```
clc

% SYMBOLIC SOLUTIONS
syms d2 d4 q1 q2 q3 X Y Z q2_1 q2_2

% D-H parameters
% i:  0       1      2   3   4
al = [0., -pi/2, -pi/2, 0., 0.];
a  = [0.,    0.,    0., 0., 0.];
d  = [0.,    0.,    d2, q3, d4];
th = [0.,    q1,    q2, 0., 0.];

% Direct Kinematics
T01 = transf_i_1_i( 1, al, a, d, th )
T12 = transf_i_1_i( 2, al, a, d, th )
T23 = transf_i_1_i( 3, al, a, d, th )
T34 = transf_i_1_i( 4, al, a, d, th )
T02 = T01*T12
T03 = T02*T23
T04 = T03*T34

P04 = T04(1:3,4)
P04m = [X Y Z]

% INVERSE KIN
q_3 = (-2*d4 + sqrt( 4*d4^2 - 4*( d4^2 + d2^2 - P04m(1)^2 - P04m(2)^2 -
 P04m(3)^2 ) ))/2

sin_th2 = sqrt( (P04m(1)^2 + P04m(2)^2 - d2^2) / (q_3 + d4)^2 )
q_2_1   = -atan2( sin_th2, sqrt(1-sin_th2^2) )
q_2_2   = -q_2_1

alph  = atan2( -P04m(1), P04m(2) )
beta1 = atan2( (q3+d4)*sin(q2_2), d2 )
beta2 = atan2( (q3+d4)*sin(q2_1), d2 )

q_1_1 = alph + beta1
q_1_2 = alph + beta2


% Jacobian
xi     = [1, 1, 0];
z_0_i  = [T01(1:3,3), T02(1:3,3), T03(1:3,3)]
P_dist = [P04-T01(1:3,4), P04-T02(1:3,4), P04-T03(1:3,4)]

% Rotational velocity:
J_o = xi .* z_0_i

% Linear velocity:
J_v = ~xi .* z_0_i + xi .* cross( z_0_i, P_dist, 1 )

% Complete geometric Jacobian;
```

```
J = [J_o; J_v]
```

```
T01 =

[cos(q1), -sin(q1), 0, 0]
[sin(q1),  cos(q1), 0, 0]
[      0,        0, 1, 0]
[      0,        0, 0, 1]
```

```
T12 =

[ cos(q2), -sin(q2), 0,  0]
[       0,        0, 1, d2]
[-sin(q2), -cos(q2), 0,  0]
[       0,        0, 0,  1]
```

```
T23 =

[1,  0, 0,  0]
[0,  0, 1, q3]
[0, -1, 0,  0]
[0,  0, 0,  1]
```

```
T34 =

[1, 0, 0,  0]
[0, 1, 0,  0]
[0, 0, 1, d4]
[0, 0, 0,  1]
```

```
T02 =

[cos(q1)*cos(q2), -cos(q1)*sin(q2), -sin(q1), -d2*sin(q1)]
[cos(q2)*sin(q1), -sin(q1)*sin(q2),  cos(q1),  d2*cos(q1)]
[       -sin(q2),         -cos(q2),        0,           0]
[              0,                0,        0,           1]
```

```
T03 =

[cos(q1)*cos(q2),  sin(q1), -cos(q1)*sin(q2), - d2*sin(q1) -
 q3*cos(q1)*sin(q2)]
[cos(q2)*sin(q1), -cos(q1), -sin(q1)*sin(q2),   d2*cos(q1) -
 q3*sin(q1)*sin(q2)]
[       -sin(q2),        0,         -cos(q2),                        -
q3*cos(q2)]
[              0,        0,                0,
 1]
```

```
T04 =

[cos(q1)*cos(q2),  sin(q1), -cos(q1)*sin(q2), - d2*sin(q1) -
 d4*cos(q1)*sin(q2) - q3*cos(q1)*sin(q2)]
[cos(q2)*sin(q1), -cos(q1), -sin(q1)*sin(q2),   d2*cos(q1) -
 d4*sin(q1)*sin(q2) - q3*sin(q1)*sin(q2)]
[       -sin(q2),        0,         -cos(q2),                                  -
 d4*cos(q2) - q3*cos(q2)]
[              0,        0,                0,
                     1]


P04 =

- d2*sin(q1) - d4*cos(q1)*sin(q2) - q3*cos(q1)*sin(q2)
  d2*cos(q1) - d4*sin(q1)*sin(q2) - q3*sin(q1)*sin(q2)
                              - d4*cos(q2) - q3*cos(q2)


P04m =

[X, Y, Z]


q_3 =

(X^2 + Y^2 + Z^2 - d2^2)^(1/2) - d4


sin_th2 =

((X^2 + Y^2 - d2^2)/(X^2 + Y^2 + Z^2 - d2^2))^(1/2)


q_2_1 =

-atan2(((X^2 + Y^2 - d2^2)/(X^2 + Y^2 + Z^2 - d2^2))^(1/2), (1 - (X^2 + Y^2 -
 d2^2)/(X^2 + Y^2 + Z^2 - d2^2))^(1/2))


q_2_2 =

atan2(((X^2 + Y^2 - d2^2)/(X^2 + Y^2 + Z^2 - d2^2))^(1/2), (1 - (X^2 + Y^2 -
 d2^2)/(X^2 + Y^2 + Z^2 - d2^2))^(1/2))


alph =

atan2(-X, Y)


beta1 =
```

```
atan2(sin(q2_2)*(d4 + q3), d2)
```

```
beta2 =

atan2(sin(q2_1)*(d4 + q3), d2)
```

```
q_1_1 =

atan2(sin(q2_2)*(d4 + q3), d2) + atan2(-X, Y)
```

```
q_1_2 =

atan2(sin(q2_1)*(d4 + q3), d2) + atan2(-X, Y)
```

```
z_0_i =

[0, -sin(q1), -cos(q1)*sin(q2)]
[0,  cos(q1), -sin(q1)*sin(q2)]
[1,        0,         -cos(q2)]
```

```
P_dist =

[- d2*sin(q1) - d4*cos(q1)*sin(q2) - q3*cos(q1)*sin(q2), - d4*cos(q1)*sin(q2)
 - q3*cos(q1)*sin(q2), -d4*cos(q1)*sin(q2)]
[  d2*cos(q1) - d4*sin(q1)*sin(q2) - q3*sin(q1)*sin(q2), - d4*sin(q1)*sin(q2)
 - q3*sin(q1)*sin(q2), -d4*sin(q1)*sin(q2)]
[                              - d4*cos(q2) - q3*cos(q2),                 -
 d4*cos(q2) - q3*cos(q2),         -d4*cos(q2)]
```

```
J_o =

[0, -sin(q1), 0]
[0,  cos(q1), 0]
[1,        0, 0]
```

```
J_v =

[  d4*sin(q1)*sin(q2) - d2*cos(q1) + q3*sin(q1)*sin(q2),
                                                  -cos(q1)*(d4*cos(q2) +
 q3*cos(q2)), -cos(q1)*sin(q2)]
[- d2*sin(q1) - d4*cos(q1)*sin(q2) - q3*cos(q1)*sin(q2),
                                                  -sin(q1)*(d4*cos(q2) +
 q3*cos(q2)), -sin(q1)*sin(q2)]
[
  0, cos(q1)*(d4*cos(q1)*sin(q2) + q3*cos(q1)*sin(q2)) +
 sin(q1)*(d4*sin(q1)*sin(q2) + q3*sin(q1)*sin(q2)),         -cos(q2)]
```

```
J =

[                                                    0,
                                                                -
sin(q1),                0]
[                                                    0,

 cos(q1),                0]
[                                                    1,

 0,                0]
[  d4*sin(q1)*sin(q2) - d2*cos(q1) + q3*sin(q1)*sin(q2),
                                                  -cos(q1)*(d4*cos(q2) +
 q3*cos(q2)), -cos(q1)*sin(q2)]
[- d2*sin(q1) - d4*cos(q1)*sin(q2) - q3*cos(q1)*sin(q2),
                                                  -sin(q1)*(d4*cos(q2) +
 q3*cos(q2)), -sin(q1)*sin(q2)]
[
  0, cos(q1)*(d4*cos(q1)*sin(q2) + q3*cos(q1)*sin(q2)) +
 sin(q1)*(d4*sin(q1)*sin(q2) + q3*sin(q1)*sin(q2)),          -cos(q2)]
```

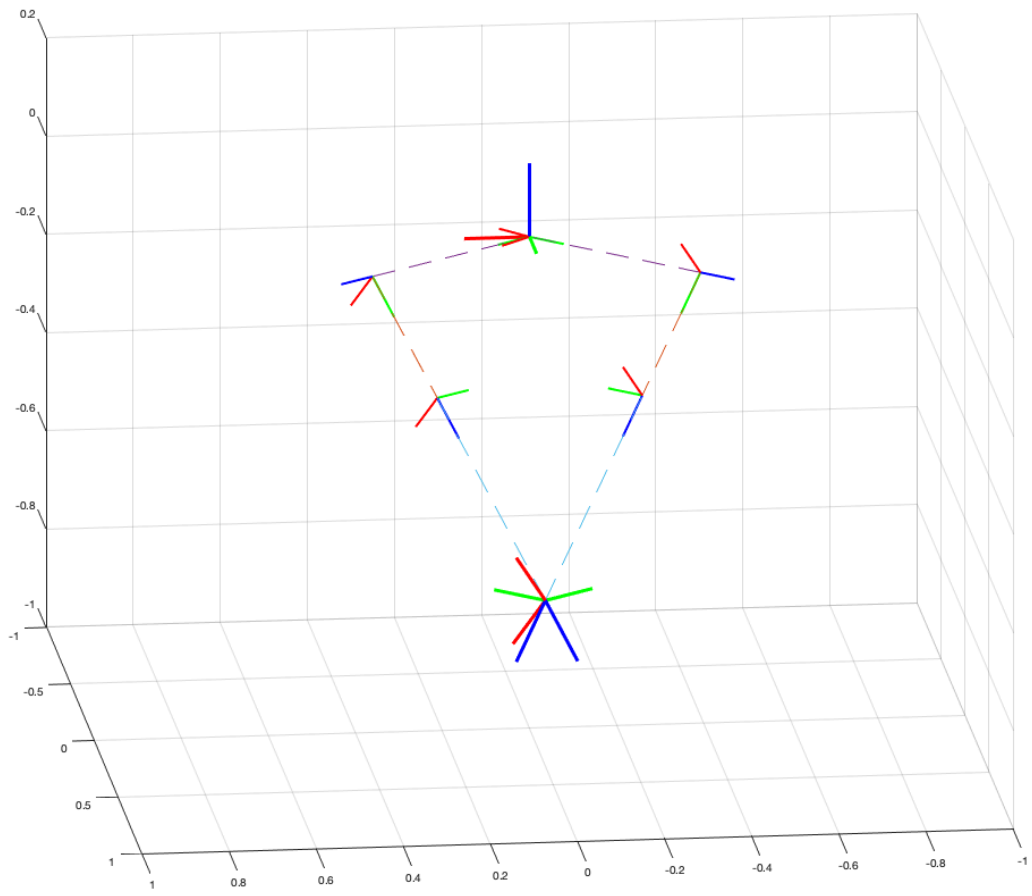
NUMERIC IMPLEMENTATION

```
draw = true;

if draw
    alfa = 340;
    beta = 140;
    l = 1;
    axs  = axes( 'XLim', [-l l], 'YLim', [-l l], 'ZLim', [-l 0.2] );
    view( alfa, beta ); grid on;
    handles(1) = axs;
end


% DH parameters
%        i:    0      1      2   3    4
AL =          [0, -pi/2, -pi/2,  0,   0];
 A =          [0,     0,     0,  0,   0];
 D = @(q3)    [0,     0,   0.5, q3, 0.5];
TH = @(q1,q2) [0,    q1,    q2,  0,   0];

q = [pi/4, -pi/4, 0.3];

% DIRECT KIN
T01 =       transf_i_1_i( 1, AL, A, D(q(3)), TH(q(1),q(2)) )
T02 = T01 * transf_i_1_i( 2, AL, A, D(q(3)), TH(q(1),q(2)) )
T03 = T02 * transf_i_1_i( 3, AL, A, D(q(3)), TH(q(1),q(2)) )
T04 = T03 * transf_i_1_i( 4, AL, A, D(q(3)), TH(q(1),q(2)) )

if draw
```

```
    [~, handlesR] = DK_draw( AL, A, D(q(3)), TH(q(1),q(2)), handles, true );
    pause()
end

% INVERSE KIN
D_fix = D(q(3))

P04 = T04(1:3,4)

q_3 = (-2*D_fix(5) + sqrt( 4*D_fix(5)^2 - 4*( D_fix(5)^2 + D_fix(3)^2 -
 P04(1)^2 - P04(2)^2 - P04(3)^2 ) ))/2

sin_th2 = sqrt( (P04(1)^2 + P04(2)^2 - D_fix(3)^2) / (q_3 + D_fix(5))^2 );
q_2_1   = -atan2( sin_th2, sqrt(1-sin_th2^2) )
q_2_2   = -q_2_1

alph  = atan2( -P04(1), P04(2) );
beta1 = atan2( (q_3+D_fix(5))*sin(q_2_2), D_fix(3) );
beta2 = atan2( (q_3+D_fix(5))*sin(q_2_1), D_fix(3) );

q_1_1 = alph + beta1
q_1_2 = alph + beta2


% inverse check
if draw
    [T1, ~] = DK_draw( AL, A, D(q_3), TH(q_1_1, q_2_1), handles, true )
    pause()
    [T2, ~] = DK_draw( AL, A, D(q_3), TH(q_1_2, q_2_2), handles, true )
end

% Geometric Jacobian
% Manipulator: REVOLUTE - REVOLUTE - PRISMATIC

xi     = [1, 1, 0, 0];
z_0_i  = [T01(1:3,3), T02(1:3,3), T03(1:3,3), T04(1:3,3)];
P_dist = [P04-T01(1:3,4), P04-T02(1:3,4), P04-T03(1:3,4), P04-T04(1:3,4)];

% Rotational velocity:
J_o = xi .* z_0_i

% Linear velocity:
J_v = ~xi .* z_0_i + xi .* cross( z_0_i, P_dist, 1 )

% Complete geometric Jacobian;
J = [J_o; J_v]


T01 =

    0.7071   -0.7071         0         0
    0.7071    0.7071         0         0
         0         0    1.0000         0
         0         0         0    1.0000
```

```
T02 =

    0.5000    0.5000   -0.7071   -0.3536
    0.5000    0.5000    0.7071    0.3536
    0.7071   -0.7071         0         0
         0         0         0    1.0000


T03 =

    0.5000    0.7071    0.5000   -0.2036
    0.5000   -0.7071    0.5000    0.5036
    0.7071         0   -0.7071   -0.2121
         0         0         0    1.0000


T04 =

    0.5000    0.7071    0.5000    0.0464
    0.5000   -0.7071    0.5000    0.7536
    0.7071         0   -0.7071   -0.5657
         0         0         0    1.0000


D_fix =

         0         0    0.5000    0.3000    0.5000


P04 =

    0.0464
    0.7536
   -0.5657


q_3 =

    0.3000


q_2_1 =

   -0.7854


q_2_2 =

    0.7854


q_1_1 =
```

```
    0.7854


q_1_2 =

   -0.9085


T1 =

    0.5000    0.7071    0.5000    0.0464
    0.5000   -0.7071    0.5000    0.7536
    0.7071         0   -0.7071   -0.5657
         0         0         0    1.0000


T2 =

    0.4348   -0.7886   -0.4348    0.0464
   -0.5576   -0.6149    0.5576    0.7536
   -0.7071         0   -0.7071   -0.5657
         0         0         0    1.0000


J_o =

         0   -0.7071         0         0
         0    0.7071         0         0
    1.0000         0         0         0


J_v =

   -0.7536   -0.4000    0.5000    0.5000
    0.0464   -0.4000    0.5000    0.5000
         0   -0.5657   -0.7071   -0.7071


J =

         0   -0.7071         0         0
         0    0.7071         0         0
    1.0000         0         0         0
   -0.7536   -0.4000    0.5000    0.5000
    0.0464   -0.4000    0.5000    0.5000
         0   -0.5657   -0.7071   -0.7071
```



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