Useful Statistical Rules

Economics 140A
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If a and b are constants and X, Y, Z are random variables.

$$\mathbb{E}[X] = \sum_{i=1}^{\infty} x_i p_i \qquad \text{(Discrete)}$$
$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x f(x) dx \qquad \text{(Continuous)}$$

$$\begin{aligned}
\mathbb{E}[a + X] &= a + \mathbb{E}[X] \\
\mathbb{E}[aX] &= a\mathbb{E}[X] \\
\mathbb{E}[aX + bY] &= a\mathbb{E}[X] + b\mathbb{E}[Y]
\end{aligned}$$

$$\begin{aligned}
Var(X) &= \mathbb{E}[(X - \mathbb{E}(X))^2] \\
Var(X) &= \mathbb{E}[X^2] - (\mathbb{E}[X])^2 \\
Var(a) &= 0 \\
Var(aX) &= a^2 Var(X) \\
Var(aX + bY) &= a^2 Var(X) + b^2 Var(Y) + 2ab\, Cov(X, Y) \\
\mathbb{E}[X] = 0 &\iff Var(X) = \mathbb{E}[X^2]
\end{aligned}$$

$$\begin{aligned}
Cov(X, Y) &= \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])] \equiv \sigma_{XY} \\
Cov(a + X, b + Y) &= Cov(X, Y) \\
Cov(aX, bY) &= ab\, Cov(X, Y) \\
Cov(X + Y, Z) &= Cov(X, Z) + Cov(Y, Z) \\
Cov(X, X) &= Var(X) \\
Cov(X, a) &= 0
\end{aligned}$$

Normal Distribution with Mean μ and variance σ^2

$$X \sim \mathcal{N}(\mu, \sigma^2) \implies \frac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$$
$$\mathbb{E}[X] = \mu \qquad Var(X) = \sigma^2$$

Uniform Distribution with endpoints $a < b$

$$Y \sim \mathcal{U}(a, b)$$
$$\mathbb{E}[Y] = \frac{1}{2}(a + b) \qquad Var(Y) = \frac{1}{12}(b - a)^2$$