Kísérlet

1 ← elemi

2

⋮

6

kocka

$$\Omega = \{\omega_1, \dots, \omega_n, \dots\}$$

diszkrét

$$A = \{\text{párost kapunk}\} = \{2, 4, 6\}$$

$$A \subset \Omega \xrightarrow{P} \mathbb{R} \quad [0,1]$$

$$\frac{K_A}{N}$$

K_A ← gyak

$\frac{K_A}{N}$ ← rel gyak

$$\Omega \supset A \longmapsto P(A) \geq 0$$

$$\in [0,1]$$

$P(\Omega)=1 \quad P(\emptyset)=0 \quad \underbrace{P(A+B)=P(A)+P(B)}_{A\cdot B=0}$

$$P(\omega_k)=P(\omega_l)$$

$(\Omega, \mathcal{A}, P)$ valségi mező

Kombinatorikus Klasszikus

$\frac{1}{N}$ $\omega_1, \ldots, \omega_N$

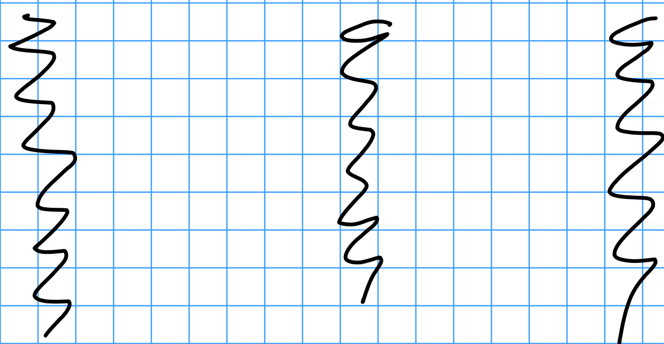
$\frac{1}{N}$

Double

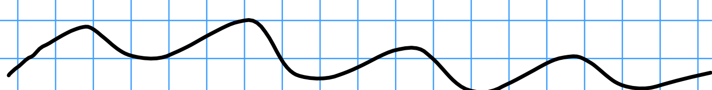
$$(x+y)+z = x+(y+z)$$

$x \qquad y \qquad z$

$$0.4 + (-0.5) + 0.1$$



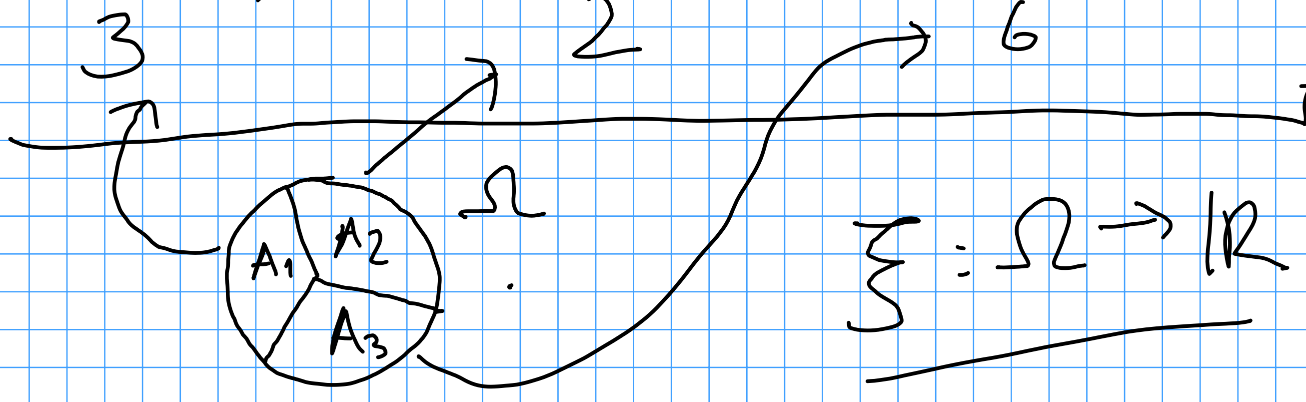
$N = 10^6$



$$A \longrightarrow P(A)$$

$p \qquad v$

$\frac{1}{3} \to -1 \qquad \frac{1}{2} \to 0 \qquad \frac{1}{6} \to 4$



$\underline{\xi : \Omega \to \mathbb{R}}$

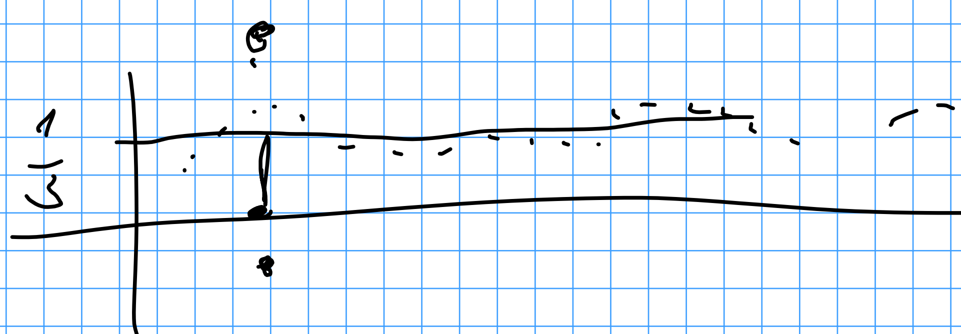
$$\frac{k_1 + k_2 \qquad + k_N}{N}$$

$$\frac{\tilde{k}_{-1} \cdot -1 + \tilde{k}_0 \cdot 0 + \tilde{k}_4 \, 4}{N} =$$

$$\frac{\tilde{k}_{-1}}{N}(-1) + \frac{\tilde{k}_0}{N}\, 0 + \frac{\tilde{k}_4}{N}\, 4$$

$$E(\xi) = p_{-1}(-1) + p_0 \cdot 0 + p_4 \cdot 4 =$$

$$\frac{1}{3}(-1) + \frac{1}{2}\, 0 + \frac{1}{6} \cdot 4 = \boxed{\frac{1}{3}}$$



$(\text{tipp} - k)^2$ átlag

$|\text{tipp} - k|$ medián

$V_1 \ldots \quad V_n, \ldots$

$p_1 \quad p_n, \ldots \quad \sum p_k = 1$

Kísérlet

Addig dobunk amíg 6-est nem kapunk

V

$1 - \frac{1}{6}$

$2 - \frac{5}{6} \cdot \frac{1}{6}$

$3 - \frac{5}{6} \frac{5}{6} \frac{1}{6}$

$\vdots$

p – param. Geom.

$\xi \quad p$ – par

$n \quad \left(\frac{5}{6}\right)^{n-1} \cdot \frac{1}{6} \qquad (1-p)^{n-1} p$

$$E(\xi) = \sum_{k=1}^{\infty} k \cdot \left(\frac{5}{6}\right)^{k-1} \frac{1}{6} \qquad \sum k(1-p)^{k-1} p = \frac{1}{p}$$

$$P(A|B) = P(A)$$

$$\boxed{\text{flen}}$$

$$P(A \cdot B) = P(A)P(B)$$

$$\begin{array}{cc} \text{cédulák} & \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \\ & \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \\ & \begin{pmatrix} 1 & 0 & 1 \end{pmatrix} \\ & \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \end{array}$$

$\xi \to 1$ jegy

$\eta \to 2$ jegy

$$A_x \qquad A_y$$

$$\{\xi = x\}, \{\eta = y\}$$

$$\forall x, \forall y \quad A_x, A_y$$

ξ, η	0	1	
	0	1	$\leftarrow U$
	$\frac{1}{2}$	$\frac{1}{2}$	$\leftarrow P$

kontingens

$\xi \backslash \eta$	0	1	
0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	

$$P(\xi = 0) \cdot P(\eta = 0) \overset{?}{=} P(\xi = 0, \eta = 0)$$

Pl. Feldobunk 10 (n) - szer

Fej: 0, 1, …, 9, 10

$k \leftrightarrow \binom{10}{k} p^k (1-p)^{10-k}$

p - a fej valsége

V p

(n, p) Binomiális v.v.

$$E(\xi) = np$$

$$\sum_{0}^{n} \underset{\downarrow\ \text{v.}}{k} \underbrace{\binom{n}{k} p^k (1-p)^{n-k}}_{\downarrow\ p}$$

$\xi_1 \quad \xi_2 \quad \xi_n$

$$E(\xi_k) = (1-p)\,0 + p \cdot 1$$

Bernoulli - v.v

$$E(\xi) = E(\overbrace{\xi_1 + \dots + \xi_n})$$

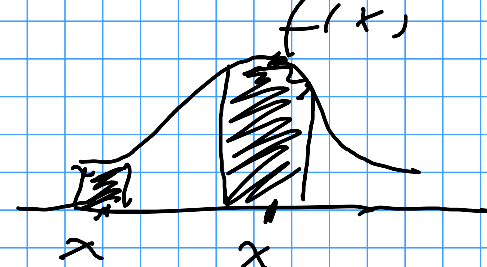
$$E(\xi_1) + \dots + E(\xi_n) = np$$

$x_1 \dots \quad x_n, \dots$

$p_1 \quad\quad p_n \dots$

$\rightarrow$ valeloszlás

$$\sum x\, p_x$$



$$\int_{-\infty}^{\infty} x \cdot f(x)\, dx \qquad \int_{-\infty}^{\infty} f = 1$$

$a = 0 \quad b = 11$

$\frac{(b-a)^2}{12} \rightarrow \frac{(11-1)^2}{12} = \frac{100}{12}$

$8,333 \quad \sqrt{8,333}$

$\frac{a+b}{2} = 5.5$

Bin

$\xi = \xi_1 + \dots + \xi_n$

$D^2(\xi) \overset{flen}{=} D^2(\xi_1) + \dots + D^2(\xi_n)$

$p(1-p)$

$\frac{1}{4}$

$\sqrt{\frac{11}{4}} = \sqrt{2,25} \approx 1,5$

$E(\xi) = m$

$E(\xi - m)^2 = D^2(\xi)$

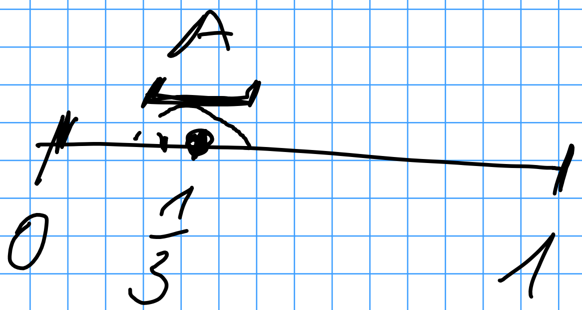
$P(\xi = \frac{1}{3})$

$\xi \sim U(0,1)$

$\xi \sim N(0,3)$

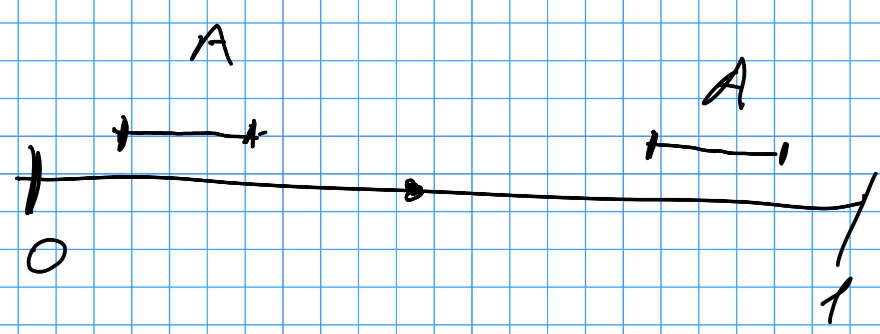
$Bin(n,p)$

$Geom(p)$



$\{\frac{1}{3}\} \subset A$

$P(\{\frac{1}{3}\}) \leq P(A)$



$1 \rightarrow \frac{1}{2}$

$2 \rightarrow \frac{1}{3}$

$3 \rightarrow \frac{1}{6}$