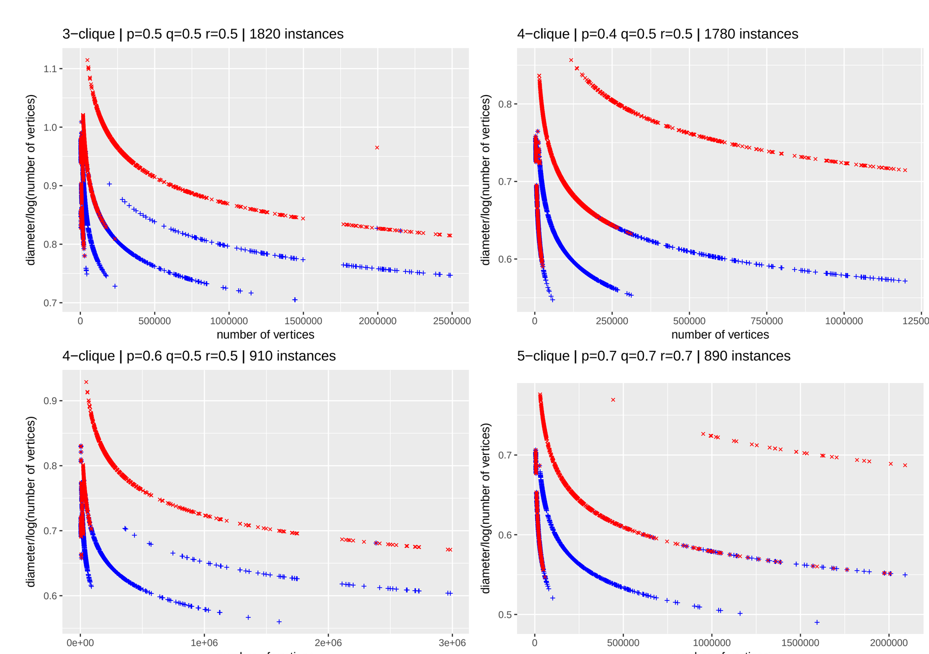
Scale-free property

In [BM14] the authors proved the scale free property in case of $N = 3$, for the weight of vertices and for the degree, in [FP16] the same was proved for $N \geq 3$. We generated a few instances of the N-clique model for $N = 3, 4$ with stepsize 10^7. In the following figures the theoretical "power-law" lines are intentionally omited, to make easier the pictorial comparison with the N-star model.



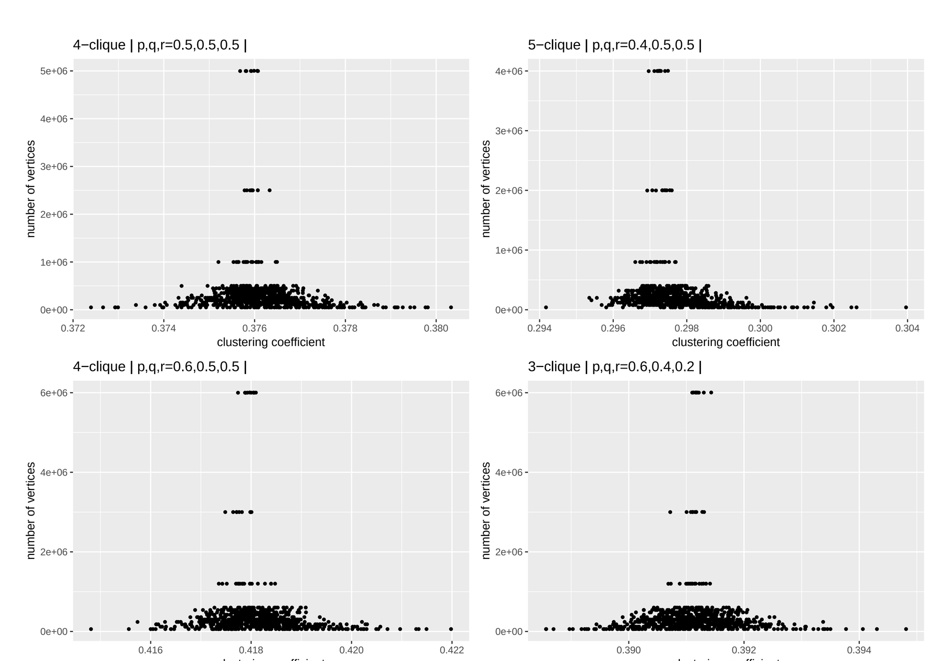
Small-world property

Finding the exact diameter of large graphs is a computationally intensive task. We implemented the method proposed in [Cre+10]. The algorithm iteratively refines the lower and upper bound for the given graph using breadth first searches. We generated ca. 1000 instances of the N-clique models for various parameters, and determined the additive 2-approximations of the diameter, i.e. an interval at most length 2 consisting the diameter. In the figures the red and blue marks are the upper and lower bounds for $\frac{diam(G(V,E))}{\log(|V|)}$. From practical point of view one can conclude that the diameter is in $O(\log(|V|))$.



Clustering coefficient

The local clustering coefficient of a node v, is the proportion of the connected vertex pairs and all possible pairs from the adjacency of v, see [WS98]. The exact computation based on counting all triangles in a network, see [Lat08]. We implemented a simple $O(|V|maxdeg^2)$ algorithm. The figures below show that the model has relatively high average clustering coefficient (as the cliques), moreover some kind of convergence can be observed.

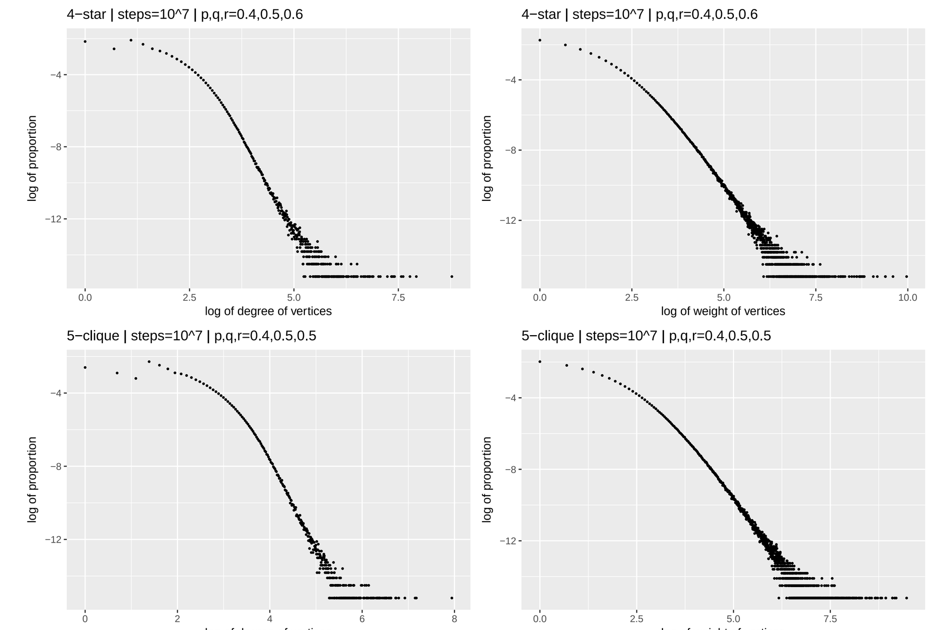


The N-star model

It is a centralized variant of the N-clique model: at each step a (possibly new) vertex and $N-1$ old vertices interact forming an N-star, i.e an N-tree with a central vertex and $N-1$ leaves. The process of the evolution is as follows: Start with an N-star, with weigths one all of its substars. At each step N vertices interact. W.p. p add a new vertex v to the graph and select an $N-1$ element set K from the old vertices: w.p. r from the $N-1$ stars proportional to their weights or w.p. $1-r$ from the existing vertices uniformly, then set $K = K \cup v$. W.p. $1-p$ select N element set K from the old vertices: w.p. q select from the N stars proportional to their weights or w.p. $1-q$ from the existing vertices uniformly. Note that if no central node in K ($(p, 1-r)$ and $(1-p, 1-q)$ branches) select it uniformly from the *old* vertices of K. At last add the N-star generated by K to the graph and increase all of its substars weights by one.

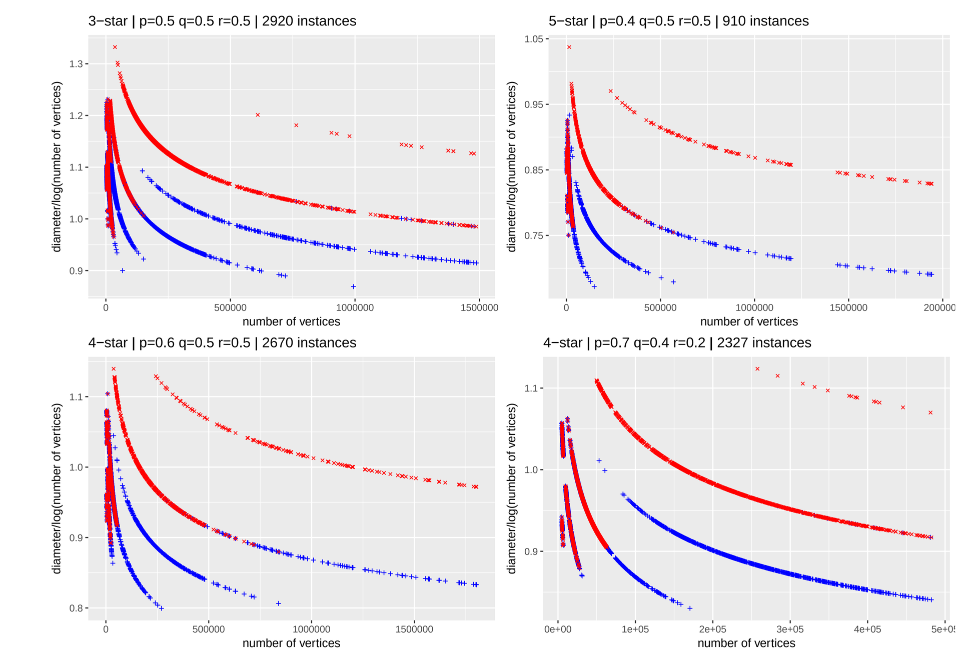
Scale-free property

There are partial results on the power-law distribution of weights of the vertices: [Faz+15]. The weight of a vertex adds up from two parts: peripheral and central one. Fixing one of them, convergence proprties can be proved.



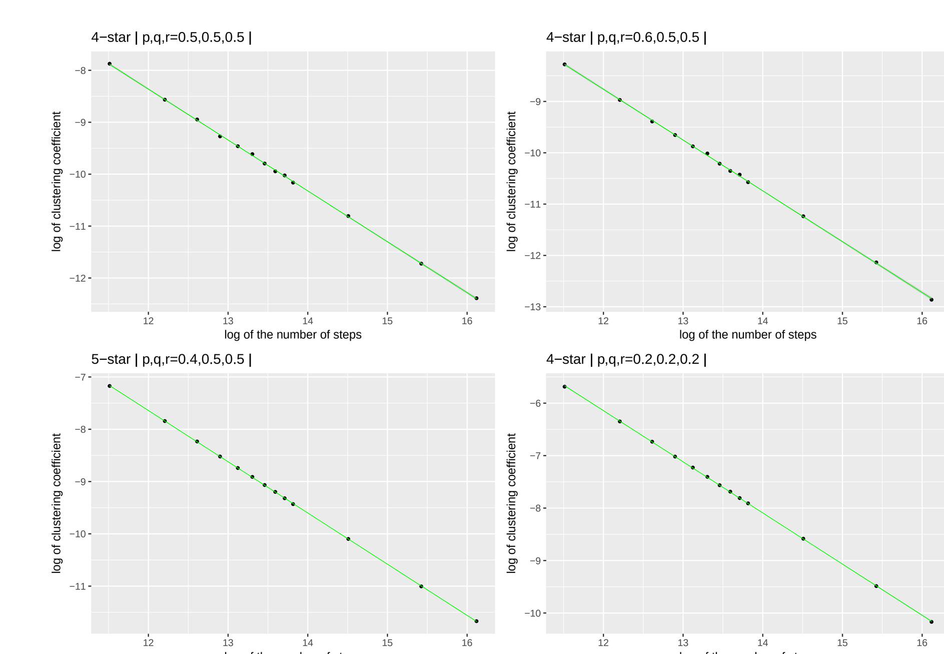
Small-world property

Based on the simulation data one can conjecture, that the N-star model diameter is in $O(\log(|V|)$.



Clustering coefficient

The model has extremely low clustering cofficient. In the figures the black marks represent the average of the observed coefficents for selected stepsizes. Log-log scale used, and green line were fitted.



References

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