Coordinate Geometry

1. **Problem statement :** Find the area of a rhombus of its vertices are $\left(3,0\right)$, $\left(4,5\right)$, $\left(-1,4\right)$ and $\left(-2,-1\right)$taken in order

   Solution:
   The input vertices for this problem are given as

$$\mathbf{A} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{1}$$

   Area of a rhombus $= \frac{1}{2}$ (product of its diagonals) for the given vertices of the rhombus, we can find the vectors **B** and **D** formed by its diagonals as follows:

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \tag{2}$$

   the length of the another diagonal **A** and **C** are can be obtained by

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{3}$$

   To find the area of the rhombus, we can take the cross product of these two diagonal vectors which are given by

$$\|\mathbf{a} \times \mathbf{b}\| = \left\| \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right\| = |a_1 b_2 - a_2 b_1| \tag{4}$$

   By using above formula

$$\left\| \left(\mathbf{B} - \mathbf{D}\right) \times \left(\mathbf{A} - \mathbf{C}\right) \right\| = \left\| \begin{pmatrix} 6 \\ 6 \end{pmatrix} \times \begin{pmatrix} 4 \\ -4 \end{pmatrix} \right\| = 48 \tag{5}$$

   Area of rhombus $= \frac{1}{2}$ (Cross product of diagonals) $= \frac{48}{2} = 24$

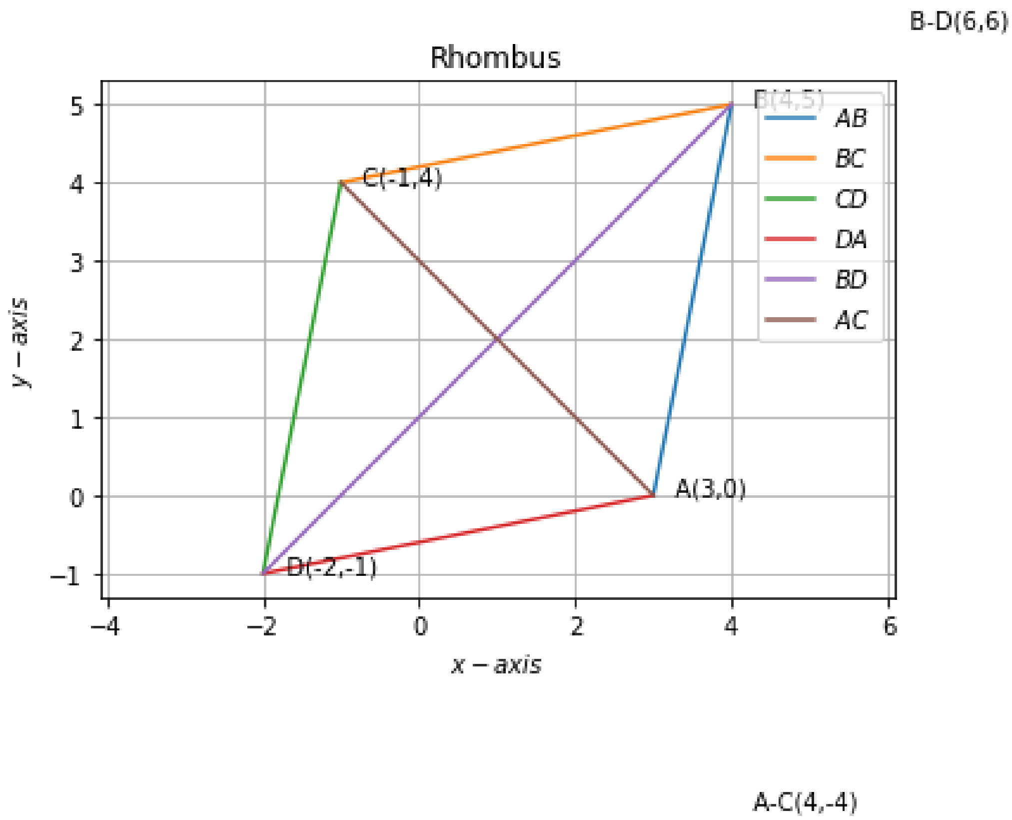

Figure 1