```
clc

% SYMBOLIC SOLUTIONS
syms q1 q2 q3 a2 d4 X Y Z A B C


% D-H parameters
% i:  0   1       2      3  4
al = [0., 0., -pi/2, -pi/2, 0.];
a  = [0., 0.,   -a2,    0., 0.];
d  = [0., q1,    0.,    0., d4];
th = [0., 0.,    q2,    q3, 0.];



% Direct Kinematics
T01 = transf_i_1_i( 1, al, a, d, th )
T12 = transf_i_1_i( 2, al, a, d, th )
T23 = transf_i_1_i( 3, al, a, d, th )
T34 = transf_i_1_i( 4, al, a, d, th )
T02 = T01*T12
T03 = T02*T23
T04 = T03*T34

P04  = T04(1:3,4)
P04m = [X Y Z]

% INVERSE KIN
q_2 = atan2( -P04m(2), -P04m(1) )

if cos(q2) ~= 0
    sin_q3 = (P04m(1) - cos(q2)*a2) / (-cos(q2)*d4)
else
    sin_q3 = (P04m(2) - sin(q2)*a2) / (-sin(q2)*d4)
end

cos_q3 = sqrt(1-sin(q3)^2)
q3_1   = atan2( sin(q3), cos(q3) )
q3_2   = -q3_1 - pi

q1_1 = P04m(3) + d4*cos(q3)
q1_2 = P04m(3) - d4*cos(q3)


% Geometric Jacobian
xi = [0, 1, 1];

z_0_i = [T01(1:3,3), T02(1:3,3), T03(1:3,3)]
P_dist  = [P04-T01(1:3,4), P04-T02(1:3,4), P04-T03(1:3,4)]

J_o   = xi .* z_0_i
```

```
J_v   = ~xi .* z_0_i + xi .* cross( z_0_i, P_dist, 1 )

J = [J_v; J_o]
```

T01 =

```
[1, 0, 0,  0]
[0, 1, 0,  0]
[0, 0, 1, q1]
[0, 0, 0,  1]
```

T12 =

```
[cos(q2), -sin(q2), 0, 0]
[sin(q2),  cos(q2), 0, 0]
[      0,        0, 1, 0]
[      0,        0, 0, 1]
```

T23 =

```
[ cos(q3), -sin(q3), 0, -a2]
[       0,        0, 1,   0]
[-sin(q3), -cos(q3), 0,   0]
[       0,        0, 0,   1]
```

T34 =

```
[1,  0, 0,  0]
[0,  0, 1, d4]
[0, -1, 0,  0]
[0,  0, 0,  1]
```

T02 =

```
[cos(q2), -sin(q2), 0,  0]
[sin(q2),  cos(q2), 0,  0]
[      0,        0, 1, q1]
[      0,        0, 0,  1]
```

T03 =

```
[cos(q2)*cos(q3), -cos(q2)*sin(q3), -sin(q2), -a2*cos(q2)]
[cos(q3)*sin(q2), -sin(q2)*sin(q3),  cos(q2), -a2*sin(q2)]
[       -sin(q3),         -cos(q3),        0,          q1]
[              0,                0,        0,           1]
```

T04 =

```
[cos(q2)*cos(q3),  sin(q2), -cos(q2)*sin(q3), - a2*cos(q2) -
 d4*cos(q2)*sin(q3)]
[cos(q3)*sin(q2), -cos(q2), -sin(q2)*sin(q3), - a2*sin(q2) -
 d4*sin(q2)*sin(q3)]
[       -sin(q3),        0,         -cos(q3),                   q1 -
 d4*cos(q3)]
[              0,        0,                0,
 1]


P04 =

- a2*cos(q2) - d4*cos(q2)*sin(q3)
- a2*sin(q2) - d4*sin(q2)*sin(q3)
                  q1 - d4*cos(q3)


P04m =

[X, Y, Z]


q_2 =

atan2(-Y, -X)


sin_q3 =

-(X - a2*cos(q2))/(d4*cos(q2))


cos_q3 =

(1 - sin(q3)^2)^(1/2)


q3_1 =

atan2(sin(q3), cos(q3))


q3_2 =

- pi - atan2(sin(q3), cos(q3))


q1_1 =

Z + d4*cos(q3)


q1_2 =
```

```
Z - d4*cos(q3)


z_0_i =

[0, 0, -sin(q2)]
[0, 0,  cos(q2)]
[1, 1,        0]


P_dist =

[- a2*cos(q2) - d4*cos(q2)*sin(q3), - a2*cos(q2) - d4*cos(q2)*sin(q3), -
d4*cos(q2)*sin(q3)]
[- a2*sin(q2) - d4*sin(q2)*sin(q3), - a2*sin(q2) - d4*sin(q2)*sin(q3), -
d4*sin(q2)*sin(q3)]
[                      -d4*cos(q3),                       -d4*cos(q3),
 -d4*cos(q3)]


J_o =

[0, 0, -sin(q2)]
[0, 0,  cos(q2)]
[0, 1,        0]


J_v =

[0,   a2*sin(q2) + d4*sin(q2)*sin(q3),                          -
d4*cos(q2)*cos(q3)]
[0, - a2*cos(q2) - d4*cos(q2)*sin(q3),                          -
d4*cos(q3)*sin(q2)]
[1,                                 0, d4*sin(q3)*cos(q2)^2 +
 d4*sin(q3)*sin(q2)^2]


J =

[0,   a2*sin(q2) + d4*sin(q2)*sin(q3),                          -
d4*cos(q2)*cos(q3)]
[0, - a2*cos(q2) - d4*cos(q2)*sin(q3),                          -
d4*cos(q3)*sin(q2)]
[1,                                 0, d4*sin(q3)*cos(q2)^2 +
 d4*sin(q3)*sin(q2)^2]
[0,                                 0,                                      -
sin(q2)]
[0,                                 0,
 cos(q2)]
[0,                                 1,
    0]
```

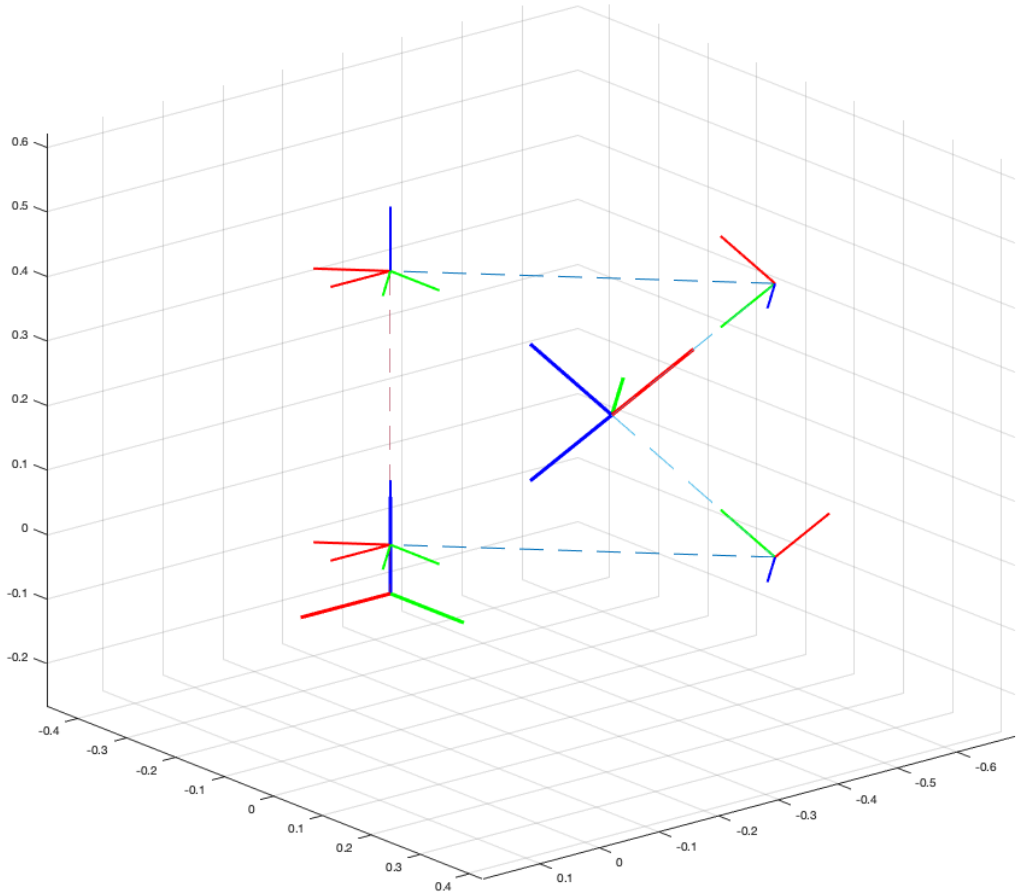
NUMERICAL SOLUTION

```
clc

draw = true;

if draw
    alfa = 340;
    beta = 140;
    l = 1;
    axs  = axes( 'XLim', [-l l], 'YLim', [-l l], 'ZLim', [-l l] );
    view( alfa, beta ); grid on;
    handles(1) = axs;
end

% DH parameters
%        i:    0      1    2   3
AL =          [0., 0., -pi/2, -pi/2,  0.];
 A =          [0., 0.,  -0.5,    0.,  0.];
 D = @(q1)    [0., q1,    0.,    0., 0.3];
TH = @(q2,q3) [0., 0.,    q2,    q3,  0.];

q = [0.5, -pi/4, -pi/4];

% DIRECT KIN
T01 =       transf_i_1_i( 1, AL, A, D(q(1)), TH(q(2),q(3)) );
T02 = T01 * transf_i_1_i( 2, AL, A, D(q(1)), TH(q(2),q(3)) );
T03 = T02 * transf_i_1_i( 3, AL, A, D(q(1)), TH(q(2),q(3)) );
T04 = T03 * transf_i_1_i( 4, AL, A, D(q(1)), TH(q(2),q(3)) )


if draw
    DK_draw( AL, A, D(q(1)), TH(q(2),q(3)), handles, true );
    pause()
end

% INVERSE KIN
P04 = T04(1:3,4)
D_fix = D(q(1));

q2 = atan2( -P04(2), -P04(1) )

if cos(q2) ~= 0
    sin_q3 = (P04(1) - cos(q2)*A(3)) / (-cos(q2)*D_fix(5));
else
    sin_q3 = (P04(2) - sin(q2)*A(3)) / (-sin(q2)*D_fix(5));
end

cos_q3 = sqrt(1-sin_q3^2);
q3_1 = atan2( sin_q3, cos_q3 )
q3_2 = -q3_1 - pi

q1_1 = P04(3) + D_fix(5)*cos_q3
```

```
q1_2 = P04(3) - D_fix(5)*cos_q3

% inverse check
if draw
    [T1, ~] = DK_draw( AL, A, D(q1_1), TH(q2,q3_1), handles, true )
    pause()
    [T2, ~] = DK_draw( AL, A, D(q1_2), TH(q2,q3_2), handles, true )
end


T04 =

    0.5000   -0.7071    0.5000   -0.2036
   -0.5000   -0.7071   -0.5000    0.2036
    0.7071         0   -0.7071    0.2879
         0         0         0    1.0000


P04 =

   -0.2036
    0.2036
    0.2879


q2 =

   -0.7854


q3_1 =

   -0.7854


q3_2 =

   -2.3562


q1_1 =

    0.5000


q1_2 =

    0.0757


T1 =

    0.5000   -0.7071    0.5000   -0.2036
   -0.5000   -0.7071   -0.5000    0.2036
```

```
    0.7071         0   -0.7071    0.2879
         0         0         0    1.0000
```

```
T2 =

   -0.5000   -0.7071    0.5000   -0.2036
    0.5000   -0.7071   -0.5000    0.2036
    0.7071         0    0.7071    0.2879
         0         0         0    1.0000
```



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