GOETHE UNIVERSITÄT
FRANKFURT AM MAIN

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FORSCHUNGSPRAKTIKUM I UND II: LÄNGSSCHNITTDATENANALYSE IN R

Fixed effects
session v

AGENDA

- Decomposition of variance into within and between part
- The logic of Fixed Effects (FE) models
- Benefits and limitations of FE
- Comparison of FE and First Difference models

THE POPULARITY OF FIXED EFFECTS IN SOCIOLOGY

Source: Hill, Davis, Roos & French (2020). [Limitations of fixed-effects models for panel data](). Sociological Perspectives, 63(3): 359

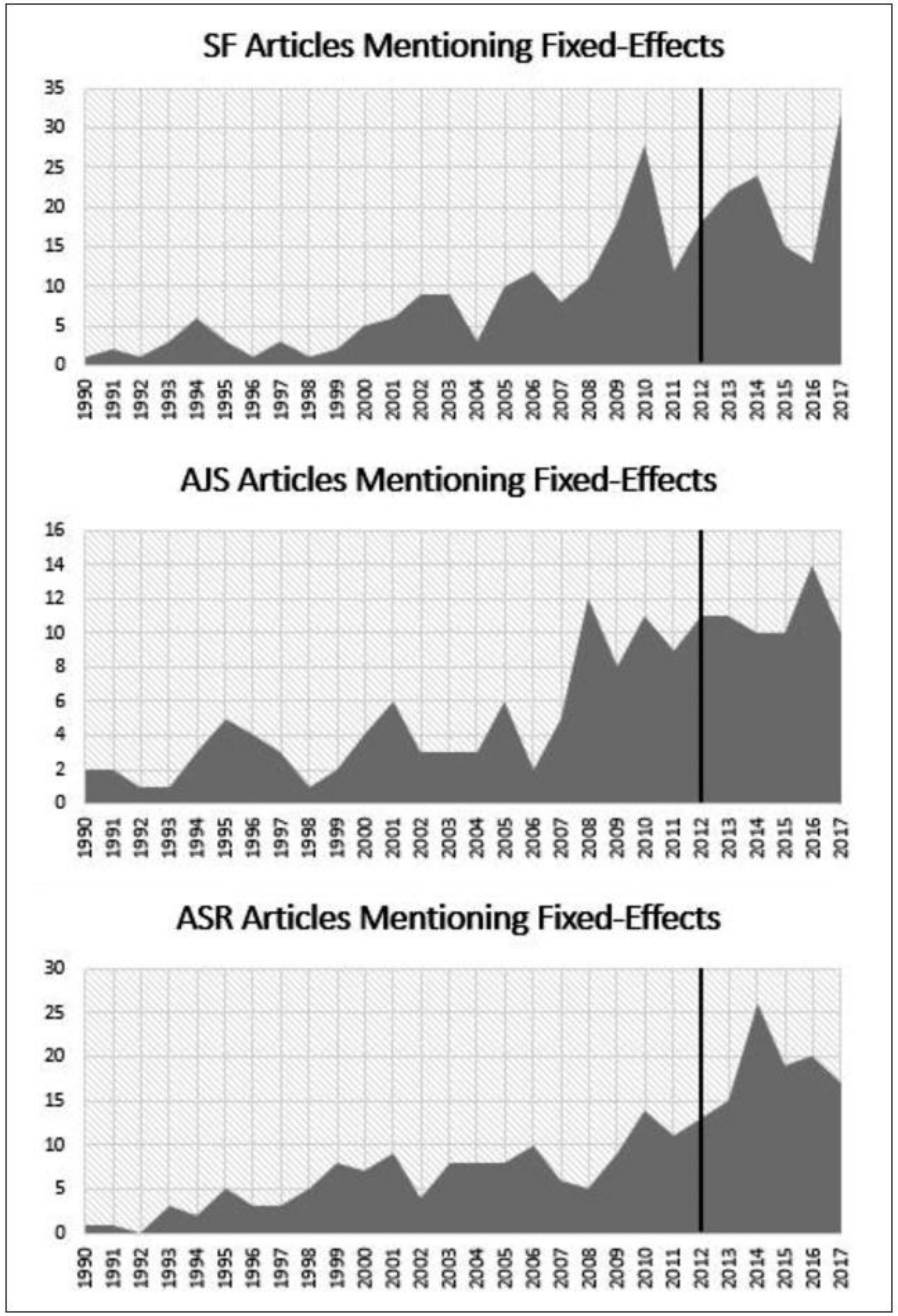
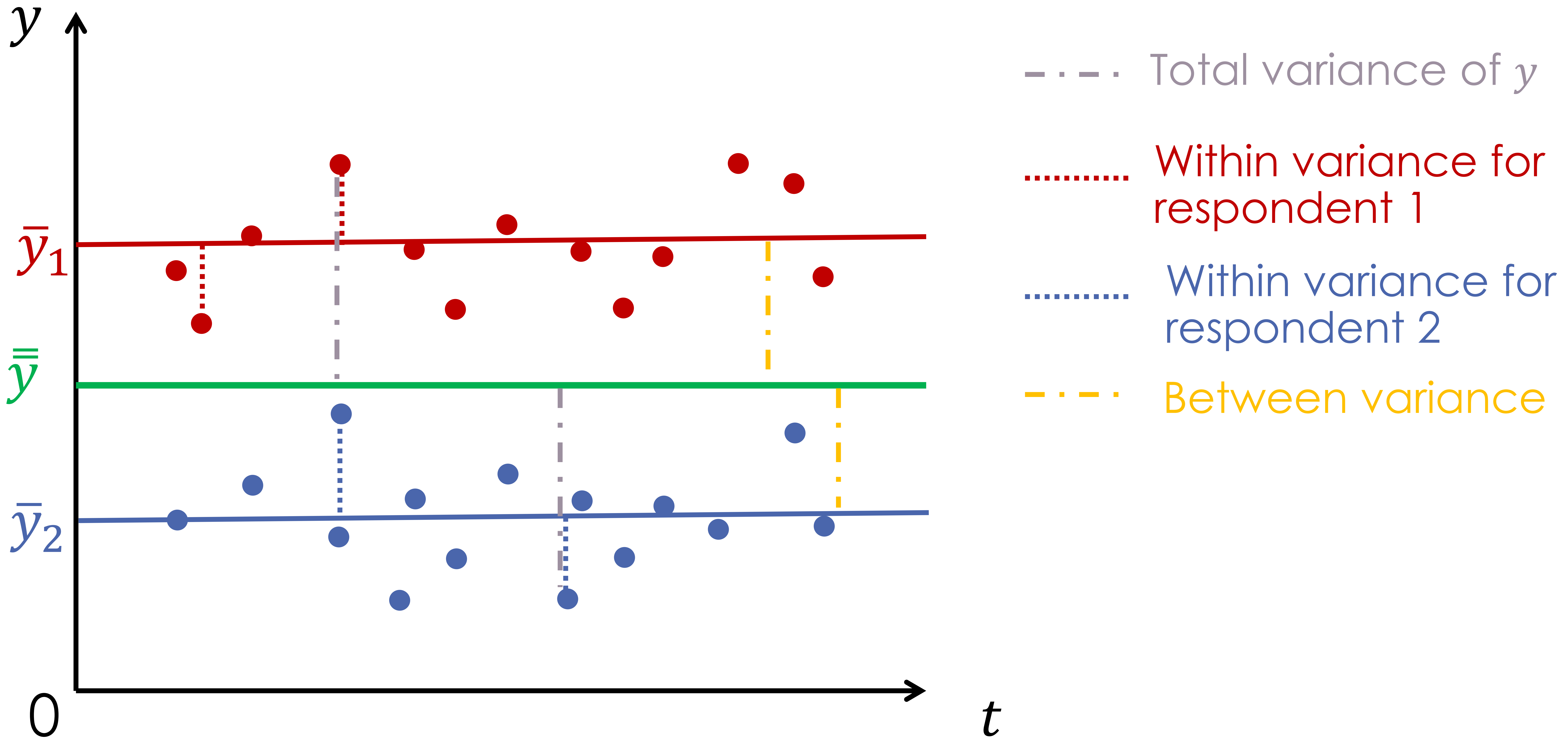


Figure 1. The growing number of articles mentioning fixed-effects (1990–2017) with demarcation for the current study period (2012–2017) by *Social Forces* (SF), *American Journal of Sociology* (AJS), and *American Sociological Review* (ASR).

WITHIN AND BETWEEN VARIANCE IN PANEL DATA

WITHIN AND BETWEEN VARIANCE



WITHIN AND BETWEEN VARIANCE

- Within variance: within one individual over time → $(y_{it} - \bar{y}_i)$
- Between variance: between individuals → $(\bar{y}_i - \bar{\bar{y}})$

ID	Year	y_{it}
1	2009	0.58
1	2010	0.88
1	2011	0.04
2	2009	0.66
2	2010	0.22
2	2011	0.5
3	2009	0.3
3	2010	0.3
3	2011	0.3

Individual-specific mean $(\bar{y}_i)$

0.50

0.46

- 0.30

Overall mean $(\bar{\bar{y}})$

0.42

WITHIN AND BETWEEN VARIANCE

- $\bar{\bar{y}} = 0.42$
- $\bar{y}_1 = 0.50$
- $\bar{y}_2 = 0.46$
- $\bar{y}_3 = 0.30$

ID	Year	y_{it}
1	2009	0.58
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Overall variance: $(y_{it} - \bar{\bar{y}})$	Within variance: $(y_{it} - \bar{y}_i)$	Between variance: $(\bar{y}_i - \bar{\bar{y}})$
0.16	0.08	0.08
0.46	0.38	0.08
-0.38	-0.46	0.08
0.24	0.2	0.04
-0.2	-0.24	0.04
0.08	0.04	0.04
-0.12	0	-0.12
-0.12	0	-0.12
-0.12	0	-0.12

UNOBSERVED HETEROGENEITY

RECAP: OMITTED VARIABLE BIAS

- OLS yields biased effects if confounding variables are omitted
- Omitted variables ≙ *unobserved heterogeneity*
- So… How can we use panel data to estimate unbiased effects if there is correlated unobserved heterogeneity?

PANEL DATA MODEL

- Adding index for time: $y_{it} = \beta_0 + \beta_1 x_{1it} + \cdots + \beta_k x_{kit} + \varepsilon_{it}$
- Differentiating between time-constant and time-varying variables:
  - $y_{it} = \beta_0 + \beta_1 x_{1it} + \cdots + \beta_k x_{kit} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + e_{it}$
  - $i = 1, \dots, n$ units (e. g. persons)
  - $t = 1, \dots, T$ observations (e. g.: person-years)
  - k time-varying variables x
  - l time-constant variables z
- Decomposition of error term: $\varepsilon_{it} = u_i + e_{it}$

UNOBSERVED EFFECTS MODEL

- $y_{it} = \beta_0 + \underbrace{\beta_1 x_{1it} + \cdots + \beta_k x_{kit} + e_{it}} + \underbrace{\gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i}$

  - Time varying characteristics
  - Variables (x), for example: grades, attitudes, income, …
  - Idiosyncratic error (e_{it}): All sources of time-varying variance not captured by x, treated similar to error term in OLS

  - Time constant characteristics
  - Variables (z), for example: country of birth, date of graduation, … (?)
  - Unobserved heterogeneity (u_i): All time-constant sources of variation not captured by z

UNOBSERVED HETEROGENEITY: EXAMPLE

- True model: $death_{it} = \beta_0 + \beta_1 coffee_{it} + \beta_2 gender_i + u_i + e_{it}$
- Gender not observed: $death_{it} = \beta_0 + \beta_1 coffee_{it} + u_i + e_{it}$
- The error term is now correlated with the variables in the model (remember session ii)
- u_i includes $gender$, a confounding variable that correlates with $coffee$ and $death$ (in this example)

CORRELATED UNOBSERVED HETEROGENEITY

- Analogous to OLS , unobserved effects model yields biased estimates if error terms (u_i or e_{it}) correlate with variables in the model
- Solution: Control everything that is time-constant of each unit (here: person)
- $death_{it} = \beta_0 + \beta_1 coffee_{it} + u_i + e_{it}$
- u_i as something “typical” for person i
- Part of u_i might be observed, but other parts might not
- How can we control for such stable idiosyncrasies?

FIXED EFFECTS

LEAST SQUARE DUMMY VARIABLES

- Let say person i is a person which has been interviewed several times
- One solution: Control for individual i

→Add a dummy for individual i (1: interviews of individual i, 0: interviews of all other respondents)

- Because we observe every person multiple times, we could add dummies for *all persons* without exhausting degrees of freedom

LEAST SQUARE DUMMY VARIABLES

- Include a dummy variable for each person (not person-year!)
- $death_{it} = \beta_0 + \beta_1 coffee_{it} + \gamma_1 \delta_1 + \cdots + \gamma_n \delta_n + u_i + e_{it}$
- Such a model is called a Least Square Dummy Variables (LSDV) regression
- Model yields so-called *fixed effects estimates*
- *Fixed* Effects because each unit has a specific fixed effect on the dependent variable

FIXED EFFECTS-TRANSFORMATION

- Including dummy variables for each unit might not always be feasible
- Another way to obtain results: Fixed Effects-Transformation
- Instead of controlling u_i, we eliminate it from the regression function

MEANS

- $t = 1$: $death_{i1} = \beta_0 + \beta coffee_{i1} + u_i + e_{i1}$
- $t = 2$: $death_{i2} = \beta_0 + \beta coffee_{i2} + u_i + e_{i2}$

- Mean: $\overline{death}_{i.} = \beta_0 + \beta\overline{coffee}_{i.} + \bar{u}_i + \bar{e}_{i.}$

→ $\overline{death}_{i.} = \beta_0 + \beta\overline{coffee}_{i.} + u_i + \bar{e}_{i.}$

TIME-DEMEANING AT T=1

- $(t = 1) \; - mean:$
- $\left(death_{i1} - \overline{death}_{i.}\right)$

$$= (\beta_0 + \beta coffee_{i1} + u_i + e_{i1}) - \left(\beta_0 + \beta \overline{coffee}_{i.} + u_i + \bar{e}_{i.}\right)$$

$$= \beta(coffee_{i1} - \overline{coffee}_{i.}) + (u_i - u_i) + (\bar{e}_{i1} - \bar{e}_{i.})$$

$$= \beta(coffee_{i1} - \overline{coffee}_{i.}) + (\bar{e}_{i1} - \bar{e}_{i.})$$

TIME-DEMEANING AT T=2

- $(t = 2) \ - mean:$
- $\left(death_{i2} - \overline{death}_{i.}\right)$

$$= (\beta_0 + \beta coffee_{i2} + u_i + e_{i2}) - \left(\beta_0 + \beta\overline{coffee}_{i.} + u_i + \bar{e}_{i.}\right)$$

$$= \beta(coffee_{i2} - \overline{coffee}_{i.}) + (u_i - u_i) + (\bar{e}_{i2} - \bar{e}_{i.})$$

$$= \beta(coffee_{i2} - \overline{coffee}_{i.}) + (\bar{e}_{i2} - \bar{e}_{i.})$$

TIME-DEMEANING

- Time-demeaning of panel data
- $\left(death_{it} - \overline{death_{i.}}\right) = \beta(coffee_{it} - \overline{coffee_{i.}}) + (\bar{e}_{it} - \bar{e}_{i.})$
- All estimates are based on within-unit variation over time
- All between-unit variance (time stable difference between persons) is removed from the data

FIXED EFFECTS TRANSFORMATION

- $t = 1$: $\quad y_{i1} = \beta_0 + \beta_1 x_{1i1} + \cdots + \beta_k x_{ki1} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + e_{i1}$
- $t = 2$: $\quad y_{i2} = \beta_0 + \beta_1 x_{1i2} + \cdots + \beta_k x_{ki2} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + e_{i2}$
- $t = T$: $\quad y_{iT} = \beta_0 + \beta_1 x_{1iT} + \cdots + \beta_k x_{kiT} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + e_{iT}$
- Mean:

$$\bar{y}_{i.} = \beta_0 + \beta_1 \bar{x}_{1i.} + \cdots + \beta_k \bar{x}_{ki.} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + \bar{e}_{i.}$$

FIXED EFFECTS TRANSFORMATION

- Mean: $\bar{y}_{i.} = \beta_0 + \beta_1 \bar{x}_{1i.} + \cdots + \beta_k \bar{x}_{ki.} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + \bar{e}_{i.}$
- $t - mean$: $(y_{it} - \bar{y}_{i.}) =$
  $(\beta_0 + \beta_1 x_{1it} + \cdots + \beta_k x_{kit} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + e_{it}) -$
  $(\beta_0 + \beta_1 \bar{x}_{1i.} + \cdots + \beta_k \bar{x}_{ki.} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + \bar{e}_{i.})$

$\rightarrow (y_{it} - \bar{y}_{i.}) = \beta_1 (x_{1it} - \bar{x}_{1i.}) + \cdots + \beta_k (x_{kit} - \bar{x}_{ki.}) + \gamma_1 (z_{1i} - z_{1i}) + \cdots + \gamma_l (z_{li} - z_{li}) + (u_i - u_i) + (e_{it} - \bar{e}_{i.})$

$\rightarrow (y_{it} - \bar{y}_{i.}) = \beta_1 (x_{1it} - \bar{x}_{1i.}) + \cdots + \beta_k (x_{kit} - \bar{x}_{ki.}) + (e_{it} - \bar{e}_{i.})$

$\rightarrow \ddot{y}_{it} = \beta_1 \ddot{x}_{1it} + \cdots + \beta_k \ddot{x}_{kit} + \ddot{e}_{it}$

RECAP: TRANSFORMING THE DATA

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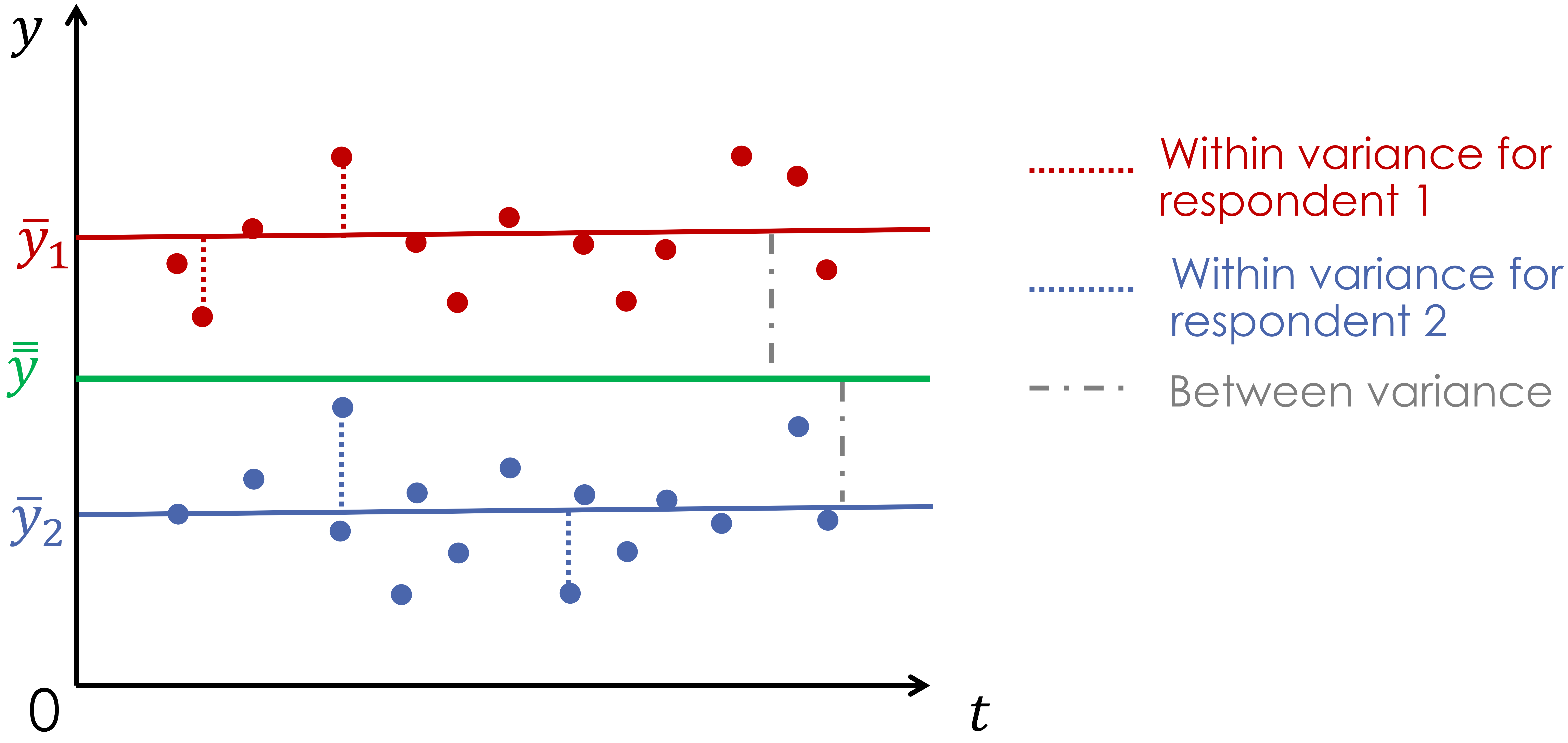
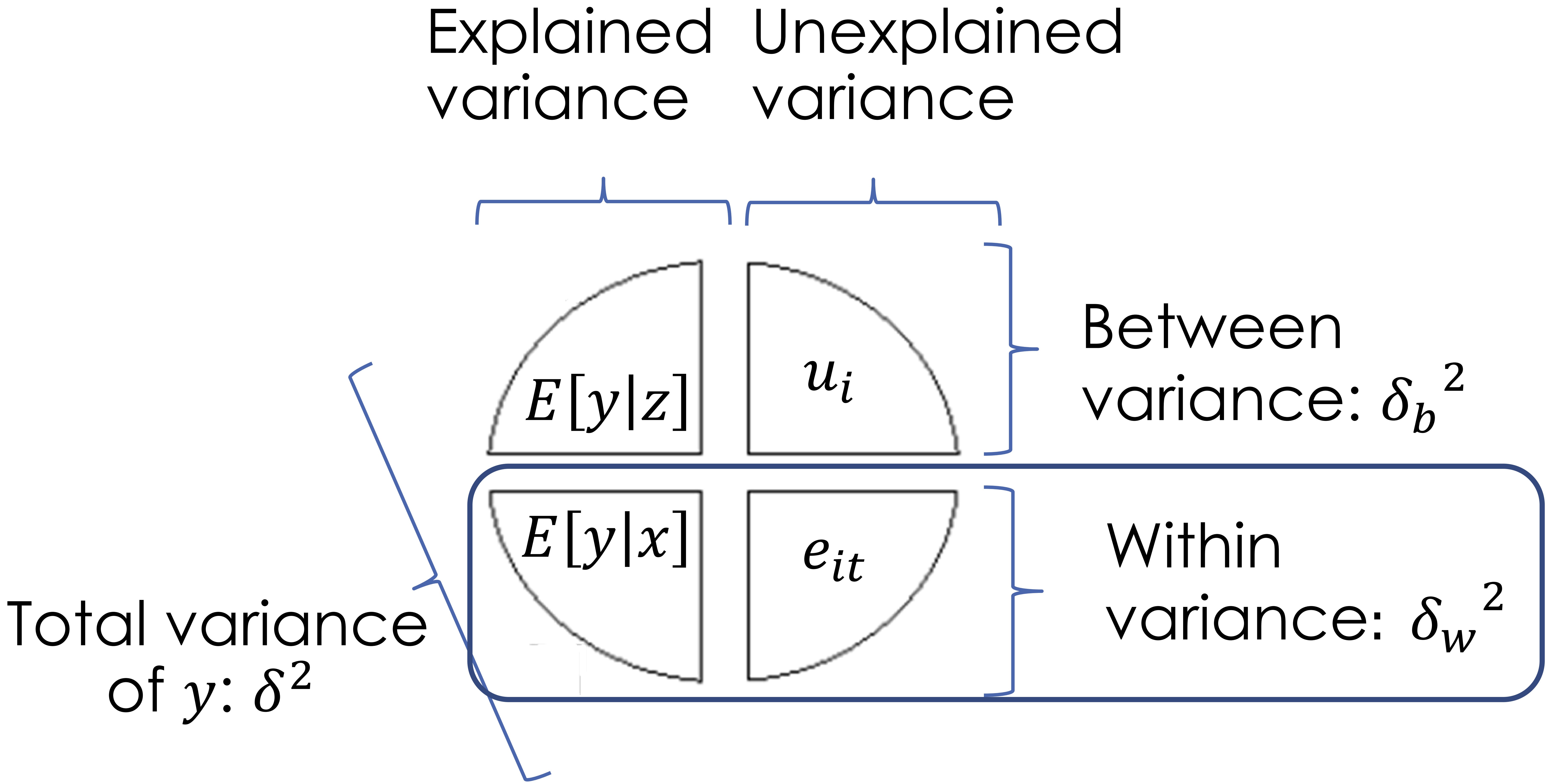
RECAP: WITHIN AND BETWEEN VARIANCE



COMPOSITION OF y



WHY DO DEMEANED FE AND LSDV YIELD THE SAME EFFECTS?

- Both are practically linear models using the OLS estimator
- Including dummies for units partials out the effects of individuals
- What is left is independent of all differences between individuals or in other words: rid of between variance
- Dummy variables capture all (also unmeasured) time-constant characteristics of individuals
- Thus, you also get the FE estimates when you control for the unit-specific means of each variable

WHY DO FE AND LSDV *NOT* YIELD THE SAME STANDARD ERRORS?

- *N* seems to be the same (data points)
- But time-demeaning actually costs degrees of freedom because it uses information from the data (the unit-specific means)
- … Or LSDV: each dummy costs one degree of freedom
- Running a linear model with manually demeaned data does not account for this
- Hence, significance tests need to be corrected manually
- If they are not, OLS with manually demeaned variables yields underestimated standard errors

MODELLING TIME TRENDS IN FE

ONE- VS. TWO-WAY FE

- Person FE: Average change in y if x increases by one unit *over time*
- Time FE: Average change in y if x increases by one unit *between cases*
- Two-way FE: Average difference in within-person changes in y at time point t for each one unit increase in x at t, averaged over all t

→"*two-way FE model unhelpfully combines within-unit and cross-sectional variation in a way that produces un-interpretable answers.*" (Kropko & Kubinec 2020: 1)

FIXED EFFECTS INDIVIDUAL SLOPES

- FE assume parallel trends between treated and untreated
- I.e.: Both groups would follow the same over-time trend in y if x wouldn't change
- For example: Does marriage increase hourly wage for men? → Men who eventually get married show steeper wage growth even before marriage
- See Rüttenauer & Ludwig (2020)

LIMITS OF FIXED EFFECTS

See Hill, Davis, Roos & French (2020). Limitations of fixed-effects models for panel data. *Sociological Perspectives* 63 (3) 357 - 369.

LOW STATISTICAL POWER

- Observations without temporal variation do not contribute to FE estimator by design

→Reduced sample size

→Low statistical power (high standard errors)

- Observations with little temporal variation contribute little to FE estimator

→Coefficients are based on small number of observations

→Limited reliability ("Silly estimators", Beck & Katz 2001: 494)

→Increased Type II error rate (false negative)

- *Statistically significant FE estimate likely robust, but non-significant FE may be due to low power*

EXTERNAL VALIDITY

- Observations with little temporal variation contribute little to FE estimator

→Model of temporal changes only apply to a specific subgroup of observations

→Subgroup might differ from broader population (i.e. sample might no longer be representative)

→P-values might have less statistical meaning

- FE are treatment effects on treated (only units with change are observed), OLS (theoretically) are average treatment effects

OTHER ISSUES OF FE

- Estimates less reliable with less time periods
- Repeated measurement error (overly conservative estimates)
- FE useless for estimating time stable differences
- Unclear which variables are time-stable or varying
- FE only control *time-stable* effects of time-stable variables

LIMITS TO CAUSAL INFERENCE

1. Time-varying confounders (erogeneity assumption)
2. Reverse causality (y affecting x)
3. Lagged effects (past x affecting current y)

- All would be solved by including all time-varying confounders, but how realistic is that?

SUMMING UP

SUMMARY

- FE eliminate any between-unit variance from the data

→Estimates only based on within-unit variation

→Automatically control for unobserved heterogeneity (everything time-constant)

- … Which is a huge step forward for estimating unbiased effects in many cases

SUMMARY

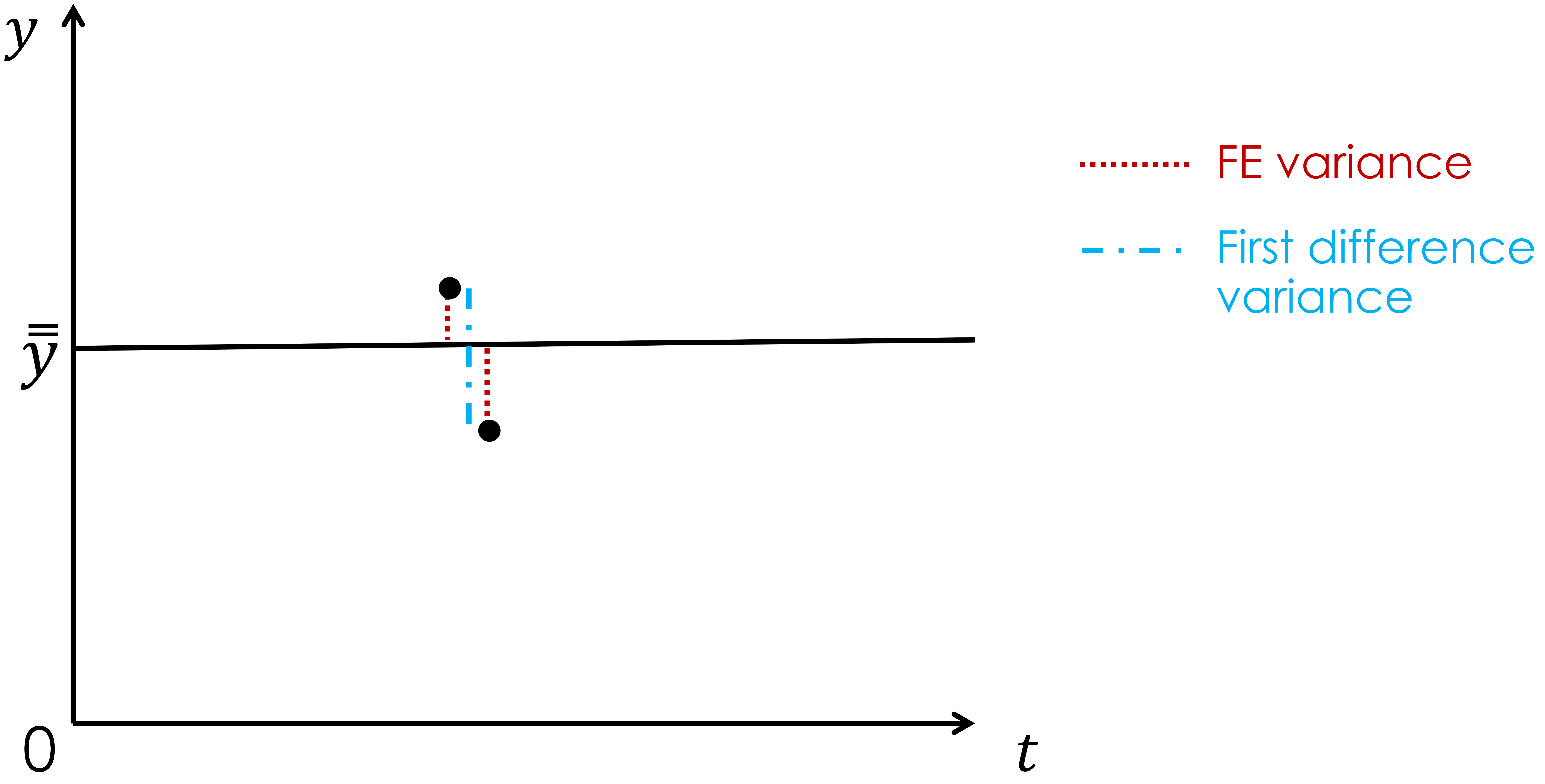
- However, time-constant variables drop out (effects of constants cannot be estimated, just like OLS)
- … But interactions between time-constant and time-varying variables can still be estimated → Does the effect of x depend on z?
- Often more crucial: Many aspects might not be totally constant but empirically vary only little over time
- FE "*may kill some of the omitted variables bias bathwater, but they also remove much of the useful information in the baby, the variable of interest.*" (Angriest & Pischke 2009: 225)

FIRST DIFFERENCE

FIRST DIFFERENCE ESTIMATION

- Depended variable is the change in y compared to the time before
- … which is explained by changes in x compared to the time before
- For $t = 2$ this yields the same results as FE
- The sum of the deviation of two data points from their mean equals the difference between those data points
- When $t > 2$, results will differ

FE AND FIRST DIFFERENCE WITH T=2



FIRST DIFFERENCE ESTIMATION

- t: $y_{it} = \beta_0 + \beta_1 x_{1it} + \cdots + \beta_k x_{kit} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + e_{it}$
- $t-1$: $y_{it-1} = \beta_0 + \beta_1 x_{1it-1} + \cdots + \beta_k x_{kit-1} + \gamma_1 z_{1i} + \cdots + \gamma_l z_{li} + u_i + e_{it-1}$

- Difference:

$(y_{it} - y_{it-1}) = \beta_1 (x_{1it} - x_{1it-1}) + \cdots + \beta_k (x_{kit} - x_{kit-1}) + (e_{it} - e_{it-1})$

$\rightarrow \Delta y_{it} = \beta_1 \Delta x_{1it} + \cdots + \beta_k \Delta x_{kit} + \Delta e_{it}$

FIXED EFFECTS VS. FIRST DIFFERENCE

- FE model deviations from the unit-specific mean at each time point, but otherwise ignore the temporal aspect
- That means for FE it does not matter *when* a particular value was observed
- FD, on the other hand, model changes in y between two consecutive time points
- Hence, the temporal order is important for FD

FIXED EFFECTS VS. FIRST DIFFERENCE

- FE and FD both eliminate between-unit variance (control for unobserved heterogeneity)
- And both cannot estimate effects of time-constant variables
- FD automatically control general time trend, FE do not (but can be added to model by including dummies for time-points)
- FD is based on fewer observations because data point at t drops out when there is no observation at $t - 1$

FIXED EFFECTS VS. FIRST DIFFERENCE

ID	t	y_{it}	x_{it}	$\bar{y}_{i.}$	$\bar{\boldsymbol{x}}_{\boldsymbol{i}.}$
1	1	6	0	8	3
1	2	8	3	8	3
1	3	6	2	8	3
1	4	10	5	8	3
1	5	10	5	8	3

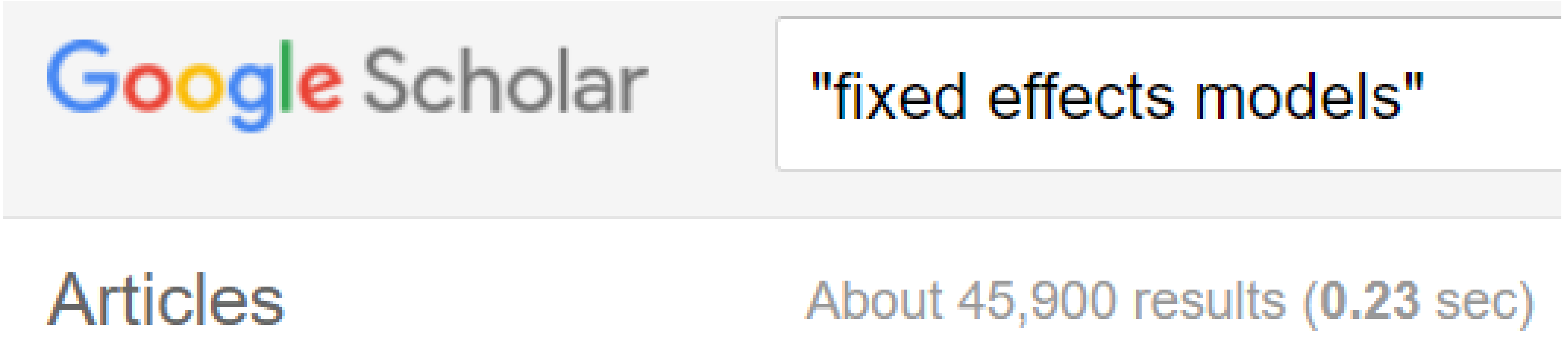
FIXED EFFECTS VS. FIRST DIFFERENCE

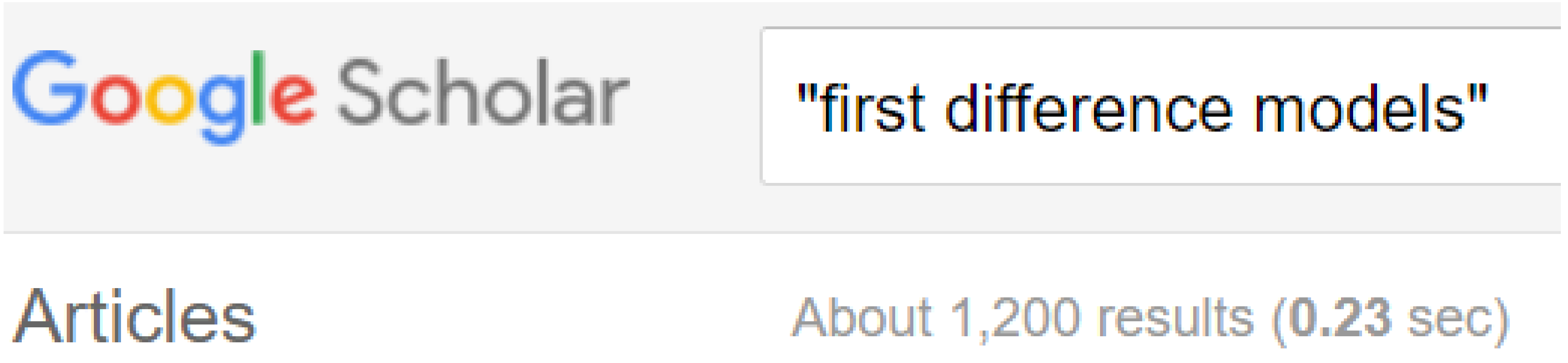
ID	t	y_{it}	x_{it}	$\bar{y}_{i.}$	$\bar{\boldsymbol{x}}_{\boldsymbol{i}.}$	$y_{it} - \bar{y}_{i.}$	$x_{it} - \bar{\boldsymbol{x}}_{\boldsymbol{i}.}$
1	1	6	0	8	3	-2	-3
1	2	8	3	8	3	0	0
1	3	6	2	8	3	-2	-1
1	4	10	5	8	3	2	2
1	5	10	5	8	3	2	2

FIXED EFFECTS VS. FIRST DIFFERENCE

ID	t	y_{it}	x_{it}	$\bar{y}_{i.}$	$\bar{\boldsymbol{x}}_{\boldsymbol{i}.}$	$y_{it} - \bar{y}_{i.}$	$x_{it} - \bar{\boldsymbol{x}}_{\boldsymbol{i}.}$	$y_{it} - y_{it-\mathbf{1}}$	$x_{it} - x_{it-\mathbf{1}}$
1	1	6	0	8	3	-2	-3	.	.
1	2	8	3	8	3	0	0	2	3
1	3	6	2	8	3	-2	-1	-2	-1
1	4	10	5	8	3	2	2	4	2
1	5	10	5	8	3	2	2	0	0

THAT BEING SAID…





LITERATURE

- Chapter 4.1 (pages 126 ff.) in: Andreß, Golsch, & Schmidt (2014). Applied panel data analysis for economic and social surveys. Springer Science & Business Media
- Brüderl (2010). Kausalanalyse mit Paneldaten. Pages 963-994 in: Handbuch der sozialwissenschaftlichen Datenanalyse. VS Verlag für Sozialwissenschaften
- Study applying Fixed Effects: Czymara & Dochow (2018). Mass media and concerns about immigration in Germany in the 21st century: individual-level evidence over 15 years. European Sociological Review, 34(4), 381-401