

Plural Logic and Sensitivity to Order

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Abstract: Sentences that exhibit sensitivity to order (e.g. *John and Mary arrived at school in that order* and *Mary and John arrived at school in that order*) present a challenge for the standard formulation of plural logic. In response, some authors have advocated new versions of plural logic based on more fine-grained notions of plural reference, such as serial reference [Hewitt 2012] and articulated reference [Ben-Yami 2013]. The aim of this article is to show that sensitivity to order should be accounted for without altering the standard formulation of plural logic. In particular, sensitivity to order does not call for a more fine-grained notion of plural reference. We point out that the phenomenon in question is quite broad and that current proposals are not equipped to deal with the full range of cases in which order plays a role. Then we develop an alternative, unified account, which locates the phenomenon not in the way in which plural terms can refer, but in the meaning of special expressions such as *in that order* and *respectively*.

1 Introduction

Plural logic is a widely accepted framework in which the meaning and logic of basic plural expressions can be elegantly captured and analysed [Boolos 1984; Yi 1999, 2005, 2006; Oliver and Smiley 2001, 2004, 2005, 2013; Rayo 2002; Linnebo 2003, 2013; McKay 2006]. The primitive notions are those of plural reference and plural predication. In a sentence like *Annie, Bonnie, and Connie cooperate*, the plural subject *Annie, Bonnie, and Connie* is taken to refer plurally to Annie, Bonnie and Connie, and the sentence is true just in case they collectively satisfy the predicate *cooperate*. A challenge to this view arises in connection with sentences that exhibit sensitivity to order:

- (1) John and Mary arrived at school in that order.
- (2) Mary and John arrived at school in that order.

It is not evident whether the standard formulation of plural logic has the resources to provide a satisfactory account of the truth conditions of such sentences.

This has recently led some authors to advocate more sophisticated versions of plural logic. Hewitt [2012] embraces the notion of serial reference (i.e. plural reference to things in an order) as a primitive notion. Ben-Yami [2013] puts forth a view according to which plural terms can articulate their referents in ways to which predicates are sensitive. Both accounts propose, in effect, a more fine-grained notion of plural reference and of the corresponding notion of plural predication.¹

Our goal in this article is to show that sensitivity to order should be accounted for without altering the standard formulation of plural logic. In particular, sensitivity to order does not call for a more fine-grained notion of plural reference. We begin by pointing out that the phenomenon in question is quite broad and that the accounts of Hewitt and Ben-Yami are not equipped to deal with the full range of cases in which order plays a role. Then, inspired by Kay [1989], Oliver and Smiley [2004, 2013], and Chaves [2012], we develop a general account that locates the phenomenon not in the way in which plural terms can refer, but in the meaning of special expressions such as *in that order* and *respectively*. This proposal has broad scope and accounts for the phenomenon in full generality. Once the semantic contribution of these expressions is properly understood, it becomes clear that more fine-grained notions of plural reference and predication are unnecessary and have anyway limited application. We similarly reject an alternative account that appeals to multigrade predicates [Oliver and Smiley 2004, 2013].

2 Enrichments of plural logic

In some cases, the order of the terms forming a plural expression is relevant to the truth conditions of the sentence. Modifying it changes the truth conditions, as we saw in (1) and (2). The same phenomenon is observed here:

- (3) John and Mary talked to Bill and Sue, respectively.
- (4) Mary and John talked to Bill and Sue, respectively.

¹Hewitt [2012] illustrates some of the potentially far-reaching consequences of the adoption of a fine-grained notion of plural reference, e.g. an ontologically innocent account of predication, a plural interpretation of full second-order logic, and a theoretical alternative to multisets.

In the intended reading, (3) is true if and only if John talked to Bill and Mary talked to Sue. In contrast, (4) is true if and only if Mary talked to Bill and John talked to Sue.

Sensitivity to order is a puzzle for the standard formulation of plural logic, which is based on a simple conception of plural reference. Under this conception, the semantic contribution of a plural term is to supply some objects, those collectively referred to by the term. So *John and Mary* and *Mary and John* make the same semantic contribution, since both refer to John and Mary. How, then, can one explain why (1) and (2), and (3) and (4), differ in truth value?

Hewitt (2012) and Ben-Yami (2013) have recently suggested ways of addressing this issue. While they retain the idea that a plural term refers to many things at once, they propose more fine-grained notions of plural reference and plural predication. Let us examine each proposal in turn.

2.1 Serial reference

For Hewitt, the semantic contribution of a plural term is not exhausted by supplying the things to which it refers. A plural term can encode additional semantic information so that it refers to some things in a certain order. The resulting notion of plural reference, *serial reference*, offers an account of sensitivity to order. Here is an example [Hewitt 2012: 301]:

(5) We came here via (in order) Peterborough, York, and Newcastle.

This is a purported case of serial reference: the list *Peterborough, York, and Newcastle* refers to Peterborough, York, and Newcastle, and it refers to these towns in the order in which they are listed. The sentence is true, roughly, just in case the referents of *we* reached the place indicated by *here* via the locations in the list in order of mention: first Peterborough, then York, and lastly Newcastle. This is why modifying the order of the terms in the list changes the truth conditions.

Hewitt [2012: 301] argues that serial reference extends to plural demonstratives and pronouns:

Sometimes the plural demonstratives “those” and “these” give the appearance of being serial: “these are the three prizewinners in the largest pumpkin competition”; “those are the stages of the surgical procedure.” [...] Think about an anaphoric use: “Ms. Smith’s, Professor McTaggart’s, and Mr. Singh’s; *those* are the three prizewinners in the largest pumpkin competition.” Here, “those” surely inherits the ordering of the serial noun phrase it is anaphoric for.

So, for Hewitt, sensitivity to order is explained in terms of serial reference and the sensitivity of certain predicates to this way of referring.

2.2 Articulated reference

Ben-Yami puts forward an alternative proposal based on the notion of articulated reference. For him, plural terms can articulate their referents in particular ways. Articulation “concerns the way an expression refers to a plurality of particulars: the reference of an expression is articulated a certain way if the plurality is referred to by means of referring to specific sub-pluralities of it” [Ben-Yami 2013: 91]. Here is a passage in which the idea is elaborated:

A referring expression can refer to a plurality by virtue of containing other referring expressions that refer to some of that plurality. For instance, *Jack and Jill* refers to Jack and Jill because it contains the name *Jack*, used to refer to Jack, and the name *Jill*, used to refer to Jill. We can say that the reference of *Jack and Jill* is articulated into reference to Jack and reference to Jill. By contrast, if these pail-companions are your children, then *your children* refers to the same children as does *Jack and Jill*, but its reference is not articulated. Similarly, [...] *the boys and the girls* and *my children, your children and her children* are, in the assumed context, co-referential, but their reference is articulated in different ways [...] because the former, for instance, contains a part referring just to Tom, Jack and Peter, while the latter does not. [Ben-Yami 2013: 89]

In general, as in these examples, the articulation is driven by the syntactic structure of the plural term. So, for instance, a plural term articulates its referents into two pluralities *xx* and *yy* only if it contains sub-expressions that refer to *xx* and to *yy*. However, there are exceptions in cases of anaphora. Ben-Yami claims that anaphoric reference preserves articulation, thus plural pronouns can articulate their referents despite being syntactically simple. In the following examples, the occurrences of *they* inherit the articulation of *my children and yours* and *the boys and the girls*, respectively [Ben-Yami 2013: 97]:

- (6) We can suggest to my children and yours that they play against each other.
- (7) We can suggest to the boys and the girls that they play against each other.

How does the notion of articulated reference help to account for sensitivity to order? For Ben-Yami, the order of articulation is part of the articulation itself. The complex terms *John and Mary* and *Mary and John* co-refer and articulate their referents into the same subpluralities. But they do not articulate their referents in the same way: each complex term articulates its referents in an order that differs from the other. Since some predicates are sensitive to articulation, substitution of co-referential terms that articulate their referents in different orders might fail to preserve truth.

Of course, there are cases in which the order does not matter, as in the following, equivalent sentences:

(8) John and Mary cooperate.

(9) Mary and John cooperate.

Ben-Yami explains:

Although articulation due to a structured phrase always articulates the reference in a certain order, this order is usually irrelevant to the meaning of what is said; but phrases such as *in that order* or *respectively* can make use of this order and affect the meaning of what is said. [2013: 91]

So, for Ben-Yami, sensitivity to order is explained in terms of articulation of reference and the sensitivity of certain predicates to the order encoded in the articulation.

2.3 Common features

The two proposals just outlined bear obvious similarities. Both serial reference and articulated reference enrich plural reference by specifying ways in which a plural term can refer to its referents. Correspondingly, both proposals assume that some predicates discriminate between different ways in which a plural term refers to its referents. Thus the additional dimension of reference (the order) is significant for the truth conditions of sentences involving these predicates. In this sense, both proposals modify the standard formulation of plural logic: plural predication is now relative to the referents of plural terms and to the specific way in which such terms refer. This second component of predication is absent in the standard formulation of plural logic.

An apparent difference between the two proposals resides in how the additional dimension of reference is determined. For Hewitt, reference to some things in an order is a new primitive: some plural terms just refer in an order. For Ben-Yami, the articulation of reference is an additional argument of verbal expressions and it is tied to the structure or semantic information associated with a plural term. As we have seen above, articulation can be inherited via anaphora, but its primary source is syntactic. A plural term like *the children* does not articulate its referents into subpluralities. A fortiori, it does not articulate them in any order.

An interesting feature of both proposals is that they might provide the resources to account for another semantic phenomenon which also poses a *prima facie* problem for the standard formulation of plural logic: sensitivity to repetition. Like order, repetition in a plural term can affect truth conditions. Deleting a repetition may change the truth value of a sentence [Ben-Yami 2013: 98; Hewitt 2012: 300]:

(10) The sum of 1, 1, and 2 is 4.

(11) The sum of 1 and 2 is 4.

(12) The first five Fibonacci numbers are 0, 1, 1, 2, and 3, respectively.

(13) ??The first five Fibonacci numbers are 0, 1, 2, and 3, respectively.

Examples of sensitivity to repetition are not confined to mathematics. Consider the next examples (uttered, say, in 2010):

(14) The last three winners of the Roland Garros men's singles are Nadal, Federer, and Nadal, in that order.

(15) ??The last three winners of the Roland Garros men's singles are Nadal and Federer, in that order.

Just like sensitivity to order, sensitivity to repetition challenges the standard formulation of plural logic. If the semantic contribution of a plural term is to supply its referents, then *Nadal and Federer* and *Nadal, Federer, and Nadal* (or *1 and 2 and 1, 1, and 2*) make the same semantic contribution: both expressions refer to Nadal and Federer (or to 1 and 2). So the sentences above should, *prima facie*, have the same truth conditions.

With a richer notion of plural reference, one might be able to treat sensitivity to repetition on a par with sensitivity to order. As one postulates that a plural term could refer to some things in an order, one might postulate that a plural term can refer to some things with repetition [Hewitt 2012: 300]. This would be yet another way in which a plural term can refer to its referents. Analogously, the proponent of the notion of articulated reference might postulate that repetition too is a component of the articulation of a plural term. Both order and repetition would then matter for the articulation [Ben-Yami 2013: 98].

By enriching the notion of plural reference, Hewitt and Ben-Yami offer two accounts of sensitivity to order and repetition that can be incorporated within an extension of plural logic. As we show in the next section, however, these accounts turn out to have limited scope with respect to the broad range of cases in which order plays a role.

3 The problem of scope

While the proposals of Hewitt and Ben-Yami account for the data presented above, they have limited scope. Order plays a role in a broader class of cases, and Hewitt and Ben-Yami's accounts cannot be extended to deal with them. Indeed, as we explain, the source of this limitation lies in the key feature of these accounts, which locates sensitivity to order *in plural terms*, like lists.

The problem of scope is underscored by the existence of a general account of sensitivity to order, which we develop in section 5. This account explains in a unified

way the cases dealt with by Hewitt and Ben-Yami and those that lie outside the scope of their proposals. The general account makes no use of serial or articulated reference. As in standard plural logic, the semantic function of a plural term is that of referring to several things at once, in no particular order. Order enters the truth conditions only as required by the semantics of special expressions, such as *respectively* and *in that order*.

The existence of this general account supports the view that the various cases in which order plays a role form a unified phenomenon. Since the proposals of Hewitt and Ben-Yami cover only special cases of this phenomenon, the general account we put forward is clearly preferable. For the same reason, the general account is also preferable to the following, hybrid position: serial or articulated reference would be operative in the cases presented in section 2, while the resources afforded by the general account would come into play only with respect to the cases that are outside the scope of serial reference and articulated reference.

The main limitation of serial reference and articulated reference appears in cases where two (or more) orders are relevant:

- (16) Annie, Bonnie, and Connie arrived in the order they were called.

What is intuitively relevant here is not the order of mention, but the order of arrival of Annie, Bonnie, and Connie, and the order in which they were called. So two potentially conflicting orders play a role in the truth conditions. But with serial reference and articulated reference, order plays a role essentially through syntactically plural terms. Sentence (16) contains a single plural term (the list *Annie, Bonnie, and Connie*), so it involves a single order that could affect truth conditions. As a result, neither serial reference nor articulated reference (at least as they stand) could account for the semantics of these sentences.

This fact can be made vivid by considering more complex verb phrases containing explicit descriptions of order:

- (17) Annie, Bonnie, and Connie arrived in the order they were called and left in the order they completed the exam.

Here, there are four semantically relevant orders: the order in which Annie, Bonnie, and Connie arrived, the order in which they were called, the order in which they left, and the order in which they completed the exam. But there would be only one order associated with the plural term that is the subject of the sentence. Thus, serial reference and articulated reference do not have sufficient resources to account for sentences of this kind.

Moreover, a general semantics of order should ideally be able to account for data involving a number of other expressions, such as superlatives [Kay 1989] and simple definites:

(18) The three brightest students got the three highest grades, respectively.

(19) The students arrived in the order they were called.

But as we show below, serial reference and articulated reference are not equipped to deal with some of these cases.

3.1 Limitations of serial reference

Let us consider an advocate of serial reference who would like to provide a general semantics of order, covering the variety of cases in which order plays a role. There are two options. One option is to embrace the hybrid approach sketched above. This means appealing to serial reference to account for cases of the kind surveyed in section 2 and to resort to an alternative explanation for other cases. However, the hybrid approach is unattractive given the existence of a unified account (the one we develop in section 5). The other option is to provide a theoretical alternative to the unified account built around the notion of serial reference. This option, we contend, faces a dilemma.

Do syntactically plural terms always refer in an order? Suppose not. Then one is, in effect, assuming that plural terms are ambiguous between serial and non-serial uses. Consider this sentence:

(20) Annie, Bonnie, and Connie arrived at school.

If lists can refer non-serially, (20) looks like a paradigmatic case in which reference can be non-serial. However, if no order is encoded in the way in which *Annie, Bonnie, and Connie* refers, it is hard to explain compositionally why it makes sense to continue the sentence by adding *but not in that order*:

(21) Annie, Bonnie, and Connie arrived at school, but not in that order.

The second part of (21) modifies the first part and therefore depends on the semantic contribution of *Annie, Bonnie, and Connie*. If *Annie, Bonnie, and Connie* does not refer serially, then order cannot become semantically relevant through it, which contradicts the account of sensitivity to order based on serial reference. Similar considerations apply to other kinds of plural terms, such as superlatives and simple definites, as shown by the following examples:

(22) The three brightest students joined the philosophy club, but not in the order they were invited.

(23) The students taking the exam left the room, but not in the order they arrived.

Since, as suggested by these examples, an order can systematically be made salient, with any kind of plural term, the advocate of serial reference under consideration should deny that some plural terms refer non-serially.

So let us suppose that a plural term always refers in an order, though sometimes this aspect of reference is not semantically relevant. Under this supposition, one is forced to postulate a hidden order and, as a result, one faces a charge of arbitrariness. For imagine that, in an ordinary context, one utters (24):

(24) The students taking the exam are four.

On the view under discussion, *the students taking the exam* refers to the students in a certain order. Nothing in the meaning of a definite specifies what the order could be. So the speaker must choose one order arbitrarily. And the hearer must likewise choose an order arbitrarily. In many sentences, like this one, order plays no role. So why postulate one at all? Our positive proposal will corroborate the view that there is no need to postulate an order associated with the noun phrase in cases of this sort.

To emphasize the point, imagine that, after acquiring a modicum of familiarity with set theory, a student utters this sentence:

(25) The sets are more numerous than the natural numbers.

If *the sets* refers serially, it should be conceded that the order in which the speaker is referring to the sets is not accessible. Depending on the type of the order, its existence might have deep mathematical consequences. Thus, if serial reference involved a well-ordering in this sentence, a principle of global choice would follow. However, one's hypotheses concerning the structure of the human language faculty should presumably not have such important consequences concerning mathematics. And if serial reference involved some other kind of order, the question arises as to what order that would be. But there seems to be no basis on which the question can be answered.

Lists are not immune to the problem of arbitrariness. In which order does a list refer? So far the relevant order has typically been the order of the terms in the list. However, in some cases, the order of mention is irrelevant to the truth conditions of the sentence:

(16) Annie, Bonnie, and Connie arrived in the order they were called.

Of course, despite its theoretical unfruitfulness in this context, one might insist on the assumption that the order of the list is the order of mention, and admit that cases like the one at hand should receive a different explanation. But, as we noted above, this hybrid approach is unattractive. Alternatively, one might hold that the relevant order is that explicitly described by the expression *in the order they were called*. A moment's reflection shows that this suggestion is not viable. For consider the next sentence:

- (26) Annie, Bonnie, and Connie arrived in the order they were called, but left in the order they were appointed.

Maintaining that there is a single order associated with *Annie, Bonnie, and Connie* runs against the fact that we now have two equally good candidates to determine the order of this list (i.e. *the order they were called* and *the order they were appointed*), with no way of choosing among them.

These examples emphasize the limitation pointed out above. The proposal at hand forces a single order on the noun phrase *Annie, Bonnie, and Connie*, so it cannot accommodate the fact that two or more orders are involved in the interpretation of the sentence. Additional machinery will be needed to provide a general semantics of order.

3.2 Limitations of articulated reference

For the very same reason (i.e. because it forces a single order on the noun phrase), the framework of articulated reference does not have the resources to account for sentences where two (or more) orders are relevant:

- (16) Annie, Bonnie, and Connie arrived in the order they were called.

A further limitation arises from the emphasis on the syntactic or informational structure of the noun phrase. As already noted, Ben-Yami holds that “although articulation due to a structured phrase always articulates the reference in a certain order, this order is usually irrelevant to the meaning of what is said; but phrases such as *in that order* or *respectively* can make use of this order and affect the meaning of what is said” [2013: 98]. If so, how can one explain the following example in which *respectively* operates on superlatives?

- (18) The three brightest students got the three highest grades, respectively.

The noun phrases occurring in this sentence are not articulated in a way that might help to account for its truth conditions. Indeed, Ben-Yami’s [2013: 98] elaboration of the notion of articulation implies that *the three brightest students* articulates its referents in a single plurality, that of the three brightest students. For it is the three brightest students who collectively fall under the description *three brightest students*. But to account for (18), one would need an articulation into three subpluralities, each of which contains a single student among the three brightest.

In light of this, a proponent of articulated reference aiming to propose a general account of sensitivity to order might consider severing the link between articulation and the syntactic structure of the noun phrase. One might admit the possibility that the articulation be primitive or given by means other than the syntactic structure

of the noun phrase and anaphoric links to it. This liberalization of the notion of articulated reference would make the connection between articulated reference and serial reference very tight. As a result, one would face the dilemma arising for the advocate of serial reference. If, on the one hand, articulation and order were not always present, it would be hard to explain why even in paradigmatic cases in which a noun phrase is not articulated, sentences involving it can be so continued as to make order relevant:

- (27) Annie, Bonnie, and Connie cooperated, but did not arrive to school in that order.

On the other hand, if articulation and order were always present, the account would be led to postulate hidden orders, facing a charge of arbitrariness. There would be sentences in which an order is assumed but cannot be recognized by the speaker or the hearer.

So we need an altogether different approach to account for sensitivity to order in full generality. We develop such an approach in section 5. Before doing so, however, we consider and reject a different proposal based on the notion of multigrade predicate [Oliver and Smiley 2004, 2013]. We call it the *multigrade approach*.

4 The multigrade approach

According to the multigrade approach, a list of terms occurring as the subject of a predicate is not a genuine plural term. Instead, each term occupies an argument position of the predicate. Consider again the sentences:

- (1) John and Mary arrived at school in that order.
(2) Mary and John arrived at school in that order.

If *John* and *Mary* occupy distinct argument positions, then the coarse-grained logical forms of these sentences are, respectively, $A(j, m)$ and $A(m, j)$, where A is a unary predicate representing *arrived at school in that order*. The predicate is unary in the sense that it has one argument place, though it allows multiple positions within its argument place.² Since a permutation of the argument positions does not in general preserve truth value, this approach can explain why order matters in these cases. Similarly, it can explain why it plays a role in the following sentences:

- (3) John and Mary talked to Bill and Sue, respectively.

²For an account of the distinction between argument places and argument positions, as well as motivations in its favour, see Oliver and Smiley [2004: 615-18].

- (4) Mary and John talked to Bill and Sue, respectively.

On the multigrade approach, the coarse-grained logical forms of (3) and (4) are, respectively, $T(j, m; b, s)$ and $T(m, j; b, s)$, where the comma separates argument positions, and the semicolon separates argument places. Given these logical forms, there is no difficulty in explaining why they differ in truth conditions. Each sentence is true just in case the i -th object of the first argument talked to the i -th object of the second argument.

The multigrade approach can also account for sensitivity to repetition. Since repetitions occupy different argument positions, there is no reason to suppose that eliminating one occurrence, i.e. modifying the argument structure of the predicate, will preserve the truth conditions of the sentence.

However, a significant cost for the multigrade approach (partly independent of order and repetition) is that it must rely heavily on meaning postulates. If the argument structure, and specifically the structure of the positions within argument places, is semantically relevant, one needs meaning postulates to account for inferential transitions in which the structure is modified. This is what we observe in the following examples where the last sentence is intuitively entailed by the first:

- (28) (a) Annie and Bonnie are students and best friends.
 (b) Some students are best friends.
- (29) (a) Bonnie and Connie are students.
 (b) Annie, Bonnie, and Connie met.
 (c) Annie and some students met.
- (30) (a) Annie, Bonnie, and Connie visited Paris.
 (b) Annie and Bonnie visited Paris.

These sentences may be formalized as follows:

- (28*) (a) $S(a, b) \wedge B(a, b)$
 (b) $\exists xx (S(xx) \wedge B(xx))$
- (29*) (a) $S(b, c)$
 (b) $M(a, b, c)$
 (c) $\exists xx (S(xx) \wedge M(a, xx))$
- (30*) (a) $V(a, b, c; p)$
 (b) $V(a, b; p)$

Without special postulates concerning the behaviour of the argument structure of predicates, the intuitive entailments shown in these examples are not captured. For example, one must require that if a monadic predicate such as ‘students’ and ‘best friends’ is satisfied by two entities filling its argument positions, then it is also satisfied by the plurality of those entities filling its argument place. Analogous requirements must be imposed for the other examples.

Moreover, any case in which the order of the terms of a list does not matter requires a meaning postulate to the effect that the predicate applies to all permutations of its argument positions:

(31) Annie and Bonnie met for a beer.

(32) Bonnie and Annie met for a beer.

In such cases, symmetry must be imposed by hand.

However, even if this reliance on meaning postulates is accepted, the multigrade approach, like the proposals based on serial reference and articulated reference, turns out to have limited scope. It does not have the resources to account for cases of sensitivity to order where there is no significant argument structure. Compare the following sentences:

(3) John and Mary talked to Bill and Sue, respectively.

(18) The three brightest students got the three highest grades, respectively.

The account of truth conditions offered by the multigrade approach for (3) cannot be extended to (18). In (3), there is an obvious matching of the argument positions that is relevant to the truth conditions of the sentence. But this is not the case in (18), where the plural descriptions *the three brightest students* and *the three highest grades* occupy the unique position of their corresponding argument place.

Furthermore, the account provides no insight into the truth conditions of sentences in which the relevant orders are not associated with a noun phrase:

(16) Annie, Bonnie, and Connie arrived in the order they were called.

And this is even more striking when multiple, conflicting orders are involved:

(26) Annie, Bonnie, and Connie arrived in that order, but left in the order they were appointed.

A new strategy is called for. We need a unified account of the truth conditions of the variety of examples mentioned so far. This is what we pursue in the next section.

5 A general approach

Our proposal is based on the idea that sensitivity to order is due to the meaning of special expressions, such as *in that order* and *respectively*. It is through these expressions, rather than any special mode of plural reference, that order enters the truth conditions of a sentence. Understanding the semantic contribution of these expressions provides a general account of sensitivity to order as well as sensitivity to repetition.

From the compositional point of view, descriptions of order and *respectively* appear to function as verb modifiers, restricting the extension of the property or relation expressed by the verb. Consider the extension of *arrived at school*, which includes each individual or plurality of individuals instantiating the relevant property. Intuitively, the extension of *arrived at school in order of age* should include only the pluralities of individuals in the extension of *arrived at school* whose order of arrival and whose order of age are, in some appropriate sense, the same. To make this condition precise, we first introduce some preliminary notions. Then we spell out the truth conditions of sentences involving descriptions of order, *respectively*, or repetitions. In so doing, we elucidate the restriction imposed by these expressions on the extension of the verbs they modify. The semantic function of descriptions of order and *respectively* is captured without assuming that the semantic contribution of a plural term exceeds that of supplying the entities it denotes.

5.1 Preliminary notions: indexings and matching

First, we formalize a generalization of the notion of indexing sketched in Oliver and Smiley [2004: 663]. Let xx be a plurality of objects. Let ii be some indices and $<$ a salient order over them. An indexing f of xx based on $(ii, <)$ is a partial, plural relation on ii and the subpluralities of xx , satisfying two conditions:

- it is left-total: for any i in ii , there are some yy in xx such that $f(i, yy)$;
- it covers the objects it indexes: any x in xx is among some yy for which there is an i such that $f(i, yy)$.

Given that f is left-total, we also use functional notation, i.e. $f(i)$, to denote the yy such that $f(i, yy)$. The expression $f(i)$ is thus a plural term denoting those objects. As usual in plural logic, a plural term can denote one or several objects.

The extension of such a relation can be specified by some ordered pairs. For example, if the objects xx are a , b , and c , and the indices ii are $i < j < k$, each line below characterizes a possible indexing of xx based on $(ii, <)$:

$$(i, a), (j, c), (k, b)$$

$$(i, a), (i, b), (j, a), (k, c)$$

$$(i, c), (j, c), (k, a), (k, b)$$

Next, we define a relative notion of matching of two indexings. Let R be a relation between objects xx and yy , let f be an indexing of xx based on $(ii, <_1)$, and let g be an indexing of yy based on $(jj, <_2)$. We say that f matches g with respect to R when these two conditions are satisfied:

- there is a salient order isomorphism H between $(ii, <_1)$ and $(jj, <_2)$;
- for any i in ii and j in jj , if $H(i, j)$, then $R(f(i), g(j))$.

An order isomorphism H between $(ii, <_1)$ and $(jj, <_2)$ is a bijective relation between ii and jj preserving order: whenever $H(i_1, j_1)$ and $H(i_2, j_2)$, $i_1 <_1 i_2$ if and only if $j_1 <_2 j_2$. So by definition, the matching of two indexings presupposes that there be a salient order isomorphism between the ordered indices on which they are based.³

Here is the picture. The indexings index the two pluralities of objects at hand, while the underlying order isomorphism determines a bijective correspondence between objects of the two pluralities on the basis of their indices. In particular, if two indices are related by the order isomorphism, the objects associated to those indices correspond. We introduce a relativized notion of matching (i.e. matching with respect to a relation R) since we are interested in assessing whether the corresponding objects stand in the relation R , where R may vary from case to case. As will be seen in section 5.2, this yields a uniform account of order and *respectively*.

For example, start with two pluralities, xx consisting of a , b , and c , and yy consisting of d and e . Let ii be the indices 1, 2, and 3, and let $<$ be the relation *less than*. Finally, let f be an indexing of xx based on $(ii, <)$ and let g be an indexing of yy also based on $(ii, <)$, characterized as follows:

$$f(1) = a, f(2) = b, f(3) = c$$

$$g(1) = d, g(2) = d, g(3) = e$$

In this case, we have a unique order isomorphism H between indices, namely identity. This determines the following correspondence between objects of xx and yy : a corresponds to d , b corresponds to d , and c corresponds to e . To ask whether f and g match with respect to R is then to ask whether these three conditions are satisfied: $R(a, d)$, $R(b, d)$, and $R(c, e)$. The relation R may vary. Of course, if R is identity, the functions f and g just described do not match with respect to it.

³When R is a symmetric relation, like identity, then f matches g with respect to R if and only if g matches f with respect to R . But when R is a non-symmetric relation, like the one expressed by the verb *obtain* or *help*, then the fact that f matches g with respect to R entails that g does not match f with respect to R .

5.2 An account based on indexings and matching

The key claim of our proposal is that the function of expressions like *respectively* and *in that order* is to compare certain indexings. This proposal has points of contact with Kay [1989], Oliver and Smiley [2004; 2013], and Chaves [2012]. Kay [1989: 183] holds that *respectively* can “evoke a mapping between the denotata of two plural or conjoined constituents iff that mapping is achieved via independent linear rankings of the two constituents”. Our account develops his claim, with the notion of indexing clarifying Kay’s idea of linear ranking. Oliver and Smiley [2004: 662-3] suggest the use of indexings to account for cases of repetition and for sentences containing the word *respectively*. By introducing a generalization of the notion of indexing and a notion of matching with respect to a relation R , we show that sentences containing *order*, *respectively*, or repetitions can in fact be given a unified treatment. Like Chaves [2012], who focuses on *respectively*, we maintain that adding this adverb to a sentence narrows its truth conditions. And we take descriptions of order to function similarly. In all cases, as we explain below, this requires two independently supplied indexings to match.

Let us present our proposal by giving the truth conditions of various sentences. Consider for instance:

(33) Annie, Bonnie, and Connie arrived at school in order of age.

Intuitively, there are two relevant orders here, the order in which Annie, Bonnie, and Connie arrived at school, and the order in which they were born, i.e. the order of age. The sentence is true just in case these orders are, in some appropriate sense, the same.

We can make this precise by means of the indexings corresponding to these orders. There is an indexing f of Annie, Bonnie, and Connie based on the time indices ii at which they arrived at school. The other indexing g is based on the time indices jj at which they were born. The relevant relation is that of identity, and the sentence is true just in case f matches g with respect to identity. This is the case if and only if the first person among them to arrive at school was the oldest, the second was the intermediate, and the third was the youngest.

Elaborating on the example, suppose Connie arrived at school first, followed by Bonnie, and then Annie. Then there are three time indices i_1 , i_2 , and i_3 , and an indexing f tracking the time of arrival and hence the corresponding order. So f is such that $f(i_1) = c$, $f(i_2) = b$, and $f(i_3) = a$. The time indices have a natural total order, i.e. temporal priority ($<$). As for the order of age, suppose that Connie is the oldest, Bonnie is intermediate, and Annie is the youngest. Then there are three time indices j_1 , j_2 , and j_3 , and a relation g tracking the time of birth and hence the order of age. So g is such that $g(j_1) = c$, $g(j_2) = b$, and $g(j_3) = a$. Here too the indices are naturally ordered by temporal priority ($<$), a total relation. Thus, there

is a unique order isomorphism between the two pluralities of time indices. This is given by $H(i_1, j_1)$, $H(i_2, j_2)$, $H(i_3, j_3)$. Now the sentence is true, on our account, if and only if f matches g with respect to identity. That is, for any i and j such that $H(i, j)$, $f(i) = g(j)$. In the scenario envisioned, $f(i_1) = c = g(j_1)$, $f(i_2) = b = g(j_2)$, $f(i_3) = a = g(j_3)$. The sentence is therefore true.

More generally, a sentence containing *in that order* or an explicit description of an order (e.g. *in order of age*, *in the order they were called*, etc.) is true just in case one of the relevant indexings matches the other with respect to identity. So a sentence of the form

(34) The As F in the order they G

is true if and only if f matches g with respect to identity, where f and g are salient indexings of the As tracking the order in which they F and G , respectively.

Note that our account applies in the same way to cases of non-distributive predication:

(35) The children met in order of age.

Suppose that there are three different times i_1 , i_2 , and i_3 at which some of the children met, and let f be the corresponding indexing of the children. And suppose that there are three different age groups: the seven-year-olds, the eight-year-olds, and the nine-year-olds. Let g be the corresponding indexing, i.e. the indexing associating each age among 7, 8, and 9, with the children of that age. The sentence is true if and only if $f(i_1) = g(7)$, $f(i_2) = g(8)$, $f(i_3) = g(9)$. This is the case if and only if the children who met at time i_1 are the seven-year-olds, the children who met at i_2 are the eight-year-olds, and the children who met at i_3 are the nine-year-olds.

An indexing can be given in various ways, notably by a list, by context, by a verbal expression, or by an explicit description. The expression *in that order* typically refers to the order of the list:

(1) John and Mary arrived at school in that order.

Here, an indexing f of John and Mary tracks the order in which they arrived at school. Another indexing g tracks the order in which they appear in the list *John and Mary*, i.e. the order of mention. The sentence is true just in case f matches g with respect to identity.

It is worth observing that one must countenance indexings with different families of indices. This is because not all cases involve temporal indices. Consider the following sentence:

(36) The students sat along the bench in order of height.

On one reading, the sentence is true if and only if the order of the spatial positions occupied by the students matches the order of their heights. In other words, the indexings track the positions occupied by the students and their heights. Therefore, the indices of the first indexing are given by spatial positions rather than time indices. The order of such positions is made salient by context, e.g. left-to-right from the perspective of the speaker.⁴

We have specified the truth conditions of sentences involving descriptions of order by identifying how such descriptions restrict the extension of the property or relation expressed by the verb. The plural subject supplies some objects. These objects satisfy the predicate modified by the description of order if and only if their relevant indexings match with respect to identity. An interesting feature of this proposal is that it illuminates the connection between the truth conditions of these sentences and explicit descriptions of order. That is, we may treat descriptions of order as terms standing for indexings. This explains semantic equivalences like the following:

- (16) Annie, Bonnie, and Connie arrived in the order they were called.
- (38) The order in which Annie, Bonnie, and Connie arrived is the order in which they were called.

If *the order in which Annie, Bonnie, and Connie arrived* and *the order in which they were called* stand for certain indexings and the copula demands their matching with respect to identity, the second sentence is indeed equivalent to the first.⁵ The connection between truth conditions and explicit descriptions of order is less clear on the other proposals. Moreover, if orders (however construed) play a key explanatory

⁴Ordinary cases can be handled using a finite set of indices. However, the definition of indexing we have given is more general, allowing for arbitrary pluralities of ordered indices. Some examples of sensitivity to order involve infinite collections of objects. For instance, in some relevant cosmological scenario, one might consider the following claim:

- (37) There are infinitely many galaxies and they are positioned in order of size.

Since collections of arbitrary cardinality could figure in sentences exhibiting sensitivity to order, no restriction of size should be imposed on the plurality of indices.

⁵The parallel between descriptions of order and descriptions of ‘ways’ is worth pointing out. Just as *in the order*, the expression *in the way* gives rise to semantic equivalences:

- (39) Annie drives in the way Bonnie does.
- (40) The way Annie drives is the way Bonnie drives.

Adverbials are generally thought to express (higher-order) properties that restrict the extension of the verb. This makes it plausible that descriptions of ways simply stand for such properties rather than basic individuals in the domain of discourse. The same is true of descriptions of order.

role in the analysis of sensitivity to order, postulating fine-grained notions of reference appears to be superfluous. So semantic equivalences of the kind illustrated by (16) and (38) cast additional doubt on serial reference and articulated reference.

Though, as we have just noted, some sentences exhibiting sensitivity to order can be paraphrased in terms of relations between explicit descriptions of order, this is not possible in all cases. Sentences involving *respectively* are cases in point. Nevertheless, their truth conditions can still be specified adequately on the basis of matching conditions between salient indexings:

- (3) John and Mary talked to Bill and Sue, respectively.

In this sentence, we have an indexing f of John and Mary tracking the order in which they appear in the list *John and Mary*, and an indexing g of Bill and Sue tracking the order in which they appear in the list *Bill and Sue*. The relation with respect to which the matching is evaluated is that expressed by the main verb. That is to say, the sentence is true just in case f matches g with respect to the relation of talking. Representing the order of mention by means of positive integers ordered in the natural way, we have $f(1) = j$, $f(2) = m$, $g(1) = b$, $g(2) = s$, where the order isomorphism between the indices is identity. So f matches g with respect to the relation of talking if and only if for every i among 1 and 2, $f(i)$ talked to $g(i)$. This is the case if and only if John talked to Bill and Mary talked to Sue.

More generally, a sentence containing *respectively* is true just in case one of the two relevant indexings matches the other with respect to the relation expressed by the main verb. Thus, a sentence of the form

- (41) The As R the Bs, respectively

is true if and only if f matches g with respect to the relation expressed by R , where f and g are salient indexings of the As and the Bs, respectively.

We can now clarify why our characterization of indexings involves the notion of cover instead of partition. Consider this example:

- (42) John and Bill talked to the music students and the philosophy students, respectively.

The appropriate indexings will track the order of mention of *John* and *Bill*, and the order of mention of *the music students* and *the philosophy students*. If the music students and the philosophy students overlap, the second indexing will cover the plurality of students rather than partition them. This need for covers is not surprising given the role they have been taken to play in the semantics of plurals more generally [Gillon 1987; Schwarzschild 1996].

Next, let us look at a case involving superlatives, which was puzzling for some of the accounts previously discussed:⁶

(18) The three brightest students got the three highest grades, respectively.

There are again two relevant indexings. The first, f , is an indexing of the three brightest students according to their brightness. The second, g , is an indexing of the three highest grades according to their value. The sentence is true just in case f matches g with respect to the relation of obtaining. There is an order isomorphism between the indexing of the students according to brightness and their grades. This determines which student must have gotten which grade in order for the sentence to be true.

This account allows one to make sense of the cases in which, as we observed above, multiple, potentially conflicting orders play a role in the truth conditions of a sentence. On this approach, the semantic contribution of a plural term remains that of supplying the objects plurally referred to by the term. But special expressions make indexings of these objects relevant to the truth conditions of sentences in which they appear. No refinement of the relation of plural reference is needed. As we had set out to show, the standard formulation of plural logic can accommodate sensitivity to order, once the semantic function of special expressions making order relevant is understood. The approach we have outlined avoids the puzzles that arise if one builds order into the way in which a plural term refers and makes plural predication sensitive to it.

In closing, we indicate how the approach we favour accounts for sensitivity to repetition:

(10) The sum of 1, 1, and 2 is 4.

Informal mathematical language is notoriously difficult to analyse. But for the sake of argument, let us assume that in (10), the expression *1, 1, and 2* functions as a genuine plural term, referring to the numbers 1 and 2. The structure of the list provides the appropriate indexing f relative to which the function *sum of* operates. Specifically, the indices can be taken to be the first three positive integers and f to be such that $f(1) = 1$, $f(2) = 1$, and $f(3) = 2$. The function *sum of* operates on a plurality of objects relative to an indexing of them. It sums the objects following their indices. This is to say that the semantic value of the function *sum of* applied to 1 and 2 as indexed by f is $\sum_{i=1}^3 f(i)$ (see also Oliver and Smiley [2004: 663]).

In section 2.3, we contrasted the truth conditions of (10) with those of (11):

⁶A friend of serial reference might suppose that a superlative always refers in the order “given” by the meaning of the adjective. But as we showed in sections 3.2 and 4, no such move is available to advocates of articulated reference and of the multigrade approach.

- (11) The sum of 1 and 2 is 4.

Now we can see that the contrast is due to the fact that a different indexing, g , is involved in this sentence. That is, g is an indexing of the numbers 1 and 2 such that $g(1) = 1$ and $g(2) = 2$. The semantic value of the function *sum of* applied to 1 and 2 as indexed by g is $\sum_{i=1}^2 g(i)$. In this way, we can maintain that the semantic contribution of *1, 1, and 2* and *1 and 2* is the same in both sentences. They both supply the numbers 1 and 2. What differs is their indexing, which explains why one sentence is true while the other is false.

Finally, let us analyse a non-mathematical example of sensitivity to repetition. This brings this phenomenon and sensitivity to order together, highlighting the uniformity and generality of our approach:

- (14) The last three winners of the Roland Garros men's singles are Nadal, Federer, and Nadal, in that order.

In this sentence, we have an indexing f of the last three winners of the Roland Garros men's singles tracking the order in which they won the tournament, and we have an indexing g of Nadal and Federer corresponding to the order of mention in the list *Nadal, Federer, and Nadal*. The sentence is true just in case f matches g with respect to the relation expressed by the main verb, i.e identity.

Note that a perfectly analogous explanation applies to the Fibonacci example also mentioned above:

- (12) The first five Fibonacci numbers are 0, 1, 1, 2, and 3, respectively.

The first indexing, f , tracks the definitional order of the first five Fibonacci numbers. So f is such that $f(1) = 0$, $f(2) = 1$, $f(3) = 1$, $f(4) = 2$, $f(5) = 3$. The second indexing, g , tracks the order of mention of the objects in the list *0, 1, 1, 2, and 3*. That is, g is such that $g(1) = 0$, $g(2) = 1$, $g(3) = 1$, $g(4) = 2$, $g(5) = 3$. The sentence is true if and only if the first indexing matches the second with respect to relation expressed by the main verb, i.e. identity. And this is clearly the case. Therefore sensitivity to repetition does not provide special support for serial reference or articulated reference.

6 Conclusion

As we have seen throughout this article, order plays a semantic role in a variety of cases. Two recent proposals based on the notions of serial reference and articulated reference locate order in the semantic content of plural terms like lists. While these proposals are able to account for some of the data involving lists, they turn out to have

limited scope, as does the approach based on multigrade predicates. In particular, they cannot be extended to explain cases in which two or more orders are compared.

Undoubtedly, one way of making an order salient is to use a list of terms, mentioned in a certain order. But, *pace* serial and articulated reference, this does not mean that order should be assumed to be part of the semantic content of the list. On the contrary, we have argued that the best account of sensitivity to order rejects this assumption. By locating order in the meaning of special expressions, such as *in that order* and *respectively*, the approach based on indexings and matching offers a unified account of the data within the standard formulation of plural logic. It also offers an account of sensitivity to repetition. Descriptions of order and *respectively* narrow the extension of the predicates they modify. They select pluralities on the basis of matching conditions on their relevant indexings. The semantic function of a plural term remains just that of supplying the objects it denotes.⁷

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