

MS200 tutorial 5

1. Outer products

Let

$$\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$$

- (a) Compute $\mathbf{v} \otimes \mathbf{w}$ and $\mathbf{w} \otimes \mathbf{v}$.
- (b) Describe the geometric significance of the matrix

$$\frac{\mathbf{v} \otimes \mathbf{w}}{|\mathbf{v}||\mathbf{w}|}.$$

What does it do when acting upon appropriate vectors?

2. Projections

- (a) Find the matrix P which projects 3-dimensional vectors into the plane orthogonal to

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}.$$

- (b) Project the following vectors into the plane referenced in part (a):

$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -2 \\ -4 \\ 4 \end{pmatrix}.$$

- (c) If a line through the origin is parallel to \mathbf{x} , find the distance between that line and the point described by \mathbf{a} .
- (d) Verify that $P^2 = P$. Interpret this geometrically.