Coordinate Geometry

1. **Problem statement :** Find the area of a rhombus of its vertices are $\left(3,0\right)$, $\left(4,5\right)$, $\left(-1,4\right)$ and $\left(-2,-1\right)$taken in order

   Solution:
   The input vertices for this problem are given as

$$\mathbf{A}=\begin{pmatrix}3\\0\end{pmatrix},\mathbf{B}=\begin{pmatrix}4\\5\end{pmatrix},\mathbf{C}=\begin{pmatrix}-1\\4\end{pmatrix},\mathbf{D}=\begin{pmatrix}-2\\-1\end{pmatrix} \tag{1}$$

   Area of a rhombus $=\frac{1}{2}$ (product of its diagonals) for the given vertices of the rhombus, we can find the vectors **B** and **D** formed by its diagonals as follows:

$$\mathbf{B}-\mathbf{D}=\begin{pmatrix}4\\5\end{pmatrix}-\begin{pmatrix}-2\\-1\end{pmatrix}=\begin{pmatrix}6\\6\end{pmatrix} \tag{2}$$

   the length of the another diagonal **A** and **C** are can be obtained by

$$\mathbf{A}-\mathbf{C}=\begin{pmatrix}3\\0\end{pmatrix}-\begin{pmatrix}-1\\4\end{pmatrix}=\begin{pmatrix}4\\-4\end{pmatrix} \tag{3}$$

   To find the area of the rhombus, we can take the cross product of these two diagonal vectors which are given by

$$\left\|\mathbf{a}\times\mathbf{b}\right\|=\left\|\begin{pmatrix}a_1\\a_2\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\end{pmatrix}\right\|=\left|a_1b_2-a_2b_1\right| \tag{4}$$

   By using above formula

$$\left\|\left(\mathbf{B}-\mathbf{D}\right)\times\left(\mathbf{A}-\mathbf{C}\right)\right\|=\left\|\begin{pmatrix}6\\6\end{pmatrix}\times\begin{pmatrix}4\\-4\end{pmatrix}\right\|=48 \tag{5}$$

   Area of rhombus $=\frac{1}{2}$ (Cross product of diagonals) $=\frac{48}{2}$= 24

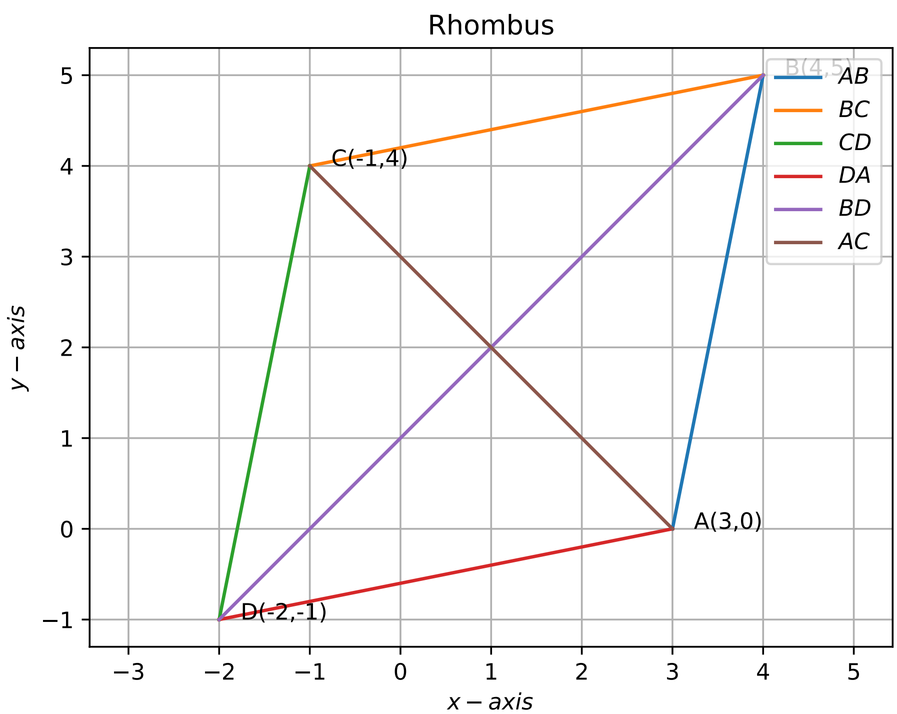
Rhombus
B(4,5)
C(-1,4)
A(3,0)
D(-2,-1)
AB
BC
CD
DA
BD
AC
y − axis
x − axis
5
4
3
2
1
0
−1
−3
−2
−1
0
1
2
3
4
5

Figure 1