

# Exercises

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22 December 2019

# Triangle exercise

1.  $\triangle ABC$  and  $\triangle AMP$  are two right triangles, right angled at  $B$  and  $M$  respectively.  $M$  lies on  $AC$  and  $AB$  is extended to meet  $P$ . Prove that:
  - 1.1  $\triangle ABC \sim \triangle AMP$
  - 1.2  $\frac{CA}{PA} = \frac{BC}{MP}$

**Solution:**

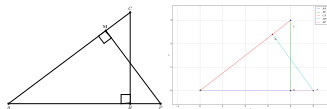


Figure 0-1: right angled triangles

From the above figure

$$\angle CAB = \angle MAP \quad (1)$$

$$\angle ABC = \angle AMP \quad (2)$$

From 1 and 2

$$\triangle ABC \sim \triangle AMP \quad (3)$$

- ▶ As corresponding sides are proportional  $\frac{CA}{PA} = \frac{BC}{MP} = \frac{AB}{AM}$

$$\frac{CA}{PA} = \frac{BC}{MP}$$

- ▶ Github link for python: [https://github.com/d-DP/geometryy/blob/master/codes/triangle/1.tri\\_exe.py](https://github.com/d-DP/geometryy/blob/master/codes/triangle/1.tri_exe.py)
- ▶ github link for  
tikz:[https://github.com/d-DP/geometryy/blob/master/figs/triangle/1.triangle\\_exercise\\_fig.tex](https://github.com/d-DP/geometryy/blob/master/figs/triangle/1.triangle_exercise_fig.tex)

# Triangle construction

2. In  $\triangle ABC$ ,  $a=8$ ,  $\angle B = 45^\circ$  and  $c-b=3.5$ . Sketch  $\triangle ABC$

**Solution:**

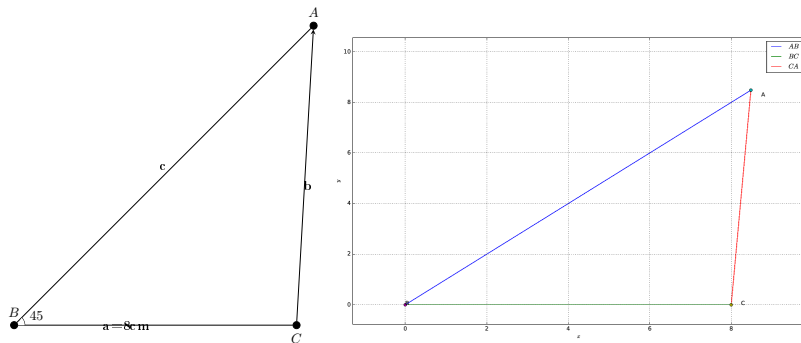


Figure 0-2: Triangle with tikz and python

Given  $a=8\text{cm}$ ,  $c-b=k$  ( $k=3.5\text{cm}$ ) Apply cosine rule

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(B) = \frac{a^2 + (b+k)^2 - b^2}{2a(b+k)}$$

$$2ab \cos B + 2ak \cos B = a^2 + k^2 + 2bk$$

$$b = \frac{a^2 + k^2 - 2ak \cos B}{2a \cos B - 2k}$$

$$b=8.49, c=11.99$$

- tikz code for above figure:

[https://github.com/d-DP/geometryy/blob/master/figs/triangle/2.triangle\\_construction\\_fig.tex](https://github.com/d-DP/geometryy/blob/master/figs/triangle/2.triangle_construction_fig.tex)

- Python code for Figure

0-2:[https://github.com/d-DP/geometryy/blob/master/codes/triangle/2.tri\\_constr.py](https://github.com/d-DP/geometryy/blob/master/codes/triangle/2.tri_constr.py)

## Quadrilateral exercise

3. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

**Solution:**

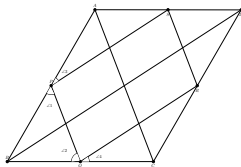


Figure 0-3: Rhombus

From  $\triangle ABC$  and  $\triangle ADC$

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \quad (4)$$

$$RS \parallel AC \text{ and } RS = \frac{1}{2}AC \quad (5)$$

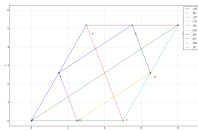


Figure 0-4: Rhombus

from 4 and 5  $PQ=RS$  ,  $PQ \parallel RS$

$$As \ PB = PQ, \angle 2 = \angle 1 \quad (6)$$

From  $\triangle APS$  and  $\triangle CQR$

- ▶  $AP=CQ, AS=CR, PS=QR$
- ▶ From SSS rule  $\triangle APS \cong \triangle CQR$

$$\angle 3 = \angle 4 \quad (7)$$

For AB, BC

$$\angle 3 + \angle SPQ + \angle 1 = 180^\circ \quad (8)$$

$$\angle 2 + \angle PQR + \angle 4 = 180^\circ$$

from 6 and 7

$$\angle 1 + \angle PQR + \angle 3 = 180^\circ \quad (9)$$

$$PS \parallel PR \quad \angle SPQ + \angle PQR = 180^\circ \implies \angle SPQ = 90^\circ$$

tikz : [https://github.com/d-DP/geometryy/blob/master/figs/quad/quad\\_exer.tex](https://github.com/d-DP/geometryy/blob/master/figs/quad/quad_exer.tex)

python : <https://github.com/d-DP/geometryy/blob/master/codes/quad/rhombus.py>



# Circle Exercise

4. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that  $\angle ACP = \angle QCD$

**Solution:**

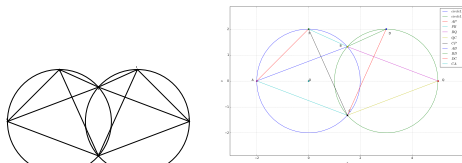


Figure 0-5: Circle

From the above figure

$$\angle PBA = \angle ACP \quad (10)$$

$$\angle DBQ = \angle QCD \quad (11)$$

$$\angle PBA = \angle DBQ \quad (12)$$

from 10,11,12  $\angle ACP = \angle QCD$

python code : <https://github.com/d-DP/geometryy/blob/master/codes/circles/3.py>

tikz: [https://github.com/d-DP/geometryy/blob/master/figs/quad/quad\\_exer.tex](https://github.com/d-DP/geometryy/blob/master/figs/quad/quad_exer.tex)

5. Draw a circle with centre B and radius 6. If C be a point 10 units away from its centre, construct the pair of tangents AC and CD to the circle.

**Solution:**

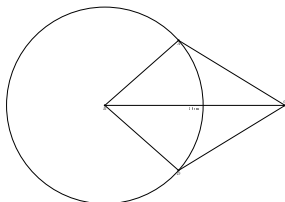


Figure 0-6: Circle with tikz



## Miscellaneous

6. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

**Solution:**

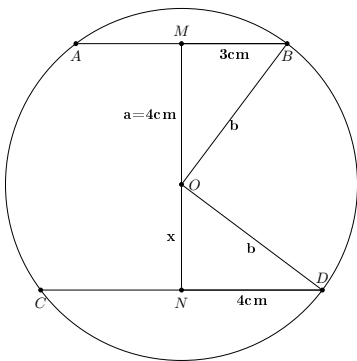


Figure 0-8: Circle

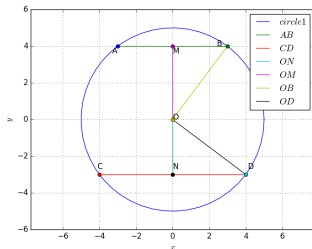


Figure 0-9: Circle

Apply Baudhayana theorem for  $\triangle MOB$  and  $\triangle NOD$

$$a^2 + (3)^2 = (b)^2$$

$$x^2 + (4)^2 = b^2$$

python : <https://github.com/d-DP/geometryy/blob/master/codes/misc.py>

tikz: <https://github.com/d-DP/geometryy/blob/master/figs/misc.tex>

tikz: <https://github.com/d-DP/geometryy/blob/master/figs/misc.tex>

tikz: <https://github.com/d-DP/geometryy/blob/master/figs/misc.tex>

# Quadrilateral construction

7. construct a quadrilateral MIST where  $MI = 8$ . If two arcs of a circle are congruent, then their 3.5,  $IS = 6.5$ ,  $\angle M = 75^\circ$ ,  $\angle I = 105^\circ$  and  $\angle s = 120^\circ$

**Solution:**

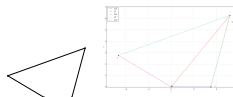


Figure 0-10: Rhombus

python : [https://github.com/d-DP/geometryy/blob/master/codes/quad/quad\\_constr1.py](https://github.com/d-DP/geometryy/blob/master/codes/quad/quad_constr1.py)  
tikz: [https://github.com/d-DP/geometryy/blob/master/figs/quad/quad\\_constr.tex](https://github.com/d-DP/geometryy/blob/master/figs/quad/quad_constr.tex)