Norms, condition numbers

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Exercise 1

Implement a function that returns the $n \times n$ matrix $A = (a_{ij})$:

$$a_{ij} = \begin{cases} 1, & \text{if } i = j, \\ -1, & \text{if } i < j, \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 2

Using the 100×100 matrix A of the previous exercise, compare the solutions of the linear systems below:

$Ax = b_1$	$Ay = b_2$

where $b_1 = -98 : 1$ and b_2 is a "perturbed" version of b_1, that is

$$b_2 = b_1$$
$$b_2(100) = b_2(100) + 1e - 6$$

Use the `backslash` operator (`\`) !

Exercise 3

Depict the set $\{x \in \mathbb{R}^2 : \|x\| = 1\}$ for 1,2 and ∞-norm!

Exercise 4

Implement functions for computing the 1,2 and ∞ norm!

Exercise 5

Compute the ∞-norm of vectors b_1, b_2 and x, y!

Exercise 6

Read the help of the function `norm`!

Exercise 7

Implement functions for computing the induced $1, \infty$ matrix norms!

Exercise 8

Compute the relative error of the right-hand side, the solution and the condition number! Use 1 and ∞ norm!

Exercise 9

Solve the system $Ax = b$ where

$$A = \begin{pmatrix} 1 & 0.99 \\ 0.99 & 0.98 \end{pmatrix}, \qquad b = \begin{pmatrix} 1.99 \\ 1.97 \end{pmatrix}.$$

Now, suppose that instead of b, we have

$$b + \delta b = \begin{pmatrix} 1.98 \\ 1.98 \end{pmatrix}$$

Solve the system $Ay = b + \delta b$! Also, compute the relative error of the right-hand side and the solution in ∞-norm. What is $cond_\infty(A)$?

Exercise 10

Implement a function that computes the $n \times n$ Hilbert-matrix!

Exercise 11

Compute the condition number of the Hilbert matrix of size 6×6! Use `hilb` and `cond`!

Exercise 12

What is the condition numbers of a random matrix of size 6×6? Experiment with different instances, use `rand` !