

MAE 544 MONLINEAR CONTROL

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@ Denivation:

A vector space an equapped with a bilinear product is called an algebra.

example (R3 with cross-product)

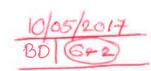
A deroEvation (D) over an algebra (A) is a mapping D: A > A such that

i) $D(\alpha\alpha + \beta b) = \alpha D(\alpha) + \beta D(b)$ for any a be A and $\alpha, \beta \in K$, where K is the underlying field.

ii) D(ab) = D(a)b+ D(b)a (Leibnetz rule)

In our case we have 19(3): $C^{0} \rightarrow \mathbb{R}$ defined as $19(6)(f) = \frac{d}{df}(f \circ x(f))\Big|_{t=0}$

cohere, $x: (-\epsilon, \epsilon) \to \mathbb{R}$, and v(a) is a tangent to x(t) of t=0, i.e. $v(a) = \begin{bmatrix} \frac{1}{2}c_1 & \frac{1}{2}c_2 \\ \frac{1}{2}t & \frac{1}{2}t \end{bmatrix} = 0$.



Tongent vectors on Smooth Manifold:

- A manifold M with a smooth storucture emposed by { (Vx, fx) | xeA}.
- -> Consider a smooth curve $T:(-\varepsilon \varepsilon) \to M$ passing through peM, i.e T(0)=p.
- Smoothness of the curve is aquivalent to smoothness of the function -
- cohere V_{α} is a neighborhood of peM.
- Also, if fact is smooth for some XEA such that pe Uz, then fact is smooth for all XEA such such that pe Uz.
- Flet, \$\Partial \text{M} \rightarrow \mathbb{R}. Then, \$\Partial \text{T} is a real-valued function defined on \$(\in \mathbb{E}, \mathbb{E}), and we can define a tangent vector up to \$M\$ at a point \$p \in M\$ as \$--

O Suppose $F: (FE, E) \rightarrow M$ is another smooth carrie on M such that $F(O) = p \in M$. It defines the same tangent vector v_p of —

for any smooth function $\Phi: M \to \mathbb{R}$. This implies that Φ and Φ are infinitesimally equivalent. Also, this defines an equivalence relationship, and as a result an tangent vector can be perceived as an equivalence

> Similar to the planar case on R, tangent rectors of a smooth manifold (M) can also be perceived as a derivation on the space of smooth functions on M.

o Let, M pe a smooth manifold of dimension m, and pEM. Then, a tangent vectors at point pell can be reased as a derivation at point p. i.e. a smooth map sp: Co(M) - & such that -i) $v_p(\alpha \mathcal{I} + \beta \mathcal{I}) = \alpha v_p(\mathcal{I}) + \beta v_p(\mathcal{I})$ for any IN E CO(M) and offer.