# Coordinate Geometry

1. **Problem statement :** Find a relation between x and y such that the point $\brak{x, y}$ is equidistant from the point $\brak{3, 6}$ and $\brak{-3, 4}$

   **Solution:**
   **Method I**
   The input parameters for this problem are given as

$$\mathbf{P} = \myvec{x \\ y}, \mathbf{A} = \myvec{3 \\ 6}, \mathbf{B} = \myvec{3 \\ -4} \tag{1}$$

   If $\mathbf{P}(x, y)$ equidistant from the points $\mathbf{A}$ and $\mathbf{B}$,

$$\norm{\mathbf{P} - \mathbf{A}} = \norm{\mathbf{P} - \mathbf{B}} \tag{2}$$

$$\implies \norm{\mathbf{P} - \mathbf{A}}^2 = \norm{\mathbf{P} - \mathbf{B}}^2 \tag{3}$$

   which can be expressed as

$$\begin{aligned} \brak{\mathbf{P} - \mathbf{A}}^\top \brak{\mathbf{P} - \mathbf{A}} &= \brak{\mathbf{P} - \mathbf{B}}^\top \brak{\mathbf{P} - \mathbf{B}} \\ \implies \norm{\mathbf{P}}^2 - 2\mathbf{P}^\top\mathbf{A} + \norm{\mathbf{A}}^2 &= \norm{\mathbf{P}}^2 - 2\mathbf{P}^\top\mathbf{B} + \norm{\mathbf{B}}^2 \end{aligned} \tag{4}$$

   which can be simplified to obtain

$$\mathbf{P} = y\mathbf{e}_1 \tag{5}$$

   where

$$y = \frac{\norm{\mathbf{A}}^2 - \norm{\mathbf{B}}^2}{2\brak{\mathbf{A} - \mathbf{B}}^\top \mathbf{e}_1} \tag{6}$$

   now substituting the A and B values in (6)

$$\brak{\mathbf{A} - \mathbf{B}}^\top = \brak{\myvec{3 \\ 6} - \myvec{-3 \\ 4}}^\top = \myvec{6 & 2} \tag{7}$$

$$\|\mathbf{A}\|^2 = 45 \tag{8}$$

$$\|\mathbf{B}\|^2 = 25 \tag{9}$$

upon substituting the values in (6) the value of $y = 5$

Hence, the desired point is $\mathbf{P}$ is $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$.

**Method II :**

If P(x,y) is the equidistant from A and B, then the

$$\mathbf{P}(\mathbf{x}, \mathbf{y}) = (\frac{\mathbf{x1} + \mathbf{x2}}{2}, \frac{\mathbf{y1} + \mathbf{y2}}{2}) \tag{10}$$

$$\mathbf{P}(\mathbf{x}, \mathbf{y}) = (\frac{3-3}{2}, \frac{6+4}{2}) = (0, 5) \tag{11}$$

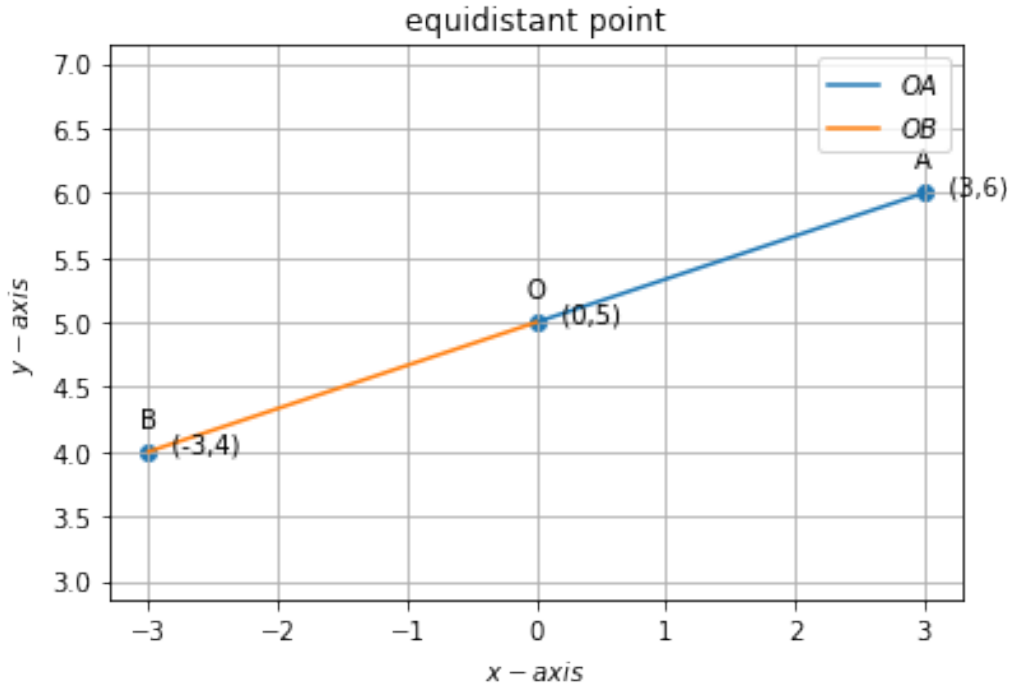


Figure 1