Exercises

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Triangle exercise

1. ABC and AMP are two right triangles, right angled at B and M respectively. M lies on AC and AB is extended to meet P. Prove that:
   1.1 $\triangle ABC \sim \triangle AMP$
   1.2 $\frac{CA}{PA} = \frac{BC}{MP}$

Solution:

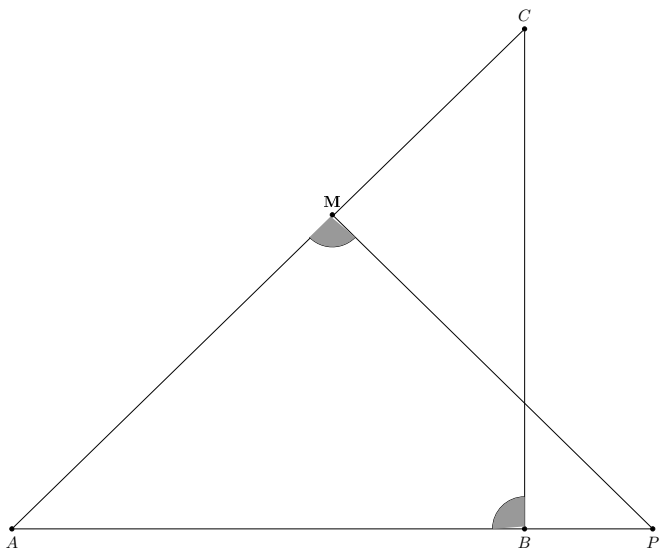

Figure 0-1: right angled triangles

From the above figure

$$\angle CAB = \angle MAP \tag{1}$$
$$\angle ABC = \angle AMP \tag{2}$$

From 1 and 2

$$\triangle ABC \sim \triangle AMP \tag{3}$$

- As correspondinng sides are proportional $\frac{CA}{PA} = \frac{BC}{MP} = \frac{AB}{AM}$

$$\frac{CA}{PA} = \frac{BC}{MP}$$

Triangle construction

2. In $\triangle ABC$,a=8,$\angle B = 45°$ and c-b=3.5. Sketch $\triangle ABC$

Solution:

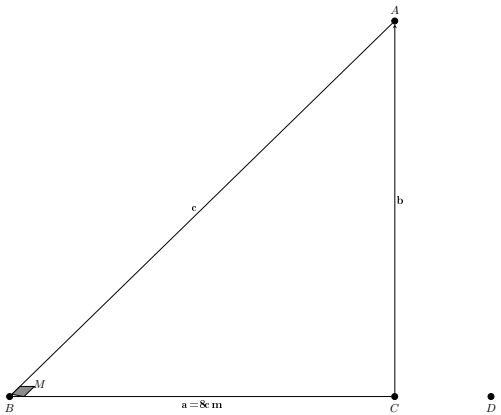

Figure 0-2: Triangle

Apply cosine rule

$$cos45° = \frac{8}{3.5 + c} \implies c = 7.813, b = 4.313$$

Python code for Figure 0-2: https://github.com/d-DP/Codes

Quadrilateral exercise

3. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Solution:

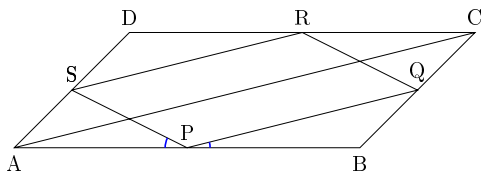

Figure 0-3: Rhombus

From $\triangle ABC$ and $\triangle ADC$

$$PQ||AC \text{ and } PQ = \frac{1}{2}AC \tag{4}$$

$$RS||AC \text{ and } RS = \frac{1}{2}AC \tag{5}$$

from 4 and 5 PQ=RS , PQ||RS

$$As \;\; PB = PQ, \angle 2 = \angle 1 \tag{6}$$

From $\triangle APS$ and $\triangle CQR$

- AP=CQ,AS=CR, PS=QR
- From SSS rule $\triangle APS \cong \triangle CQR$

$$\angle 3 = \angle 4 \tag{7}$$

For AB, BC

$$\angle 3 + \angle SPQ + \angle 1 = 180^\circ \tag{8}$$

$$\angle 2 + \angle PQR + \angle 4 = 180^\circ$$

from 6 and 7

$$\angle 1 + \angle PQR + \angle 3 = 180^\circ \tag{9}$$

$$\text{PS}\| \text{ PR } \angle SPQ + \angle PQR = 180^\circ \implies \angle SPQ = 90^\circ$$

Quadrilateral Construction

4. construct a quadrilateral MIST where MI=3.5, IS = 6.5, $\angle M = 75°, \angle I = 105°$and $\angle S = 120°$

solution:

5. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$

Solution:

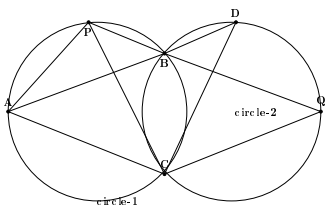

Figure 0-4: Circle

Solution:

From the above figure

$$\angle PBA = \angle ACP \tag{10}$$

$$\angle DBQ = \angle QCD \tag{11}$$

$$\angle PBA = \angle DBQ \tag{12}$$

6. Draw a circle with centre B and radius 6. If C be a point 10 units away from its centre,construct the pair of tangents AC and CD to the circle.

Solution: Taking M as centre and MB as radius, draw a circle.

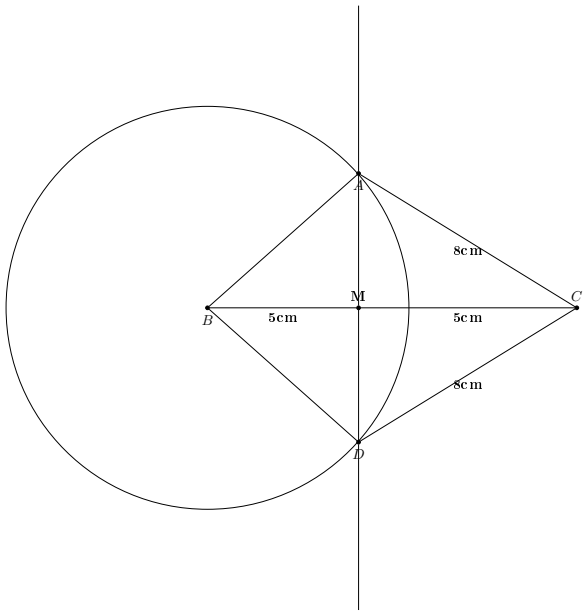

Figure 0-5: Circle

python code : https://github.com/d-DP/Codes

7. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution:

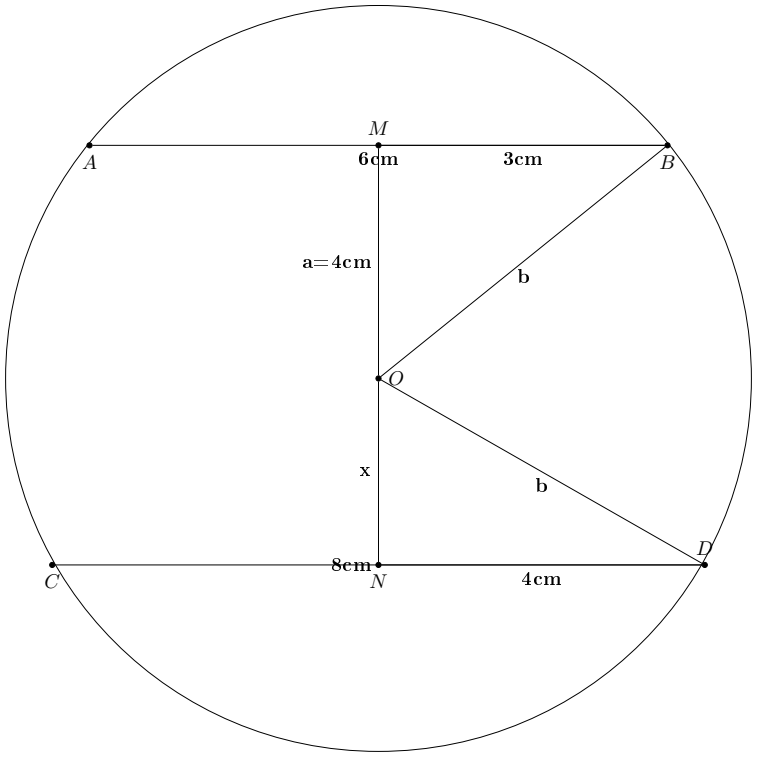

Figure 0-6: Circle

Apply Baudhayana theorem for $\triangle MOB$ and $\triangle NOD$

$$a^2 + b^2 = (3)^2$$
$$x^2 + b^2 = (4)^2$$