

4.7.7.

Density function for a normal random variable is :

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-1}{2\sigma^2} \cdot (x-\mu)^2} \quad \text{--- (1)}$$

using (1), probability for a given (x, k) pair will be :

$$P_k(x) = \frac{\pi_k \cdot \exp\left(-\frac{1}{2\sigma_k^2} \cdot (x-\mu_k)^2\right)}{\sqrt{2\pi\sigma_k^2}}$$

(2)

$$\sum_{k=1}^K \frac{\pi_k}{\sqrt{2\pi\sigma_k^2}} \cdot \exp\left(-\frac{1}{2\sigma_k^2} (x-\mu_k)^2\right)$$

Plugging in values, where issuing dividend is classified as 1 & not issuing a dividend as 0.

2/10

$$\pi_K = \pi_{\text{yes}} = 0.8$$

$$\pi_L = \pi_{\text{no}} = 0.2$$

$$x = 4$$

$$\mu_K = \mu_{\text{yes}} = 10$$

$$\mu_L = 0 = \mu_L$$

$$\sigma^2 = 36$$

$$P(y = \text{yes} | x = 4)$$

$$\frac{P(4)}{P(4)} = 0.8 \cdot \exp \left[-\frac{1}{2 \cdot 36} (4 - 10)^2 \right]$$

$$0.8 \cdot \exp \left[-\frac{1}{2 \cdot 36} (4 - 10)^2 \right] +$$

$$0.2 \exp \left[-\frac{1}{2 \cdot 36} (4 - 0)^2 \right]$$

$$= 0.8 \cdot \exp \left[-\frac{36}{2 \cdot 36} \right]$$

$$0.8 \exp \left[-\frac{36}{2 \cdot 36} \right] + 0.2 \exp \left[-\frac{16}{2 \cdot 36} \right]$$

$$= 0.8 \exp \left[-\frac{1}{2} \right]$$

$$0.8 \exp \left[-\frac{1}{2} \right] + 0.2 \exp \left[-\frac{2}{9} \right]$$

$$= 0.7519$$

$$\therefore P(\text{yes} | x = 4) = \underline{\underline{0.752}}$$