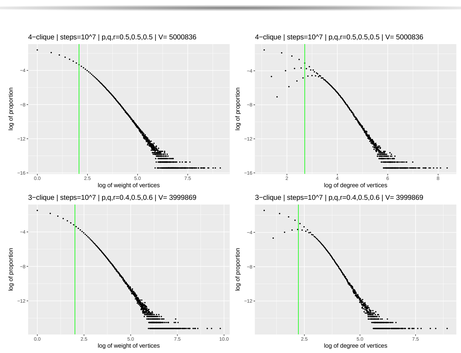
A comparison of two interaction based random graph models

Cs.Noszály, N.Uzonyi - Faculty of Informatics, University of Debrecen

The N-clique model

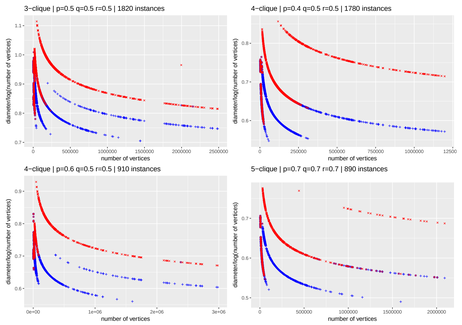
Backhausz and Móri in [BM14] introduced a new class of random graphs based on interactions of three vertices. They proved almost sure results exhibiting the scale free property. The model and the results were further generalized by Fazekas and Porvázsnyik in [FP16]. Here is a short recipe of the generation: Start with an N-clique, with weigths one all of its subcliques. At each step N vertices interact. W.p. (i.e. with probabilty) p add a new vertex v to the graph and select an $N-1$ element set K from the old vertices: w.p. r from the $N-1$ cliques proportional to their weights or w.p. $1-r$ from the existing vertices uniformly, then set $K = K \cup v$. W.p. $1-p$ select an N element set K from the old vertices: w.p. q select from the N cliques proportional to their weights or w.p. $1-q$ from the existing vertices uniformly. In ether cases add the N-clique generated by K to the graph and increase all of its subcliques weights by one.

Scale-free property



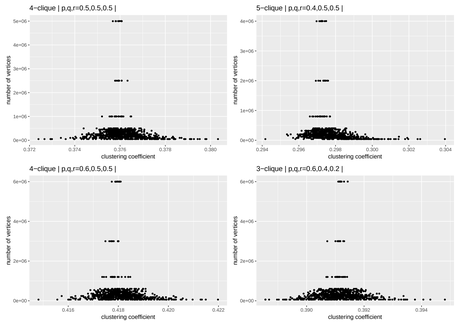
Small-world property

Finding the exact diameter of large graphs is a computationally intensive task. We implemented the method proposed in [Cre+10]. The algorithm iteratively refines the lower and upper bound for the given graph using breadth first searches. We generated ca. 1000 instances of the N-clique models for various parameters, and determined the additive 2-approximations of the diameter. In the figures the red and blue marks are the upper and lower bounds for $\frac{diam(G(V,E))}{\log(|V|)}$. From practical point of view one can conclude that the diameter is in $O(\log(|V|))$.



Clustering coefficient

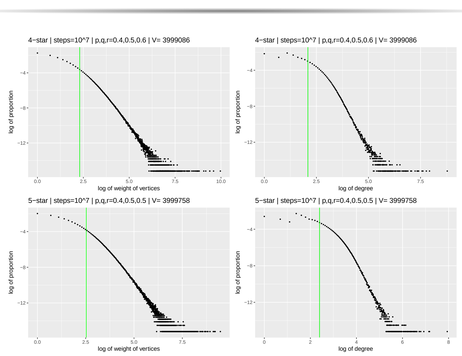
The local clustering coefficient of a node v, is a proportion of connected vertex pairs among all possible pairs from the neighbourhood of v. Its exact computations are basically triangle counting methods see [Lat08]. We implemented an $O(|V|maxdeg^2)$ algorithm.



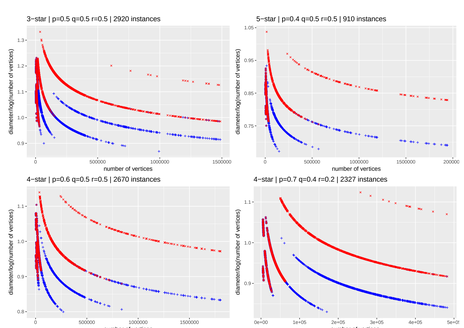
The N-star model

It is a centralized variant of the N-clique model: at each step a (possibly new) vertex and $N-1$ old vertices interact forming an N-star, i.e an N-tree with a central vertex and $N-1$ leaves. The process of the evolution is as follows: Start with an N-star, with weigths one all of its substars. At each step N vertices interact. W.p. p add a new vertex v to the graph and select an $N-1$ element set K from the old vertices: w.p. r from the $N-1$ stars proportional to their weights or w.p. $1-r$ from the existing vertices uniformly, then set $K = K \cup v$. W.p. $1-p$ select N element set K from the old vertices: w.p. q select from the N stars proportional to their weights or w.p. $1-q$ from the existing vertices uniformly. Note that if no central node in K ($(p, 1-r)$ and $(1-p, 1-q)$ branches) select it uniformly from the *old* vertices of K. At last add the N-star generated by K to the graph and increase all of its subcliques weights by one.

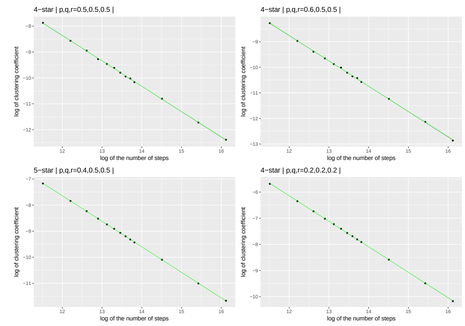
Scale-free property



Small-world property



Clustering coefficient



References

[BM14] Agnes Backhausz and Tamas F Mori. "-". In: *Acta Math. Hungar.* 143 (2014), pp. 23–43.

[Cre+10] Pierluigi Crescenzi et al. "Finding the diameter in real-world graphs". In: *Algorithms–ESA 2010* (2010), pp. 302–313.

[FP16] István Fazekas and Bettina Porvázsnyik. "-". In: *Open Mathematics* 14.1 (2016), pp. 414–424.

[Lat08] Matthieu Latapy. "Main-memory triangle computations for very large (sparse (power-law)) graphs". In: *Theor. Comput. Sci.* 407.1-3 (2008), pp. 458–473.