Coordinate Geometry

1. **Problem statement :** Find the area of a rhombus of its vertices are $\left(3,0\right)$, $\left(4,5\right)$, $\left(-1,4\right)$ and $\left(-2,-1\right)$taken in order

   Solution:

   The input vertices for this problem are given as

$$\mathbf{A} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{1}$$

   Area of a rhombus $= \frac{1}{2}$ (product of its diagonals)

   then the length of the diagonal **B** and **D** at opposite vertices are obtained by

$$\left\|\mathbf{B}-\mathbf{D}\right\|^2 = \left(\mathbf{B}-\mathbf{D}\right)^\top \left(\mathbf{B}-\mathbf{D}\right) \tag{2}$$

$$\mathbf{B}-\mathbf{D} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \tag{3}$$

$$\left(\mathbf{B}-\mathbf{D}\right)^\top = \left(6, 6\right) \tag{4}$$

$$\left\|\mathbf{B}-\mathbf{D}\right\|^2 = \sqrt{72} = 8.482 \tag{5}$$

   the length of the another diagonal **A** and **C**are can be obtained which can be simplified to obtained by

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = \left(\mathbf{A}-\mathbf{C}\right)^\top \left(\mathbf{A}-\mathbf{C}\right) \tag{6}$$

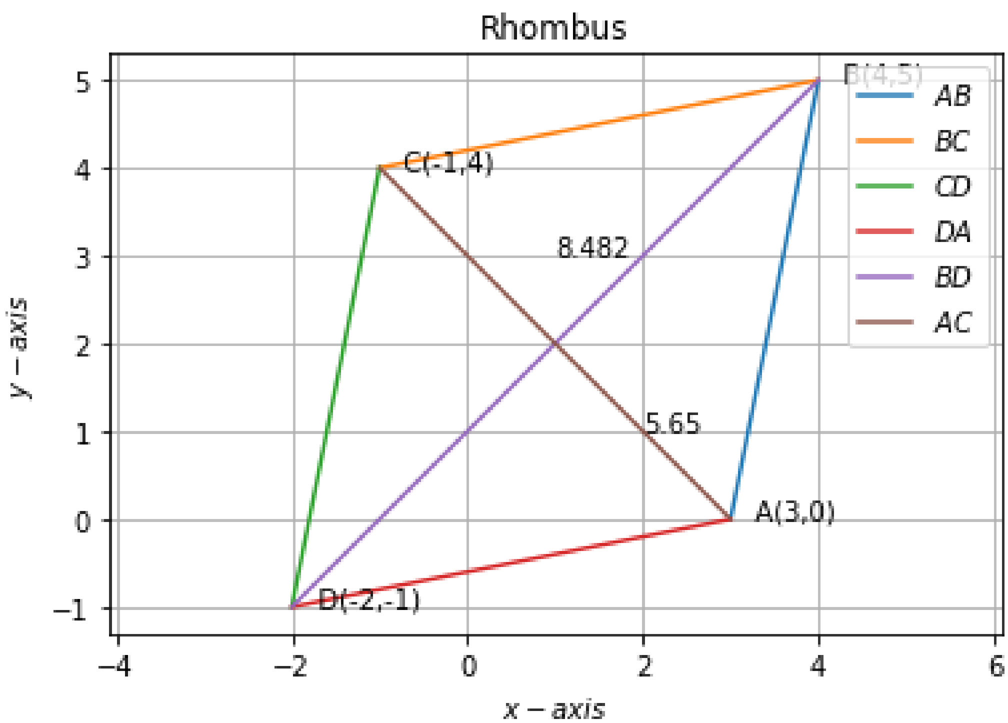
$$\mathbf{A}-\mathbf{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{7}$$

$$\tag{8}$$

$$\left(\mathbf{A}-\mathbf{C}\right)^\top = \left(4, -4\right) \tag{9}$$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = \sqrt{32} = 5.65 \tag{10}$$

$$\text{Area of a rhombus} = 23.96$$



Rhombus
B(4,5)
C(-1,4)
8.482
5.65
A(3,0)
D(-2,-1)
AB
BC
CD
DA
BD
AC
5
4
3
2
1
0
−1
−4
−2
0
2
4
6
x − axis
y − axis

Figure 1