Computational Physics (PHY241)

Homework Assignment 7

Due Date: **Thursday, March 31, 2022**

The Planck theory of thermal radiation tells us that in the (angular) frequency interval ω to $\omega + \mathrm{d}\omega$, a black body of unit area radiates electromagnetically an amount of thermal energy per second equal to $I(\omega)\,\mathrm{d}\omega$, where

$$I(\omega) = \frac{\hbar}{4\pi^2 c^2} \frac{\omega^3}{(\mathrm{e}^{\hbar\omega/k_B T} - 1)}.$$

Here $\hbar$ is Planck's constant over 2π, c is the speed of light, and k_B is Boltzmann's constant.

1. (10 points) Show that the total energy per unit area radiated by a black body is

$$W = \frac{k_B^4 T^4}{4\pi^2 c^2 \hbar^3} \int_0^\infty \frac{x^3}{\mathrm{e}^x - 1}\,\mathrm{d}x.$$

2. (10 points) Briefly explain the integration algorithm you would like to choose. At least, the corresponding finite difference expressions should be given and explained.
3. (40 points) Write a program to evaluate the integral in this expression.
4. (10 points) Discuss how accurate your answer is. Plot W as a function of the number of your chosen grid points (N) at a fixed temperature (say 300K).
5. (10 points) Make a plot showing the variation of W with respect to temperature T.
6. (20 points) Even before Planck gave his theory of thermal radiation around the turn of the 20th century, it was known that the total energy W given off by a black body per unit area per second followed Stefan's law: $W = \sigma T^4$, where σ is the Stefan–Boltzmann constant. Use your value for the integral above to compute a value for the Stefan–Boltzmann constant (in SI units) to three significant figures. Check your result against the known value, which you can find in books or on-line. You should get good agreement.