Improving Accuracy of Stress Relaxation Computation by Modeling Strain-Overshoot

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Abstract

Materials that are stiff enough to bear structural loads while also having the ability to dampen vibrations are useful in many engineering applications. Viscoelastic materials such as rubbers and foams are extensively used in protective applications due to their rate-dependent modulus and damping ratio. However, due to imperfect experimental conditions and rate sensitivity of viscoelastic material properties, an inconsistency occurs between quasistatic and dynamic measurements. In this work, we propose a combination of experimental and computational methodologies to improve the accuracy of stress-relaxation viscoelastic measurements by modeling the dynamic strain overshoot effect that occurs in commercial tensile testing setups. Using viscoelastic polydimethylsiloxane (PDMS) as the model material, we propose performing experimental stress-relaxation measurements and using Boltzmann integral and a dynamic strain-response function to estimate the actual stress-relaxation modulus of the material. We will also conduct dynamic mechanical analysis (DMA) measurements of PDMS on a custom-built DMA setup to verify the connection between dynamic modulus and actual relaxation modulus. We envision our work will help develop more accurate constitutive models to predict the mechanical behavior of viscoelastic materials, such as energy absorption in foams and rubbers and disease diagnosis in biological tissues.

Keywords: Viscoelasticity, Stress-Relaxation, Strain Overshoot, Polydimethylsiloxane, Dynamic Mechanical Analysis

Introduction

Viscoelastic materials are ubiquitous, from polymers such as rubbers, plastics, and foams to biological materials such as tissues and cork exhibit viscoelastic properties. Viscoelastic materials are materials that exhibit both viscous (fluid-like) and elastic (solid-like) behavior in response to an applied load [1]. They are widely used in a variety of applications, from the manufacture of adhesives, coatings, and elastomers to the development of cushioning and damping materials [2]. The unique combination of viscous and elastic behavior in viscoelastic materials provides several advantages over purely viscous or purely elastic materials, such as improved resistance to impacts, enhanced durability, and better energy dissipation. One of the key properties of viscoelastic materials is their time-dependent behavior. When subjected to constant strain, they undergo stress relaxation—a gradual decay of stress with time until the material fully relaxes [3].

This effect is a direct consequence of the internal viscous forces that develop within the material as it deforms, which serve to dissipate energy and reduce the overall stress level. Stress relaxation experiments are significant because they provide us with the relaxation modulus and dynamic modulus values under different loading conditions. These properties help predict the material's response to different tensile loads in various applications. The relaxation effect of the time-dependent viscoelastic materials has been largely quantified with the quasi-linear viscoelastic (QLV) model [4]. The QLV parameters help determine the Dynamic Moduli, Relaxation Time, and Hysteresis of the material, by which engineers design and optimize various structures, and components. The QLV parameters are estimated by fitting the model to a single-step response function relaxation experiment. A step function, which stipulates that the strain has been applied instantaneously, is the ideal theoretical strain history but is unattainable in practice. Due to instrumental limitations, there is always a finite time over which strain has been applied, and an overshoot of the target strain occurs to some degree in most stress relaxation experiments. It is crucial to quantify the impact of overshoot on the estimation of QLV parameters, as neglecting to reduce or counteract overshoot could result in substantial inaccuracies. Although there have been efforts to reduce the strain overshoot, there is no current model to calculate the correct viscoelastic properties after accounting for the strain overshoot. Therefore, the objective of my research project is to develop a computational model to correctly calculate the QLV parameters by including a step-response function. These corrected QLV parameters will help us accurately determine viscoelastic properties such as Dynamic Moduli and Relaxation Time of the material, and the shift in values after utilizing the step response function (1) which mimics the strain overshoot:

$$\varepsilon(\tau) = \varepsilon_0\left(1 - \frac{1}{\beta} e^{-\zeta\omega_n\tau} \sin(\beta\omega_n\tau + \theta)\right), \quad \theta = \arctan\left(\frac{n\pi}{\log\left(\frac{\varepsilon_0(-1)^{n+1}}{\varepsilon_{max}-\varepsilon_0}\right)}\right), \quad \beta = \sin\theta, \zeta = \cos\theta, w_n = \frac{n\pi}{\tau_{max}\beta} \tag{1}$$

where τ is the time in seconds, ε_0 refers to the applied constant strain value, ε_{max} is the peak of the strain overshoot, τ_{max} represents the corresponding time at ε_{max}. This model aims to establish a direct relationship between stress relaxation experiments and Dynamic Mechanical Analysis (DMA) experiments so that with only stress relaxation data, one can accurately predict the storage and loss modulus. The proposed model will contribute to the advancement of constitutive models for viscoelastic materials, and enhance the accuracy of predictions regarding the dissipative properties such as loss-tangent, loss modulus, and hysteresis.

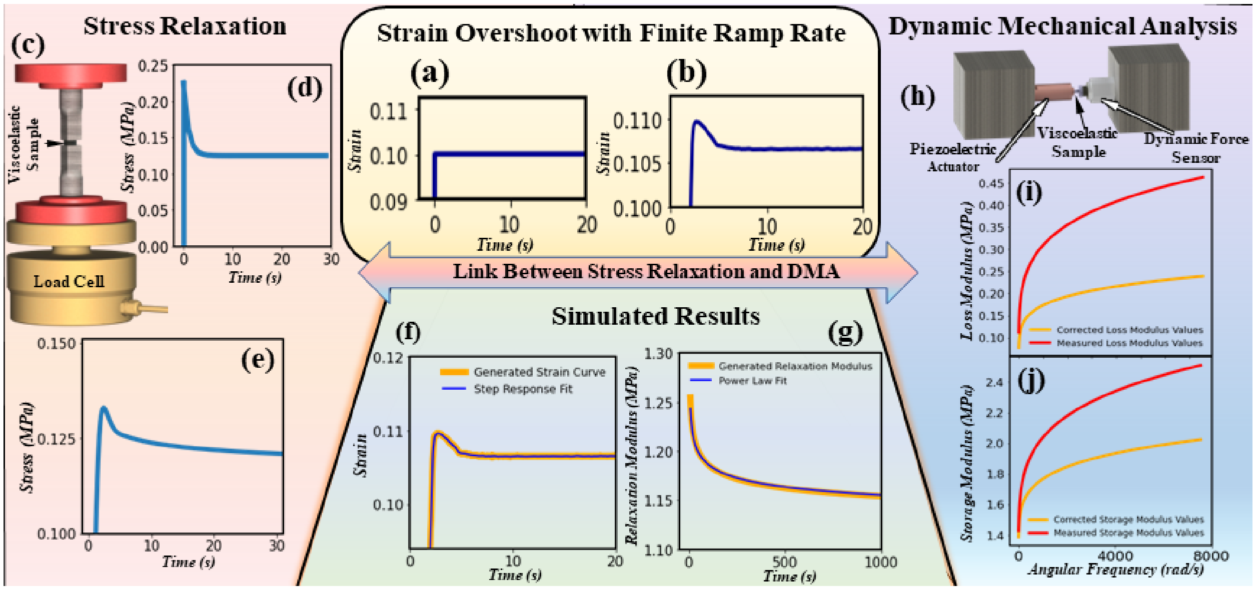

Figure 1: (a) Expected constant strain input (b) Applied strain input due to overshoot. (c) Stress-relaxation testing setup. (d,e) Expected stress response and observed stress response in relaxation experiment. (f) Simulated strain overshoot curve-fit with the step response function. (g) Simulated relaxation modulus curve-fit with power law equation. (h) Custom-built DMA setup. (i,j) Comparison between dynamic modulus values in a simulated experiment, with new values computed by our model

Methods

Previous research models incorporated directly curve-fitting the stress and strain data obtained from the relaxation experiments to obtain the QLV parameters. However, as we have discussed before, these datasets have been offset by the instrumental overshoot. For this project, I will utilize the step response function (1) which follows the strain overshoot with finite ramp time, to develop accurate stress values with the Boltzmann superposition integral (2) :

$$\sigma(t) = \int_0^t E(t-\tau)\frac{d\varepsilon(t)}{d\tau}d\tau \tag{2}$$

For the purpose of this project, I will be developing my model mainly on the Python programming language, because of my familiarity with building computational models with it and the versatility it offers with its libraries. I will collect data by conducting stress relaxation experiments on PDMS samples, utilizing the Instron testing machine at our research group's laboratory. After conducting the relaxation experiments for a specific strain rate, the noisy datasets will be cleaned with a moving average filter and standardized for analysis. For curve-fitting, I will be using the *scipy.optimize* library, which is widely used in scientific research projects. The relaxation modulus of the dataset will be obtained by dividing the experimental stress values by the input strain percentage. This curve will be masked for values before the time at which the strain attains the constant value [Fig. 1(g)]. This is done to ignore the effect of overshoot, and calculate the offset parameters. This relaxation modulus follows the power law equation (3), which is in agreement with experimental results obtained in many viscoelastic elastomers and biological materials. These power law parameters directly dictate the dynamic moduli of these viscoelastic materials:

$$E(t) = E_\infty + E_t t^{-n}, \quad E_\infty, E_t, n > 0 \tag{3}$$

where E_∞ refers to the value attained by the curve at $t = \infty$, E_t is the initial value of the dynamic modulus and n is the characteristic relaxation time exponent. I will proceed with estimating the effect of strain overshoot through the Boltzmann Superposition Integral (2), which provides the overshoot-dependent stress values for a specific relaxation modulus and strain curve. Using the power law equation for relaxation modulus (3) and step response curve (1) for strain, along with the parameters obtained from the previous curve-fits, this complex integral will provide the stress values that account for the strain overshoot. I suspect an analytical approach to solve this complex integral will lead to large time complexity. Therefore, I will approach this integral using numerical integration techniques. In Python, the SciPy library contains a *quad()* which enables us to do this computation efficiently. This calculated stress curve will be curve-fit once again with the power law relationship (3) to determine the new power law parameters. These parameters will determine the dynamic moduli which account for the strain overshoot. These two different dynamic moduli values will be compared for a specific strain rate applied on the material, and understand the degree of change with the strain overshoot. This model will also include an algorithm to predict the relationship between the different n and E_t values from the power law (3) equation when comparing the different approaches. This is done by computing the stress curves for a range of n and E_t values that represent overshoot-free calculations. Each of these curves will be subjected to the same computations as before, providing us a new corresponding n and E_t value that accounts for the overshoot effect. This project aligns with the bigger research motive of connecting DMA experiments for a wide range of frequencies, to the stress relaxation experiments in viscoelastic experiments. Through the dynamic moduli values obtained from the DMA experiments, we can backtrack to find the stress response. This past semester, I spent time conducting simulated results to quantify the effect of the strain overshoot on relaxation experiments. Strain histories with different amounts of overshoot were generated in Python using initial conditions for the stress response curve. There was a significant change in dynamic moduli values, with 26% difference between the QLV parameters. These results have encouraged my advisor and me to proceed with this project [Fig. 1(i,j)].

Timeline

From **September to October**, I will perform stress relaxation experiments with the Instron tensile testing instrument on PDMS samples, with different strain rates. From **October to November** I will analyze the data, utilize machine learning techniques, and develop the computational model to predict viscoelastic behavior with strain overshoot. From **November to January**, I will employ the model to verify dynamic moduli values with a DMA apparatus and compare them with the actual relaxation modulus. During **January and February**, I will refine the algorithms, study the discrepancies, and write the research report focusing on the results of the model. During the whole process, I will meet and seek guidance from Dr.Thevamaran and the Lab through weekly meetings.

Conclusion

Accurately determining the viscoelastic properties by accounting for the strain overshoot has far-reaching implications in both materials sciences and engineering, as well as in the field of biomechanics. Tendons, which play a crucial role in providing support and movement to our bodies, exhibit viscoelastic behavior, and understanding their mechanical response is crucial for predicting their performance under various strain rates. My proposed model will provide valuable insight into the significance of experimental overshoot and will establish a connection between DMA and stress relaxation. This is particularly important for predicting dynamic moduli, which is especially useful when only stress relaxation data is available. Furthermore, the step-response-based approach we have proposed provides an improved level of versatility, allowing for the use of various other functions as inputs to further enhance the model.

References

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