Floating-point numbers

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How can we trust in machine computations?

Exercise 1

Examine the value of the (logical) expression: $0.4 - 0.5 + 0.1 == 0$.

What is the value of $0.1 - 0.5 + 0.4 == 0$?

Exercise 2

What is the theoretical (expected) value of x after performing the following algorithm:

```
x=1/3;
for i=1:40
    x=4*x-1;
end
```

Exercise 3

Examine values of the following expressions:

$$2^{66}+1 == 2^{66},\ 2^{66}+100 == 2^{66},\ 2^{66}+10000 == 2^{66}$$

Exercise 4

What are the results of algorithms below?

```
a=0;
for i=1:5
    a=a+0.2;
end
a==1
```

```
a=1;
for i=1:5
    a=a-0.2;
end
a==0
```

Try to explain!

Exercise 5

(a) Write a code that computes the machine epsilon!

(b) Read the help of the function eps! What is the value of eps(1)?

Exercise 6

(a) Write a code that computes ε_0!

(b) What is the value of `eps(0)`?

Exercise 7

Examine the values of `realmin` and `realmax`! What is `realmin('single')` and `realmax('single')`?

For a given a, t, k_+, k_- the floating-point numbers is finite subset of the real interval $[-M_\infty, M_\infty]$

Exercise 8

Let $a = 2$, $t = 4$, $k_- = -3$, $k_+ = 2$.

(a) Plot all positive (normalized) numbers from the system!

(b) What is the value of M_∞, ε_0 és ε_1?

(c) What is the distance of two neighbouring numbers?

Exercise 9

Examine again the values of the following expressions:

$$2^{66}+1 == 2^{66},\ 2^{66}+10 == 2^{66},\ 2^{66}+100 == 2^{66},$$
$$2^{66}+1000 == 2^{66},\ 2^{66}+10000 == 2^{66}$$

Try to find the smallest $n > 0$ for which $2^{66} + n == 2^{66}$ is `false`! What is the value of `eps(2^66)`?

Exercise 10

Let $a = 2$, $t = 4$, $k_- = -3$, $k_+ = 2$. Compute the corresponding floating point numbers for:

$$0.4, \quad 0.3, \quad \frac{1}{3}, \quad 0.7, \quad \frac{1}{32}$$

Exercise 11

Examine the value of expression $0.4 - 0.5 + 0.1 == 0$! Explain! Examine the value of expression $0.1 - 0.5 + 0.4 == 0$! Explain!

Exercise 12

Let $a = 2$, $t = 4$, $k_- = -3$, $k_+ = 2$. Try to find positive $x \neq y$ floating point numbers, for which:

(f) $x + y < M_\infty$, but $x + y$ is not a floating point number.

(g) $fl(x + y) = x$.

Exercise 13

What will be the value of x after executing the code below?

```
x=1/3;
for i=1:40
    x=4*x-1;
end
```

Why is so different what we see?

Exercise 14

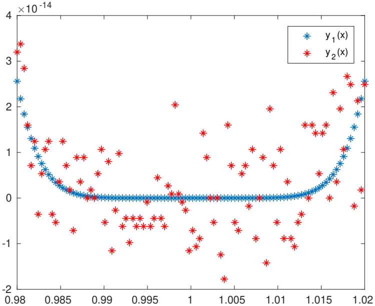
The code below modifies and restores the value of x by successive squarerooting and squareing. In theory x remains the same. What we see in practice? Why?

```
for i=1:60
    x=sqrt(x);
end
for i=1:60
    x=x^2;
end
```

Exercise 15

Plot the functions y_1 and y_2 on a small neighbourhood of 1.

$$y_1(x) = (x-1)^8,$$
$$y_2(x) = x^8 - 8x^7 + 28x^6 - 56x^5 + 70x^4 - 56x^3 + 28x^2 - 8x + 1$$



The two functions are the mathematically equivalent. Try to explain the strikin difference!

Exercise 16

Using appropriate normalizing one can avoid overflow/underflow. Let $x = [10^{200}, 1]$. Compute norm of x as described below!

(a)

$$\|x\| = \sqrt{x_1^2 + x_2^2}$$

(b)

$$c = \max\{|x_1|, |x_2|\}, \quad \|x\| = c \cdot \sqrt{\left(\frac{x_1}{c}\right)^2 + \left(\frac{x_2}{c}\right)^2}$$

Explain the observation!