

# COMPUTATIONAL PHYSICS (PHY241)

## HOMEWORK ASSIGNMENT 10

Due Date: Friday, April 22, 2022

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### Comparison of the DFT and DCT

In class, we looked at data representing the variation of the Dow Jones Industrial Average, colloquially called “the Dow,” over time. The particular time period studied in that exercise was special in one sense: the value of the Dow at the end of the period was almost the same as at the start, so the function was, roughly speaking, periodic. Here, the file called `dow2.txt` also contains data on the Dow but for a different time period, from 2004 until 2008. Over this period the value changed considerably from a starting level around 9000 to a final level around 14000.

1. Write a program in which you read the data in the file `dow2.txt` and plot it on a graph. Then smooth the data by calculating its Fourier transform, setting all but the first 2% of the coefficients to zero, and inverting the transform again, plotting the result on the same graph as the original data. As in the class exercise, you should see that the data are smoothed, but now there will be an additional artifact. At the beginning and end of the plot you should see large deviations away from the true smoothed function. These occur because the function is required to be periodic—its last value must be the same as its first—so it needs to deviate substantially from the correct value to make the two ends of the function meet. In some situations (including this one) this behavior is unsatisfactory. If we want to use the Fourier transform for smoothing, we would certainly prefer that it not introduce artifacts of this kind.
2. Modify your program to repeat the same analysis using discrete cosine transforms. You can use the functions from `scipy.fft.dct` (see [www.scipy.org](http://www.scipy.org) for details) to perform the fast cosine transforms if you wish. Again discard all but the first 2% of the coefficients, invert the transform, and plot the result. You should see a significant improvement, with less distortion of the function at the ends of the interval. This occurs because the cosine transform does not force the value of the function to be the same at both ends.

It is because of the artifacts introduced by the strict periodicity of the DFT that the cosine transform is favored for many technological applications, such as audio compression. The artifacts can degrade the sound quality of compressed audio and the cosine transform generally gives better results.

The cosine transform is not wholly free of artifacts itself however. It's true it does not force the function to be periodic, but it does force the gradient to be zero at the ends of the interval (which the ordinary Fourier transform does not). You may be able to see this in your calculations for part (b) above. Look closely at the smoothed function and you should see that its slope is flat at the beginning and end of the interval. The distortion of the function introduced is less than the distortion in part (a), but it's there all the same. To reduce this effect, audio compression schemes often use overlapped cosine transforms, in which transforms are performed on overlapping blocks of samples, so that the portions at the ends of blocks, where the worst artifacts lie, need not be used.