# Exercises

Durga Prasad

22 December 2019

# Triangle exercise

1. ABC and AMP are two right triangles, right angled at B and M respectively. M lies on AC and AB is extended to meet P. Prove that:
   1.1 $\triangle ABC \sim \triangle AMP$
   1.2 $\frac{CA}{PA} = \frac{BC}{MP}$

**Solution:**

- Consturct a right triangle ABC with a=4, c=3
- Direction vector m= A-C
- Representation of AC $n^T(x - A) = 0$ and PM $m^T(x - P) = 0$

$$\begin{pmatrix} m & n \end{pmatrix}^T M = \begin{pmatrix} m^T P \\ n^T A \end{pmatrix} \tag{1}$$

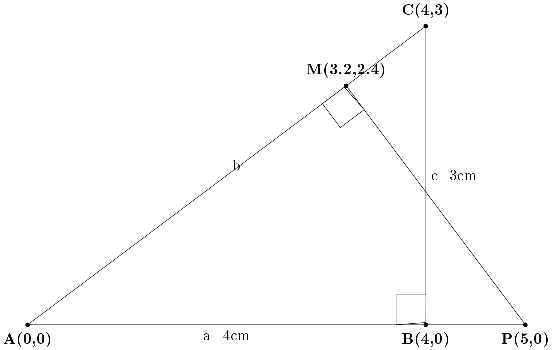


Figure 0-1: tikz figure

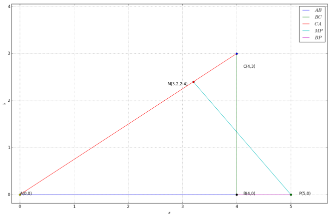


Figure 0-2: Python generated figure

- **Python:** `https://github.com/d-DP/geometryy/blob/master/codes/triangle/1.tri_exe.py`
- **Tkz:** `https://github.com/d-DP/geometryy/blob/master/figs/triangle/1.triangle_exercise_fig.tex`

From the above figure

$$\angle CAB = \angle MAP \tag{2}$$

$$\angle ABC = \angle AMP \tag{3}$$

From 1 and 2

$$\triangle ABC \sim \triangle AMP \tag{4}$$

- As correspondinng sides are proportional $\frac{CA}{PA} = \frac{BC}{MP} = \frac{AB}{AM}$

  $\frac{CA}{PA} = \frac{BC}{MP}$

# Triangle construction

2. In $\triangle ABC$ ,a=8,$\angle B = 45°$ and c-b=3.5. Sketch $\triangle ABC$

**Solution:**

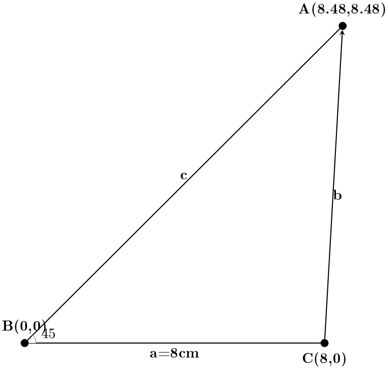


Figure 0-3: tikz figure

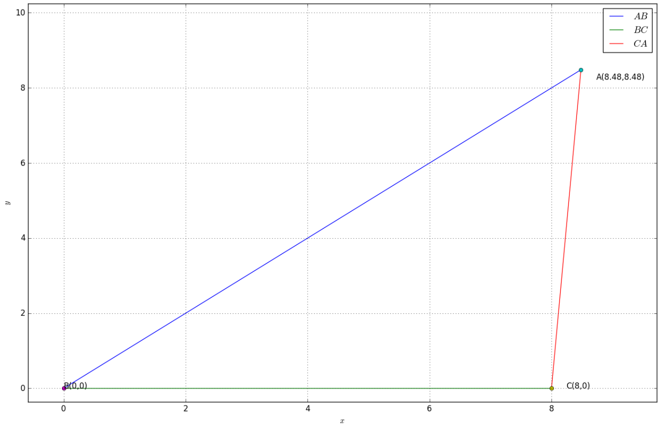


Figure 0-4: Python generated figure

Given a=8cm, c-b=k (k=3.5cm) Apply cosine rule

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos(B) = \frac{a^2 + (b+k)^2 - b^2}{2a(b+k)}$$
$$2ab\cos B + 2ak\cos B = a^2 + k + 2bk$$
$$b = \frac{a^2 + k^2 - 2ak\cos B}{2a\cos B - 2k}$$

b=8.49, c=11.99

- **tikz code:**`https://github.com/d-DP/geometryy/blob/master/figs/triangle/2.triangle_construction_fig.tex`
- **Python code:**`https://github.com/d-DP/geometryy/blob/master/codes/triangle/2.tri_constr.py`

3. Draw a circle with centre B and radius 6. If C be a point 10 units away from its centre,construct the pair of tangents AC and CD to the circle.

**Solution:**

- Draw a circle with radius r=6cm centre B.
- Apply Baudhayana theorem ABC and BCD to get AC,DC distance and find A and D coordinates.

$$B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} C = \begin{pmatrix} 10 \\ 0 \end{pmatrix} A = \begin{pmatrix} 4.5 \\ 3.96 \end{pmatrix} D = \begin{pmatrix} 4.5 \\ -3.96 \end{pmatrix}$$

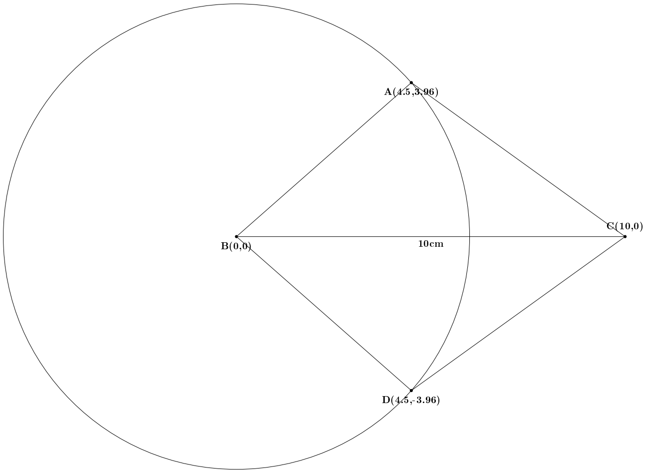


Figure 0-5: Circle with tikz

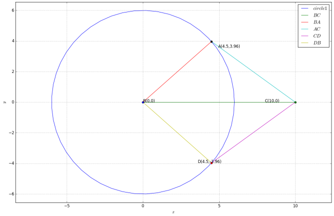


Figure 0-6: Circle with python

- **python code** :https://github.com/d-DP/geometryy/blob/master/codes/circles/circle_constr.py
- **tikz** :https://github.com/d-DP/geometryy/blob/master/figs/circles/circle_constr.tex

4. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

**Solution:**

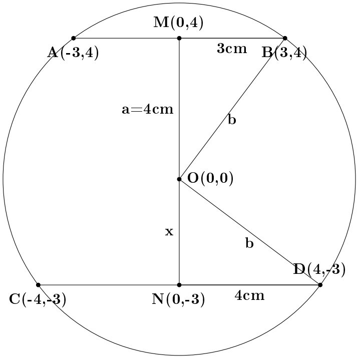


Figure 0-7: tikz figure

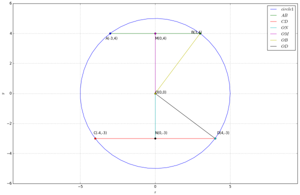


Figure 0-8: Python generated figure

Apply Baudhayana theorem for $\triangle MOB$ and $\triangle NOD$

$$a^2 + (3)^2 = (b)^2$$
$$x^2 + (4)^2 = b^2$$

- **python:** `https://github.com/d-DP/geometryy/blob/master/codes/misc.py`
- **tikz:** `https://github.com/d-DP/geometryy/blob/master/figs/misc.tex`

# Quadrilateral construction

5. construct a quadrilateral MIST where MI =3.5, IS = 6.5, $\angle M = 75°, \angle I = 105°$ and $\angle s = 120°$

**Solution:**

- Given that MI=8 and IS=6.5 Apply cosine law for $\triangle MIS$

$$c = \sqrt{a^2 + b^2 - 2ab \times \cos \angle M}$$

- Find S coordinates
  - Directon vector for ST is

$$m1 = \begin{pmatrix} 1 \\ \tan 135° \end{pmatrix}$$

  - Normal vector for MT is

$$n_1^T T = n_1^T S \tag{5}$$

  - Direction vector for ST=MT,m2=S-I
  - Normal vector

$$n_2^T T = 0 \tag{6}$$

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}^T T = \begin{pmatrix} n_1^T S \\ n_2^T M \end{pmatrix} \tag{7}$$

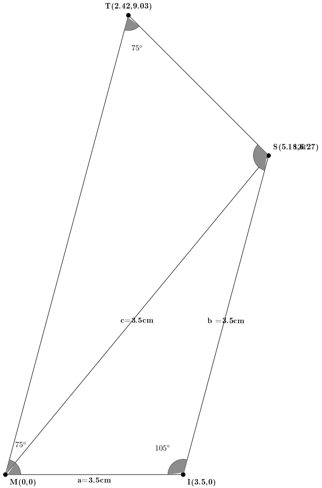


Figure 0-9: Quadrilateral with tikz

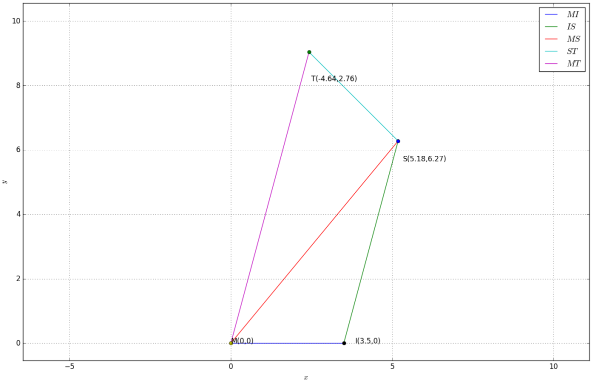


Figure 0-10: Quadrilateral with python

- **python** :https://github.com/d-DP/geometryy/blob/master/codes/quad/quad_constr1.py
- **tikz:** https://github.com/d-DP/geometryy/blob/master/figs/quad/quad_constr.tex

# Circle Exercise

6. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that $\angle ACP = \angle QCD$

**Solution:**

$$||x||^2 = r^2$$

$$P = B + \lambda m$$

$$x = B + \lambda m$$

$$||B + \lambda m||^2 = r^2$$

$$(B + \lambda m)^T (B + \lambda m) = r^2$$

$$\lambda^2 ||m||^2 + 2\lambda m^T B = r^2$$

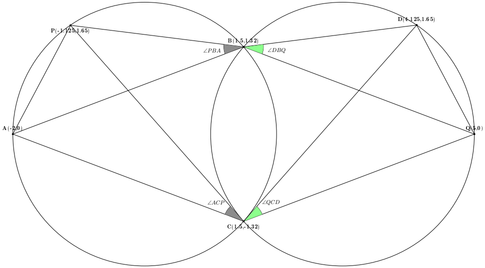


Figure 0-11: Circle tikz

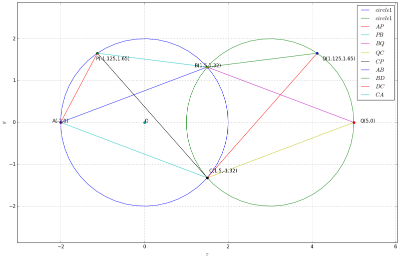


Figure 0-12: Python figure

From the above figure

$$\angle PBA = \angle ACP \tag{8}$$

$$\angle DBQ = \angle QCD \tag{9}$$

$$\angle PBA = \angle DBQ \tag{10}$$

from 10,11,12 $\angle ACP = \angle QCD$

- **python code :** `https://github.com/d-DP/geometryy/blob/master/codes/circles/3.py`
- **tikz:**`https://github.com/d-DP/geometryy/blob/master/figs/quad/quad_exer.tex`

# Quadrilateral exercise

7. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

**Solution:**

- Construct a rhombus wih following coordinates

$$A = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, B = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, C = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, D = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$$

- Find P,Q,R,S
- $P = \frac{(A+D)}{2}, Q = \frac{(D+C)}{2}, R = \frac{(C+B)}{2} and S = \frac{(A+B)}{2}$

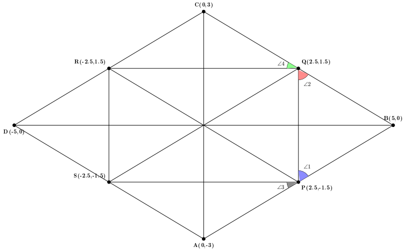


Figure 0-13: tikz figure

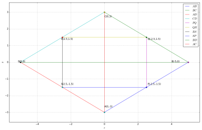


Figure 0-14: Rhombus

From $\triangle ABC$ and $\triangle ADC$

$$PQ||AC \text{ and } PQ = \frac{1}{2}AC \tag{11}$$

$$RS||AC \text{ and } RS = \frac{1}{2}AC \tag{12}$$

from 11 and 12 PQ=RS , PQ||RS

$$As \;\; PB = PQ, \angle 2 = \angle 1 \tag{13}$$

From $\triangle APS$ and $\triangle CQR$

- AP=CQ,AS=CR, PS=QR
- From SSS rule $\triangle APS \cong \triangle CQR$

$$\angle 3 = \angle 4 \tag{14}$$

For AB, BC

$$\angle 3 + \angle SPQ + \angle 1 = 180^\circ \tag{15}$$

$$\angle 2 + \angle PQR + \angle 4 = 180^\circ$$

from 13 and 15

$$\angle 1 + \angle PQR + \angle 3 = 180^\circ \tag{16}$$

$$\text{PS}||\ \text{PR}\ \angle SPQ + \angle PQR = 180^\circ \implies \angle SPQ = 90^\circ$$

tikz : `https://github.com/d-DP/geometryy/blob/master/figs/quad/quad_exer.tex`
python : `https://github.com/d-DP/geometryy/blob/master/codes/quad/rhombus.py`