VERGLEICHENDE SOZIALFORSCHUNG MIT MEHREBENENMODELLEN IN R

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Random slope models

AGENDA

- Random slope models
- More on error terms, variances and covariances

BEYOND RANDOM INTERCEPTS

RANDOM SLOPE MODELS

- Not only varying *intercepts* for each higher-level unit but also varying *slopes*

→Effect of an individual-level variable can differ across countries

- Estimated as variance → another error term
- Random slopes allows to test the generalizability of individual level relationships across countries
- Also models heteroscedasticity (not discussed)

RANDOM SLOPE MODEL

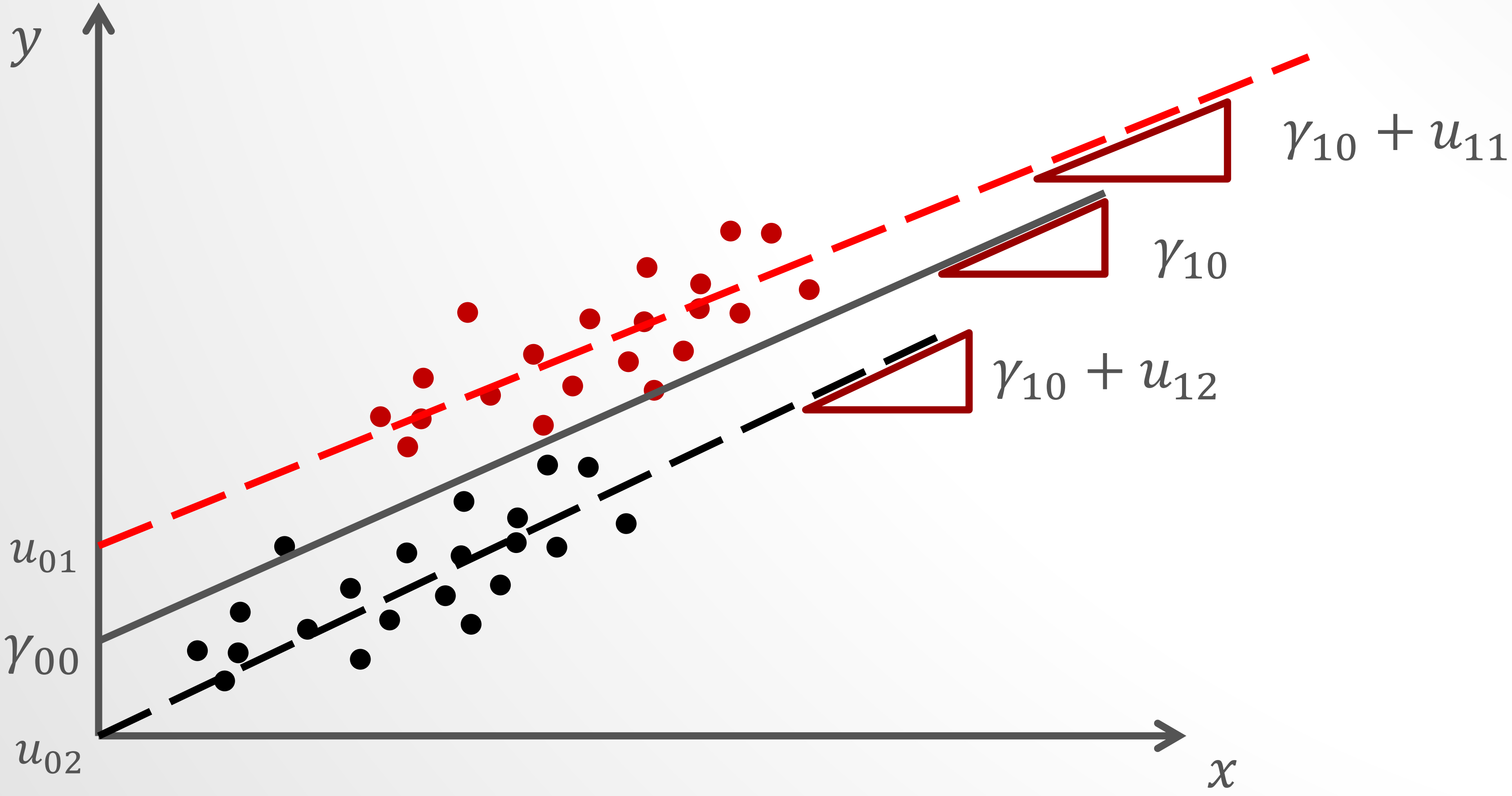
- Level 1: $y_{ij} = \beta_0 + \beta_1 x_{1ij} + \cdots + \beta_k x_{kij} + e_{ij}$
- Level 2 (random intercept): $\beta_0 = \gamma_{00} + u_{0j}$
- Level 2 (random slope): $\beta_1 = \gamma_{10} + u_{1j}$

γ_{01}: fixed effect of x_1

u_1: random effect of x_1 (varying slope)

RANDOM SLOPE MODEL

$$y_{ij} = \gamma_{00} + \gamma_{10}x_{ij} + \gamma_{01}z_j + u_{1j}x_{ij} + u_{0j} + e_{ij}$$



INTERPRETATION

- Random slope model:
$$y_{ij} = \gamma_{00} + \gamma_{10} x_{ij} + u_{1j} x_{ij} + u_{0j} + e_{ij}$$
- γ_{00} is (still) expected value across all individuals and countries
- e_{ij} is (still) the error on individual level
- New: Effect of x now differs across groups
- γ_{10} is the *average change* in y for a one unit change in x (slope of the average line)
- Put differently, γ_{00} is the *mean of y* when all x are zero (constant), whereas γ_{10} is the *average effect of x*
- What about u_{1j}?
- To be continued…

EXAMPLE

- Outcome: life satisfaction (`stflife`)
- Explanatory variables: income (`hinctnt`, individual level), GDP/c (`rgdpc`, country level)
- Question: does the relationship between income and life satisfaction vary across countries?

→In R: `lmer(stflife ~ hinctnt + rgdpc + (1 + hinctnt | cntry), = ESS02)`

Tells R to allow the effect of variable `hinctnt` to vary over `cntry`

EXAMPLE

Life satisfaction	Model 0	Model 1	Model 2	**Model 3**
Income		0.18 ***	0.18 ***	**0.19 *** **
GDP/c			0.02 ***	**0.02 *** **
Intercept	7.02***	5.99 ***	5.10 ***	**5.48 *** **
Random effects				
Intercept	0.617	0.319	0.210	**0.771**
Income				**0.008**
Covar(Intercept-Income)				**-0.07**
Residual	4.559	4.262	4.262	**4.194**

* p<0.05, ** p<0.01, *** p<0.001

ERROR TERMS

NOTATION

- Variance of β_0 between countries: $\mathrm{var}(u_{0j}) = \tau_{00}$ (random intercept)
- Variance of β_1 between countries: $\mathrm{var}(u_{1j}) = \tau_{11}$ (random slope)
- Variance of β_0 within countries: $var(e_{ij}) = {\sigma_e}^2$

ASSUMPTIONS FOR ERRORS

- First level error is normally distributed $e_{ij} \sim N(0, {\sigma_e}^2)$
- ${\sigma_e}^2$ identical across countries(homoscedasticity)
- Second level errors are normally distributed:
  - Random intercept: $u_{0j} \sim N(0, \tau_{00})$
  - Random slope: $u_{1j} \sim N(0, \tau_{11})$
- Errors on level one and two are independent

ASSUMPTIONS FOR ERRORS

- Errors on higher level not necessarily independent; they can co-vary
- For example: Covariation of random intercecpt and random slope $cov\left(u_{0j}, u_{1j}\right) = \tau_{01}$
- Multivariate normal distribution with $N_{q+1}(0, R)$
- R = Covariance matrix
- With one random slope: $N_2\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}\tau_{00} & \tau_{01}\\ & \tau_{11}\end{bmatrix}\right)$

INTERPRETATION OF (CO-)VARIANCES

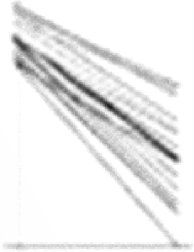
- What the model estimates are
  - Variances of the errors:
    - $\text{var}(u_{0j}) = \tau_{00}$
    - $\text{var}(u_{1j}) = \tau_{11}$
  - Covariances of these errors (if specified): $cov(u_{0j}, u_{1j}) = \tau_{01}$
- τ_{00}: Between country variance of β_0 when $x = 0$
- τ_{11}: Between country variance of β_1 when $x = 0$
- τ_{01}: Between country covariance between β_0 and β_1 when $x = 0$
- Why $x = 0$? Because, by definition, the (co-)variances differ for the different levels of x

→*(Grand mean-)center continuous x when estimating random slopes*

COVARIANCE BETWEEN INTERCEPTS AND SLOPES (τ_{01})

- $\tau_{01} > 0$ → fanning out

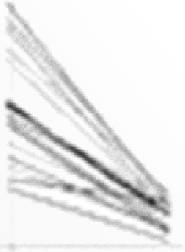
$\beta > 0$: 

$\beta < 0$: 

- Lines with larger intercepts have larger slopes
- "Those with larger values of x are *less* similar in y across countries"

- $\tau_{01} < 0$ → fanning in

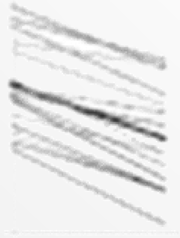
$\beta > 0$: 

$\beta < 0$: 

- Lines with smaller intercepts have larger slopes
- "Those with larger values of x are *more* similar in y across countries"

- $\tau_{01} = 0$ → no pattern

$\beta > 0$: 

$\beta < 0$: 

COVARIANCE STRUCTURES

- For example, covariance matrix for two random slopes:

$$\begin{bmatrix} \tau_{00} & \tau_{01} & \tau_{02} \\ & \tau_{11} & \tau_{12} \\ & & \tau_{22} \end{bmatrix}$$

- τ_{00}: variance of u_0 (random intercept)
- τ_{11}: variance of u_1 (random slope of x_1)
- τ_{22}: variance of u_2 (random slope of x_2)
- τ_{01}: covariance of u_0 and u_1
- τ_{02}: covariance of u_0 and u_2
- τ_{22}: covariance of u_1 and u_2

COVARIANCE STRUCTURES

- Full covariance matrix quickly becomes rather complex
- Two random slopes implies six random effect parameters
- More parameters mean much more computational time (or even no model convergence at all)
- Moreover, estimating each of the (co-)variance needs more degrees of freedom on the country level
- The might be theoretical reasons to assume that, for example, all variances and all covariances were equal across all countries

→$\tau_{00} = \tau_{11} = \tau_{22}$ and $\tau_{01} = \tau_{12} = \tau_{02}$ (Compound Symmetry)

→Only two instead of six parameters would need to be estimated

EXAMPLE

- Does the association between Attitudes toward radioactive waste and age vary across countries?
- $\beta_{age} = -0.041$
- $\tau_{00} = 43.32$
- $\tau_{11} = 0.009$
- $\tau_{01} = 0.413$

INTERPRETATION OF τ_{01}

- Attitudes of younger people more similar compared to attitudes of older people
- For example: predicted attitudes of 18 year old people range over ~20 scale points (from 46 to 66)
- … Those of 80 year old people range over ~30 scale points

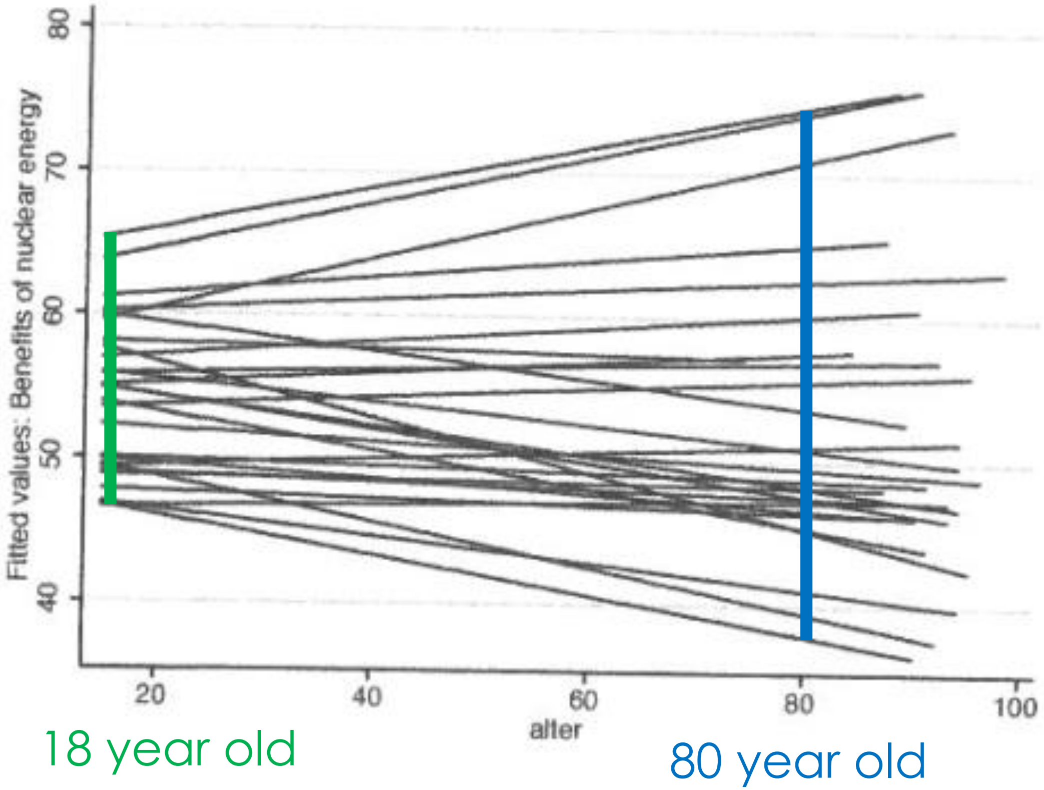


Abb. 5.8: Random-Slope-Plot für die Variable Alter

Wenzelburger, Jäckle & König (2014): 109

ANOTHER REASON FOR RANDOM SLOPES

- Adding random slopes for individual level effects can lead to more precise estimates for country level effects (Heisig, Schaeffer & Giesecke 2017)
- Differences in the effect of control variables removes noise from effects of macro-level variables
- Best performance of models that do not include random slopes for *all* individual level effects but only for the most important ones (especially with few countries)
- Balance flexibility and parsimony

INTERPRETATION OF PARAMETERS: OLS VS RANDOM EFFECTS MODEL

	OLS	Random effects model
Intercept / constant	β_0 Expected value of y ("mean") when all x are 0	γ_{00} Expected value of y ("grand mean") when all x are 0
Effect coefficient	β_1 Effect of x	γ_{10} Average effect of x across countries
Variance in the intercept	-	$\text{var}(u_0) = \tau_{00}$ *"How large is the difference in β_0 between countries?"*
Variance in the slope	-	$\text{var}(u_1) = \tau_{11}$ *"How large is the difference in the effect of x on y between countries?"*
Idiosyncratic error	e Unexplained variance of y	e Within country variance of y that is not explained by variables in the model

SUMMING UP: RANDOM EFFECTS

- Random effects tell us something about how different countries are
  - … Regarding the outcome (β_0) → random intercept
  - … Regarding the association between x and y (β_1) → random slope
- Within the RE framework, we are not so much interested in differences between two actual countries (i. e.: not in single values for u_0 or u_1)
- … But how much β_0 or β_1 (co-)vary across *all* countries → τ_{00} and τ_{11} (and τ_{01})
- This is a highly flexible modeling strategy allowing to examine a plethora of relationships and patterns in your data
- Having a model that fits the data well also has methodological benefits
- However, RE models quickly become complex
  - Difficult interpretation
  - Computationally demanding

LITERATURE

- Heisig, Schaeffer & Giesecke (2017). The costs of simplicity: Why multilevel models may benefit from accounting for cross-cluster differences in the effects of controls. American Sociological Review, 82(4), 796-827