

Coordinate Geometry

1. **Problem statement :** Find the area of a rhombus of its vertices are $(3, 0)$, $(4, 5)$, $(-1, 4)$ and $(-2, -1)$ taken in order

Solution:

The input vertices for this problem are given as

$$\mathbf{A} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \quad (1)$$

Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)

for the given vertices of the rhombus, we can find the vectors \mathbf{B} and \mathbf{D} formed by its diagonals as follows:

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \quad (2)$$

the length of the another diagonal \mathbf{A} and \mathbf{C} are can be obtained which can be simplified to obtained by

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \quad (3)$$

To find the area of the rhombus, we can then take the cross product of these two diagonal vectors are given by

$$\|\mathbf{a} \times \mathbf{b}\| = \left\| \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right\| = |a_1 b_2 - a_2 b_1| \quad (4)$$

By using above formula

$$\|(\mathbf{B} - \mathbf{D}) \times (\mathbf{A} - \mathbf{C})\| = \left\| \begin{pmatrix} 6 \\ 6 \end{pmatrix} \times \begin{pmatrix} 4 \\ -4 \end{pmatrix} \right\| = 48 \quad (5)$$

$$(6)$$

Area of rhombus = $\frac{1}{2}$ (Cross product of diagonals) = $\frac{48}{2} = 24$

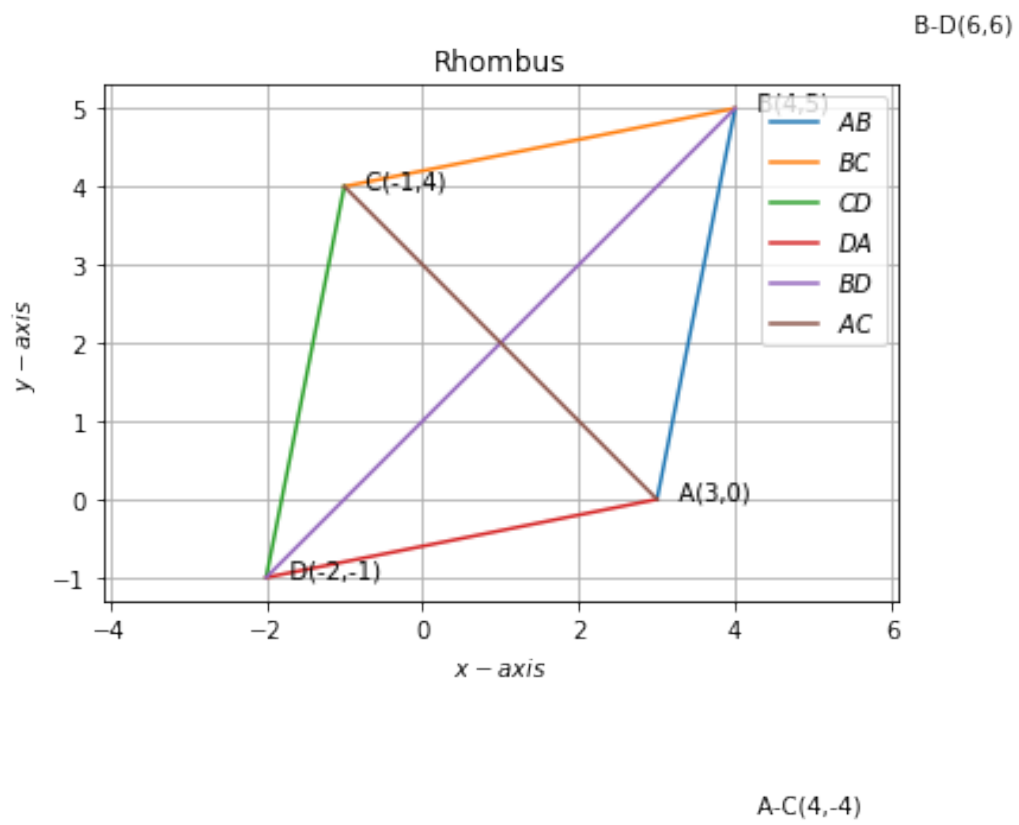


Figure 1