Supplementary Material

1 Proof of Theorem VI.1

We give the detailed definition of the transformation from $\overline{\text{AD}}$ to $\overline{\text{MWIS}}$ by replacing directed edges E_I with undirected edges $E_{I\to C}$.

Predicate 1.1 (R)**.** *$R(n_i, n_j)$ denotes if n_j is reachable from n_i by E_I. An inductive definition of predicate R is as follows,*

1. $\forall n_i \in N, R(n_i, n_i)$.
2. $\forall n_i, n_j, n_k \in N,\ R(n_i, n_j) \wedge \langle n_j, n_k\rangle \in E_I \to R(n_i, n_k)$

Predicate R is reflexive and transitive by its definition. That is, $\forall n_i, n_j, n_k \in N$ it must satisfy:

- **Reflexivity**: $R(n_i, n_i)$, i.e., every node is reachable from itself.
- **Transitivity**: if $R(n_i, n_j) \wedge R(n_j, n_k)$, then $R(n_i, n_k)$.

Definition 1.1 (T)**.** *$T : G_{\overline{AD}} \to G_{\overline{MWIS}}$, denoting the transformation from $G_{\overline{AD}} = (N, E_I, E_C)$ to $G_{\overline{MWIS}} = (N, \varnothing, E_C \cup E_{I\to C})$ by replacing directed edges E_I with undirected edges $E_{I\to C}$. Specifically, $\forall n_{a_s}, n_{a_t}, n_{b_s}, n_{b_t} \in N$, s.t. $(n_{a_t}, n_{b_t}) \in E_C \wedge R(n_{a_s}, n_{a_t}) \wedge R(n_{b_s}, n_{b_t}), (n_{a_s}, n_{b_s}) \in E_{I\to C}$.*

Then Theorem VI.1 can also be defined as follows.

Theorem. *Assuming the optimal solution of $\overline{AD}$ in $G_{\overline{AD}}$ is $N'_{\overline{AD}}$, and the one of $\overline{MWIS}$ in $G_{\overline{MWIS}} = T(G_{\overline{AD}})$ is $N'_{\overline{MWIS}}$, $N'_{\overline{AD}} = N'_{\overline{MWIS}}$.*

The proof of the Theorem is given as follows.

Proof.

1) First, we prove $N'_{\overline{\text{AD}}}$ is also an independent set of $G_{\overline{\text{MWIS}}}$. $\forall n_i, n_j \in N'_{\overline{\text{AD}}}$,

   a) According to the definition of $\overline{\text{AD}}$, as $n_i, n_j \in N'_{\overline{\text{AD}}}$, we have $(n_i, n_j) \notin E_C$.

   b) Assume $(n_i, n_j) \in E_{I\to C}$. According to the definition of T, $\exists n_{a_t}, n_{b_t}$, s.t. $(n_{a_t}, n_{b_t}) \in E_C \wedge R(n_i, n_{a_t}) \wedge R(n_j, n_{b_t})$.
   As $n_i \in N'_{\overline{\text{AD}}}$ and $R(n_i, n_{a_t})$, n_{a_t} must be in $N'_{\overline{\text{AD}}}$. Similarly, n_{b_t} is also in $N'_{\overline{\text{AD}}}$.
   Thus, both n_{a_t} and n_{b_t} are in $N'_{\overline{\text{AD}}}$. We can have $(n_{a_t}, n_{b_t}) \notin E_C$. Contradiction.
   Therefore, $(n_i, n_j) \notin E_{I\to C}$

   c) According to a) and b), $\forall n_i, n_j \in N'_{\overline{\text{AD}}}$, $(n_i, n_j) \notin E_C$ and $(n_i, n_j) \notin E_{I\to C}$. Thus, $N'_{\overline{\text{AD}}}$ is also an independent set of $G_{\overline{\text{MWIS}}}$.

2) Second, we prove $N'_{\overline{\text{MWIS}}}$ is also a solution of $\overline{\text{AD}}$ in $G_{\overline{\text{AD}}}$.

   a) $\forall n_i, n_j \in N'_{\overline{\text{MWIS}}}$, according to the definition of $\overline{\text{MWIS}}$, $(n_i, n_j) \notin E_C$.

   b) $\forall n_i \in N'_{\overline{\text{MWIS}}}$, we are going to prove that $\forall n_j \in N$ s.t. $\langle n_i, n_j\rangle \in E_I$, $n_j \in N'_{\overline{\text{MWIS}}}$ by contradiction.
   We assume $\exists n_j$ s.t. $\langle n_i, n_j\rangle \in E_I$ and $n_j \notin N'_{\overline{\text{MWIS}}}$.

i) $\forall n_k$ s.t. $(n_j, n_k) \in E_C$, as $R(n_i, n_j)$ and $R(n_k, n_k)$, according to the definition of T, we can have $(n_i, n_k) \in E_{I \to C}$. Thus, $n_k \notin N'_{\text{MWIS}}$.

ii) $\forall n_k$ s.t. $(n_j, n_k) \in E_{I \to C}$, according to the definition of T, $\exists n_{j_t}, n_{k_t} \in N$, s.t. $(n_{j_t}, n_{k_t}) \in E_C \wedge R(n_j, n_{j_t}) \wedge R(n_k, n_{k_t})$. As $R(n_i, n_j) \wedge R(n_j, n_{j_t})$, we can also have $R(n_i, n_{j_t})$ (i.e., transitivity of R), which implies $\exists n_{j_t}, n_{k_t} \in N$, s.t. $(n_{j_t}, n_{k_t}) \in E_C \wedge R(n_i, n_{j_t}) \wedge R(n_k, n_{k_t})$. Thus, $(n_i, n_k) \in E_{I \to C}$ by the definition of T, and $n_k \notin N'_{\text{MWIS}}$.

Therefore, $\forall n_k$ s.t. $(n_j, n_k) \in E_C \cup E_{I \to C}, n_k \notin N'_{\text{MWIS}}$. $N''_{\text{MWIS}} = N'_{\text{MWIS}} + \{n_j\}$ is also an independent set.

As $w : N \to \mathbb{R}_{\geq 0}$, $w(N''_{\text{MWIS}}) \geq w(N'_{\text{MWIS}})$, N'_{MWIS} is not guaranteed to be the optimal solution. Contradiction.

Therefore, $\forall n_i \in N'_{\text{MWIS}}$, if $\langle n_i, n_j \rangle \in E_I$, we must have $n_j \in N'_{\text{MWIS}}$.

c) According to a) and b) and the definition of $\overline{\text{AD}}$, N'_{MWIS} is also a solution of $\overline{\text{AD}}$ in $G_{\overline{\text{AD}}}$.

3) According to 1) and 2), we can have $N'_{\overline{\text{AD}}} = N'_{\text{MWIS}}$.

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