

VECTOR ALGEBRA

January 25, 2023

1. **Problem statement :** Evaluate the product $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$

Solution:

$$(3\mathbf{a} - 5\mathbf{b})^\top (2\mathbf{a} + 7\mathbf{b}) = 3\mathbf{a}^\top 2\mathbf{a} + 3\mathbf{a}^\top 7\mathbf{b} - 5\mathbf{b}^\top 2\mathbf{a} - 5\mathbf{b}^\top 7\mathbf{b} \quad (1)$$

Properties of Vector

$$\mathbf{a}^\top \mathbf{a} = \|\mathbf{a}\|^2 \quad (2)$$

$$\mathbf{a}^\top \mathbf{b} = \mathbf{b}^\top \mathbf{a} \quad (3)$$

By using (2) and (3)

$$(3\mathbf{a} - 5\mathbf{b})^\top (2\mathbf{a} + 7\mathbf{b}) = 6\|\mathbf{a}\|^2 + 21\mathbf{a}^\top \mathbf{b} - 10\mathbf{b}^\top \mathbf{a} - 35\|\mathbf{b}\|^2 \quad (4)$$

$$= 6\|\mathbf{a}\|^2 - 35\|\mathbf{b}\|^2 + 11\mathbf{a}^\top \mathbf{b} \quad (5)$$

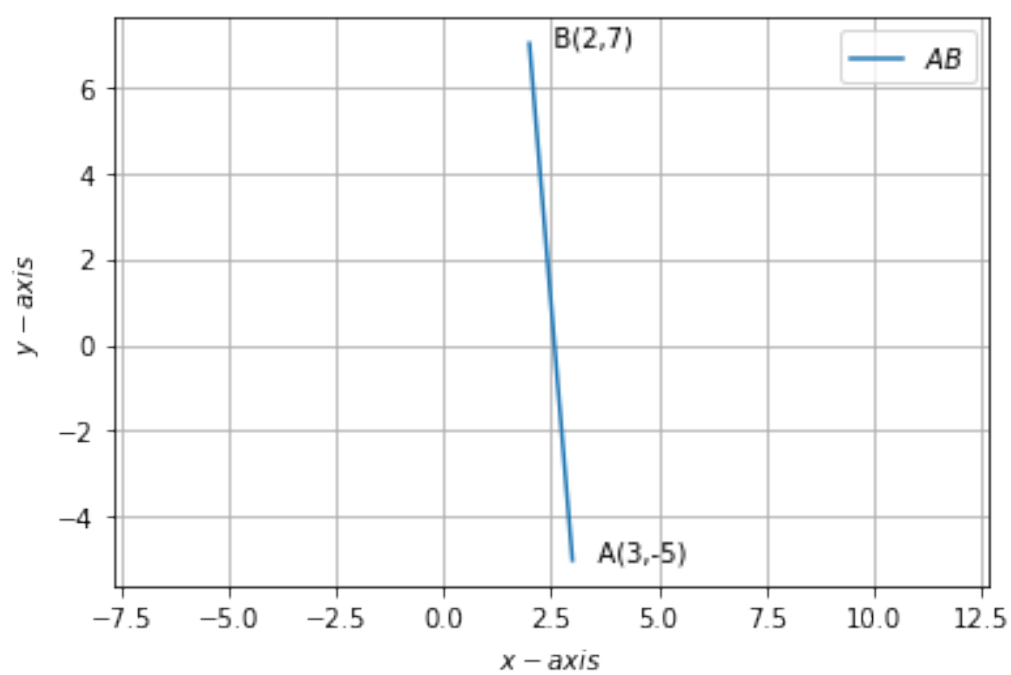


Figure 1