

Brain functional networks

Lecture 7

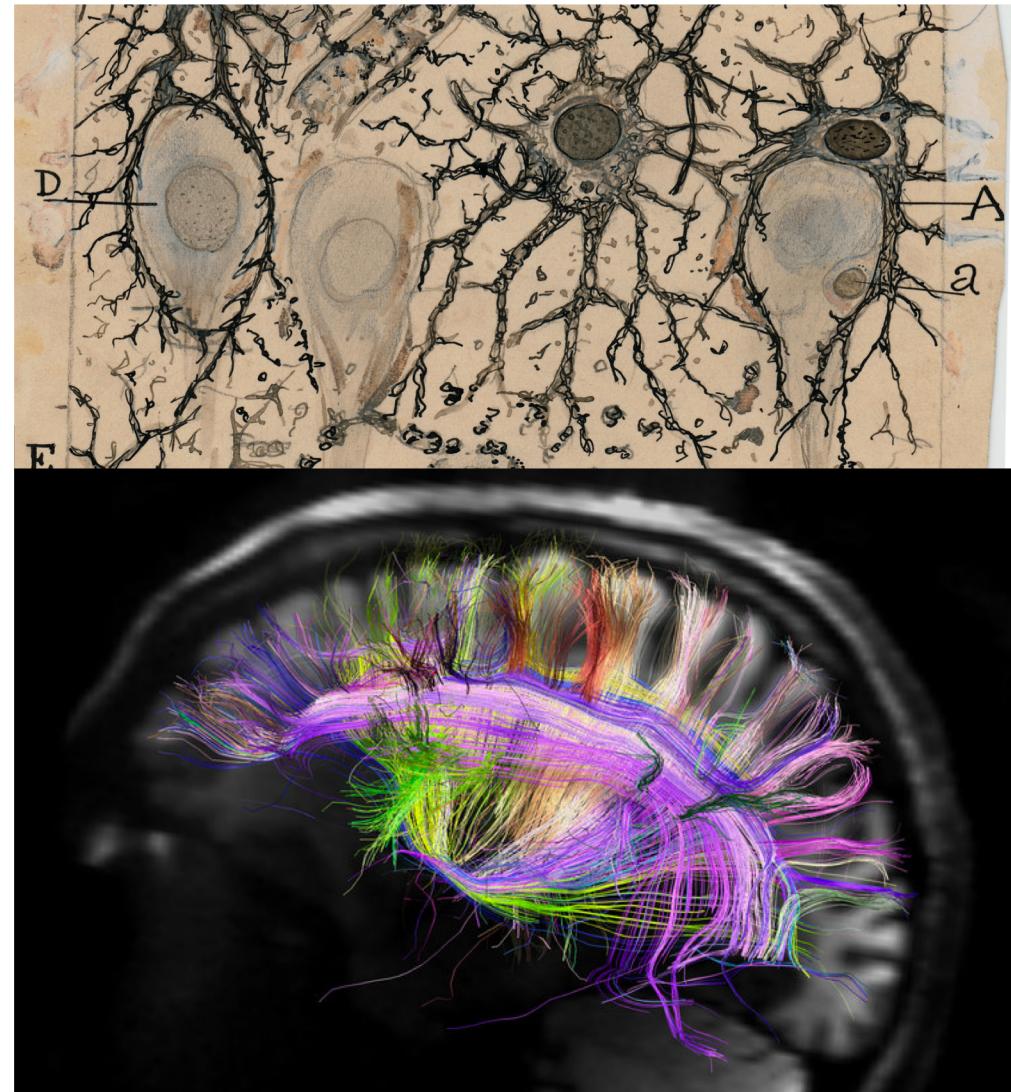
Aim and Outline

Aim is
consideration of
different types of
brain networks

- Brain network
- Structural brain connectivity
- From connections to connectome and functional neuronal network
- Functional brain connectivity
- Main definitions of graph theory

What is the brain network?

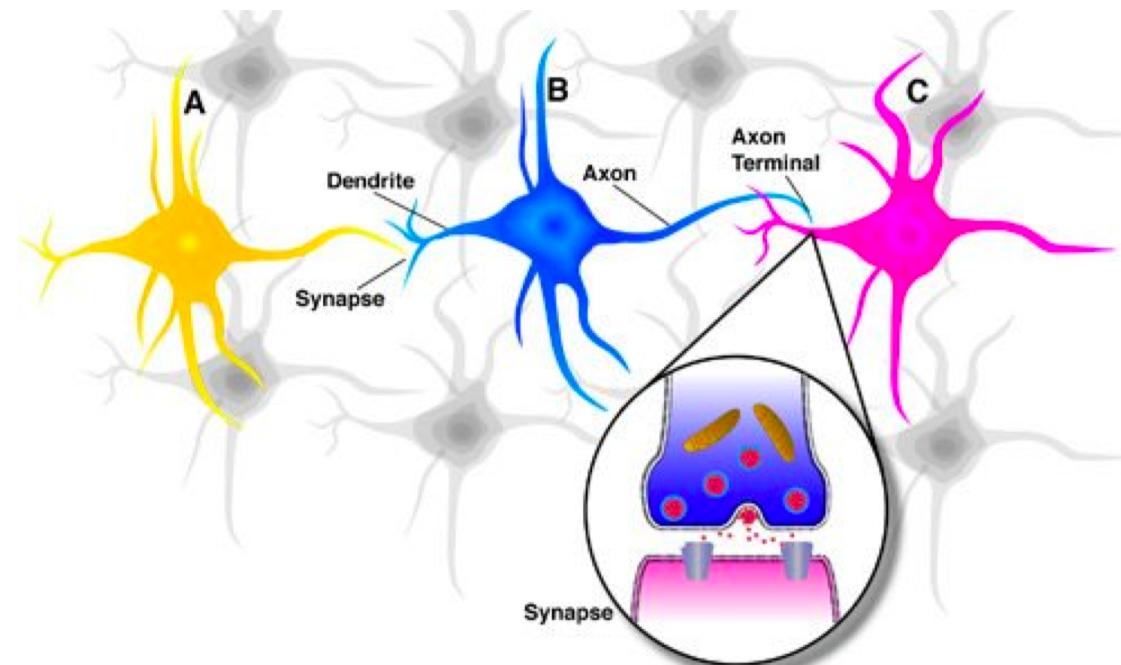
- Healthy brain function requires effective communication and integration of neural information between distributed brain regions
- The anatomical infrastructure to support interregional neural interaction is the complex network of axonal projections that is known as the human connectome
- The connectome has been proposed to understand the collective and coordinated neural phenomena underlying cognitive processes
- The complex brain functions are associated not only with the properties of individual brain regions, but emerge from their interplay within the connectome as a whole.



Structural brain connectivity

- An important first step to understanding the brain's wiring pattern and how it relates to brain function is to map its elements and connections
- Brain connections are organized on multiple spatial scales

microscale: synaptic connections between individual neurons

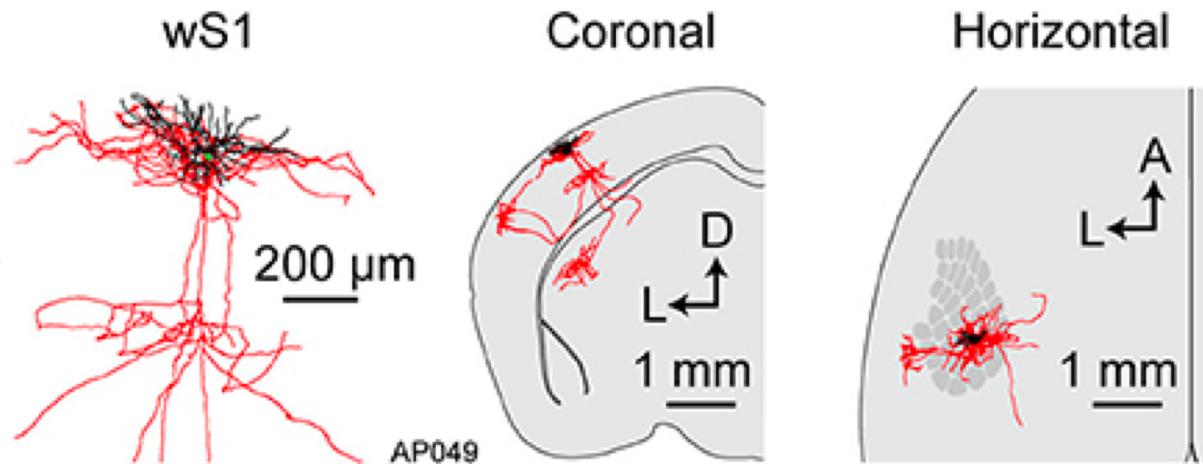
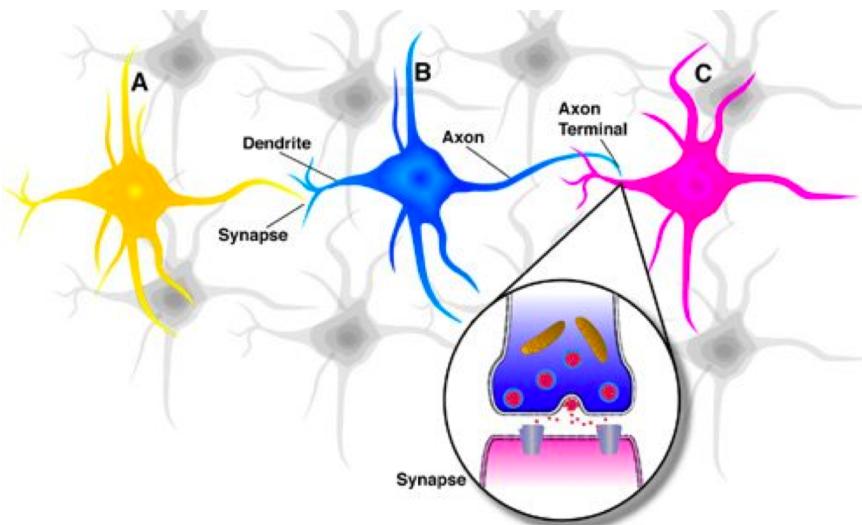


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mesoscale: axonal projections linking neuronal populations

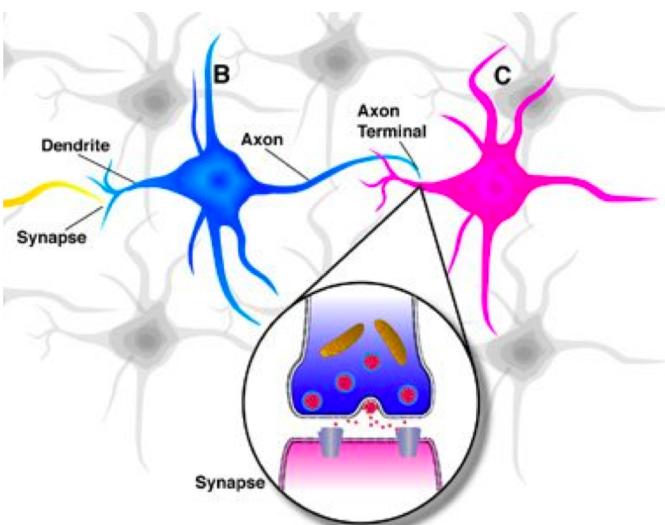
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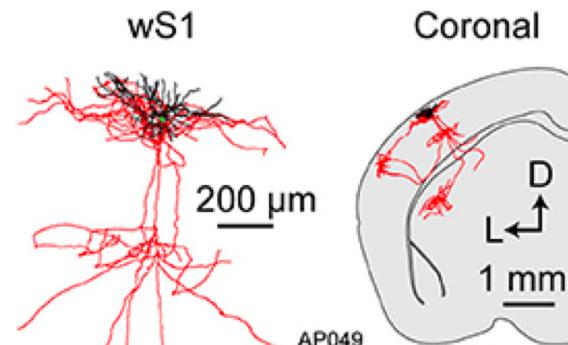
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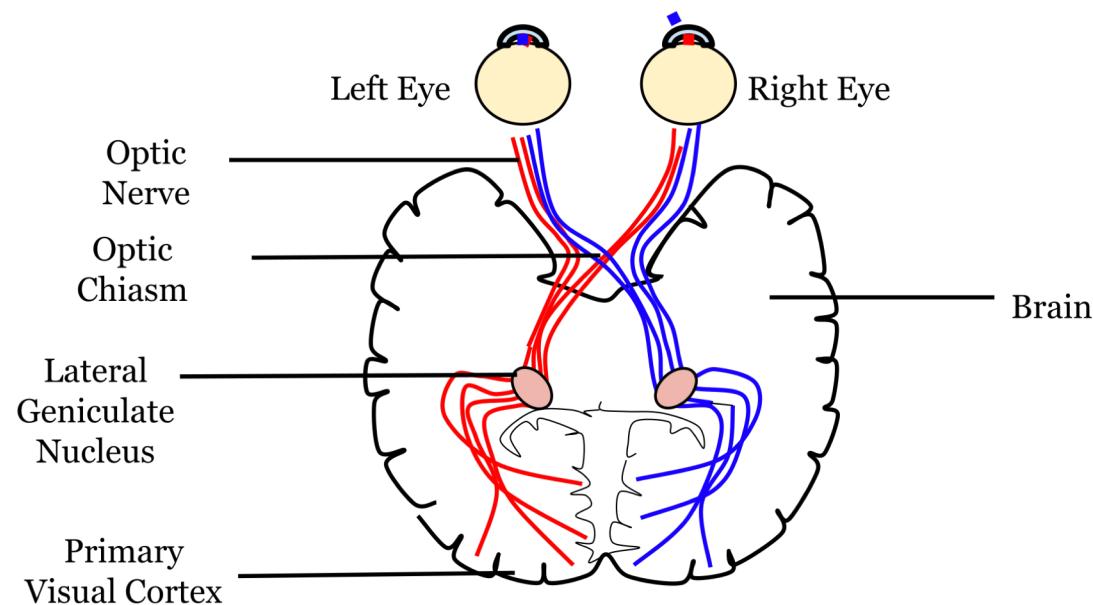
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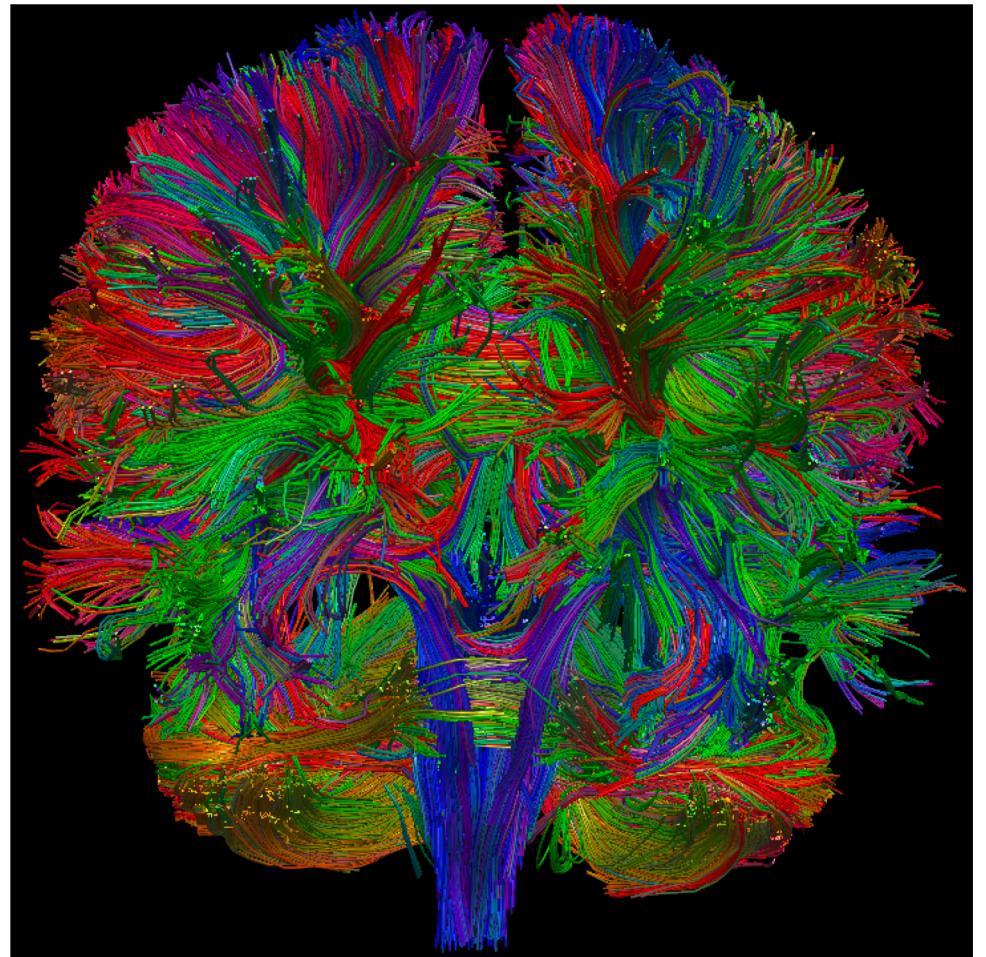


macroscale: large axonal bundles (called tracts) connecting spatially distributed and functionally specialized brain regions at the systems level



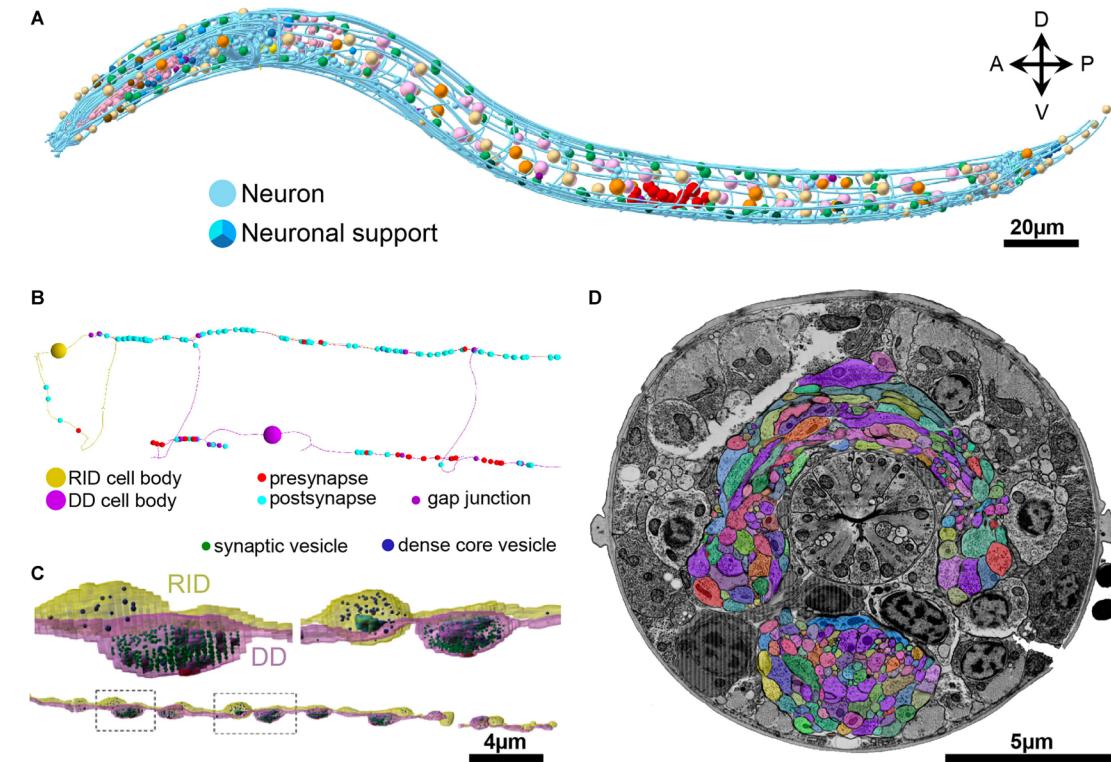
Imaging brain connectivity

- Macroscale structural connections of the brain can be reconstructed *in vivo* using diffusion-weighted imaging (DWI)
- This technique estimates the diffusion of water molecules in the brain, which is constrained by large-scale white matter fiber tracts, allowing these tracts to be delineated
- In addition to the spatial localization of macroscopic white matter connections, DWI can be used to approximate the strength or quality of structural connections. Commonly used metrics of inter-regional connectivity include the number of reconstructed fibers between two regions as a proxy of connection strength



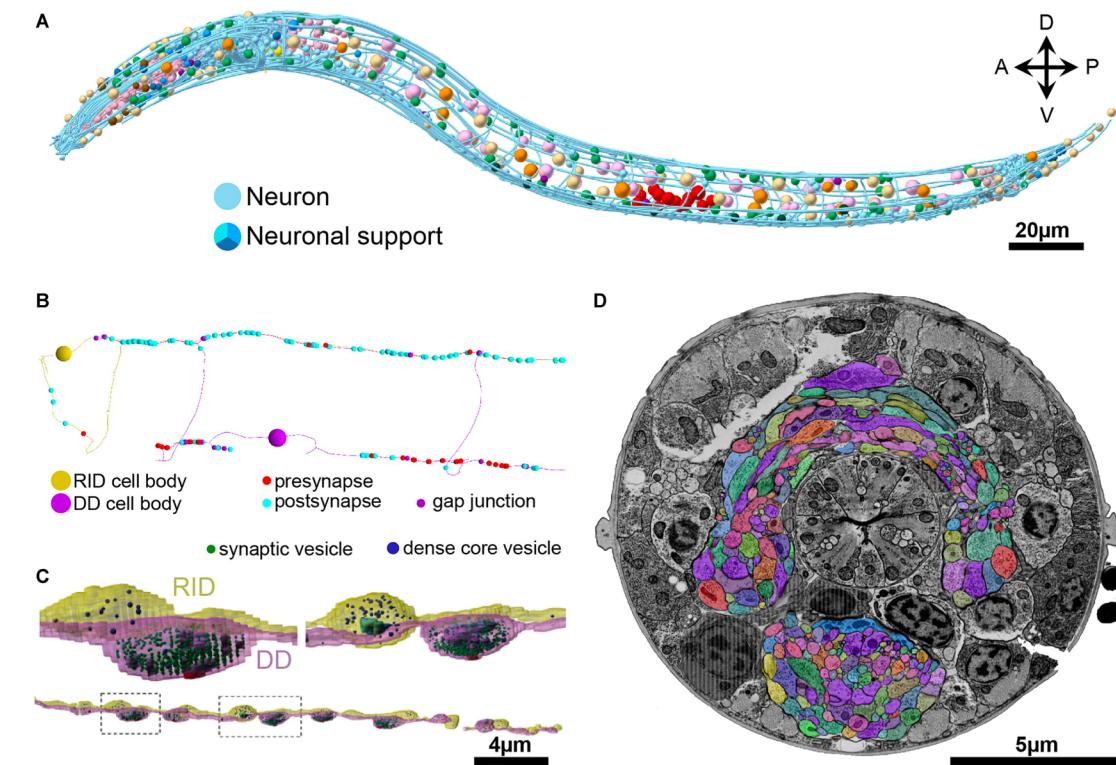
Structural brain connectivity

- The only current complete map of all interactions in a neural system is that of the roundworm *Caenorhabditis elegans*



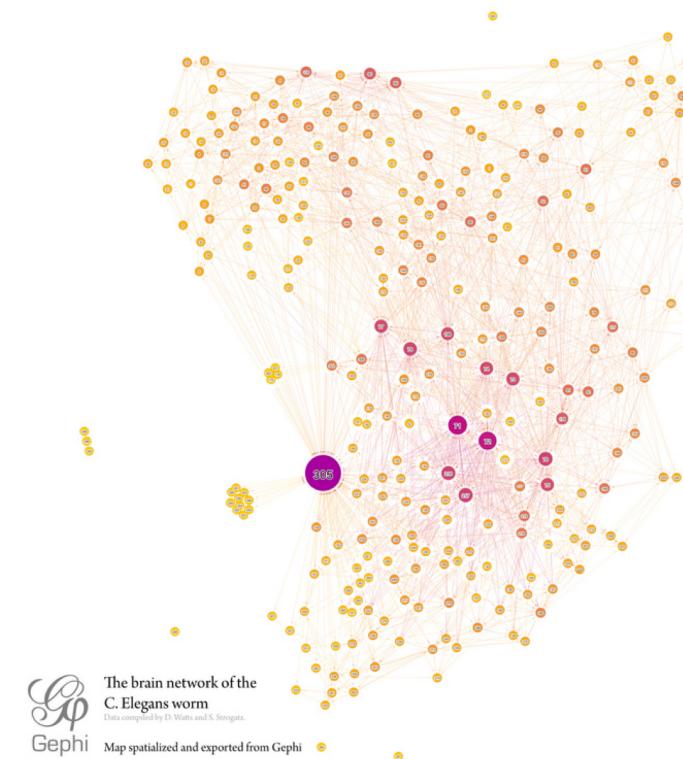
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Human brain: about 80 billion neurons and 100 trillion connections

- Due to its vast size and complexity, it is not currently possible to map the human whole-brain connectome at the scale of individual neuronal connections or even at the mesoscale of brain organization



Novel insights into human connectome organization are being provided by studies of whole-brain network reconstructions at the scale of brain regions based on data from various neuroimaging modalities

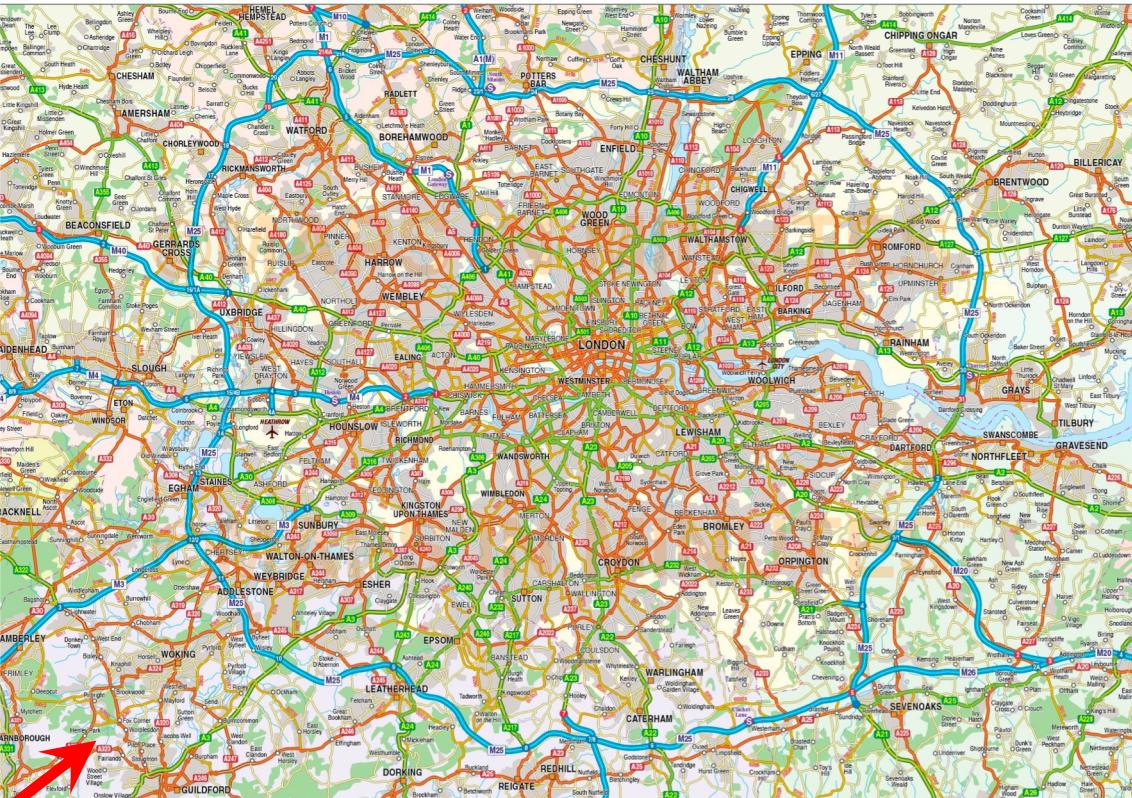
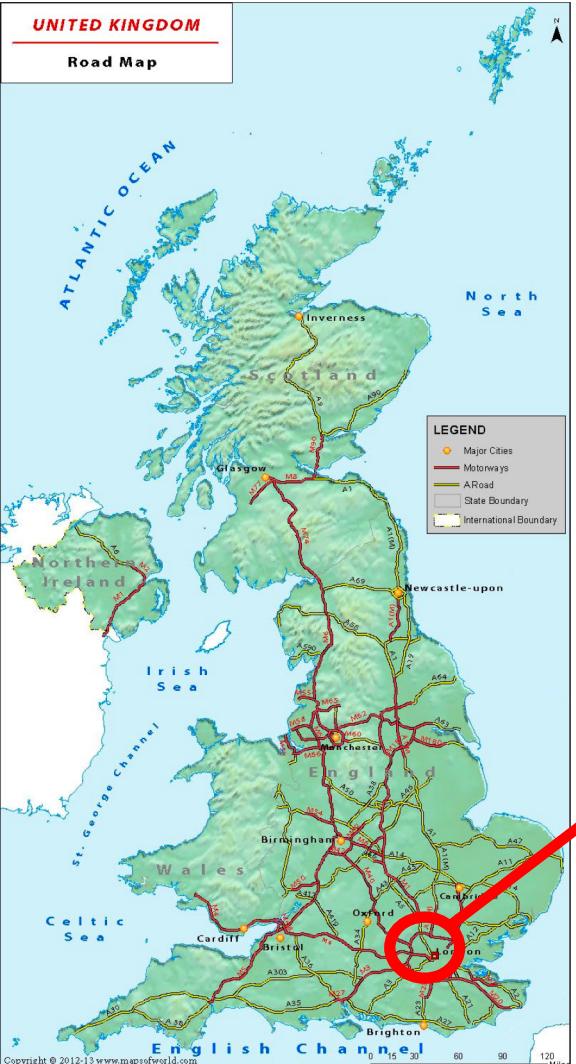
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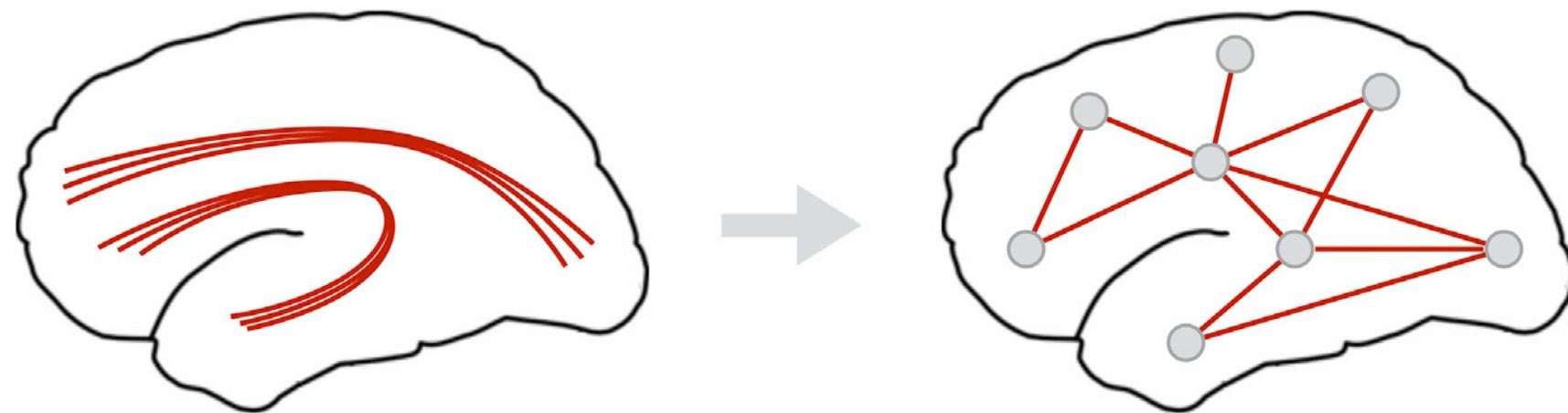
From connections to connectome: road map analogy



Therefore for the brain information flow analysing it is not necessary to create exact replica of the connectional anatomy down to the finest ramifications of dendrites and individual synaptic boutons.

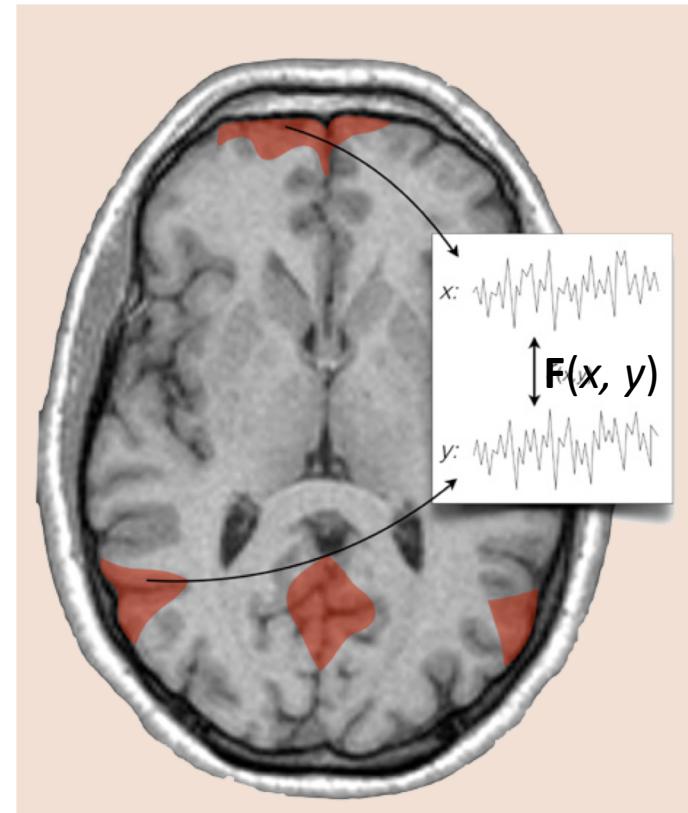
From connections to connectome and functional neuronal network

The connectome should provide a general description of the functional connections between different areas and maybe across multiple scales, including the organization of macroscopic network of functional cortical regions and their mutual connections.



Functional connectivity

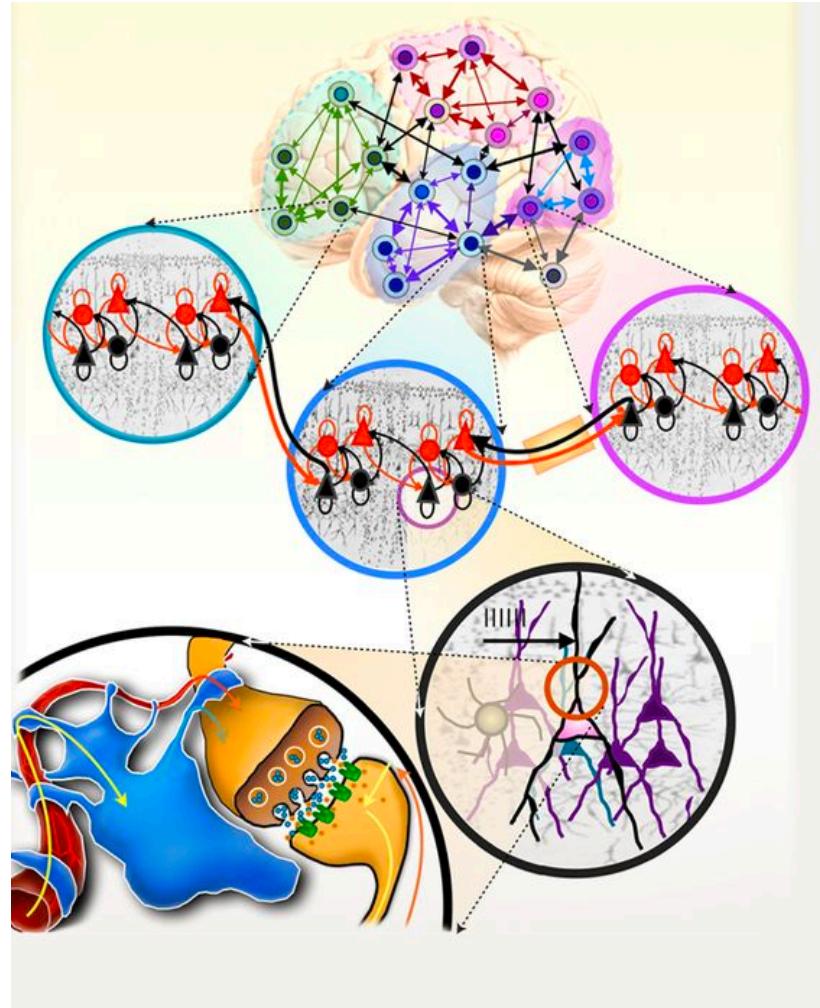
- Functional connectivity refers to the functionally integrated relationship between spatially separated brain regions and can be estimated using different neuroimaging techniques.
- Functional connectivity is related to similar patterns of activation in different brain regions regardless of the apparent physical connectedness of the regions



Effective connectivity

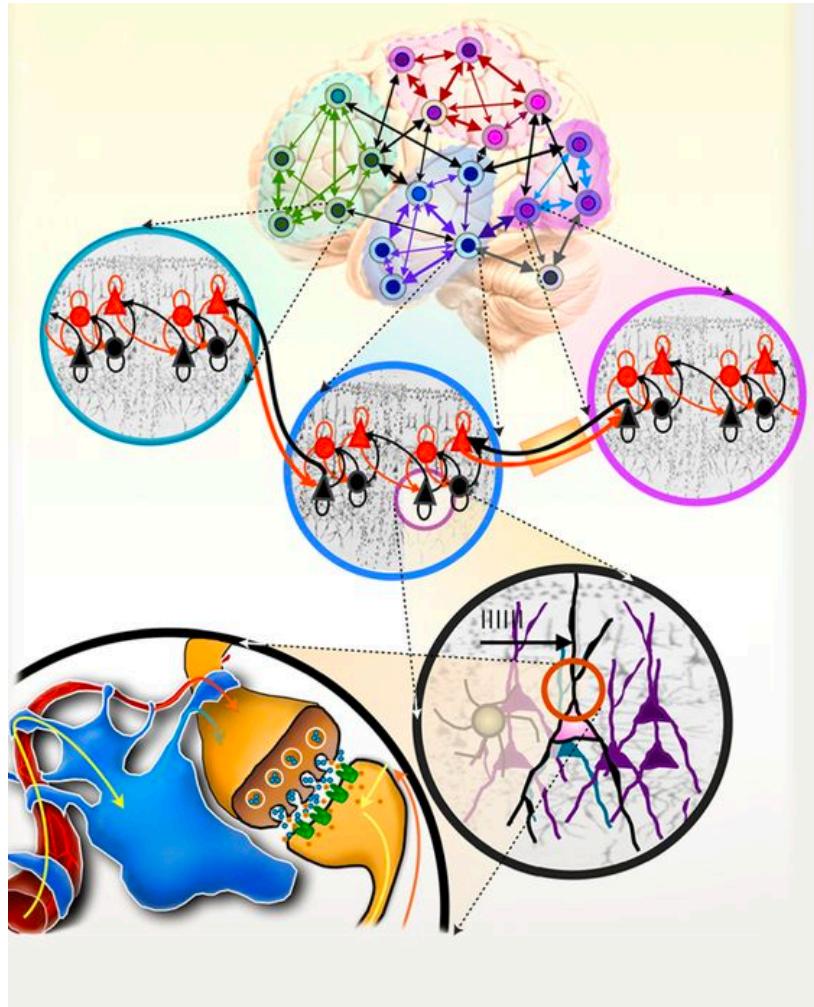
- Effective connectivity attempts to capture a network of directed causal effects between neural elements.
- it represents a generative and mechanistic model that accounts for the observed data, selected from a range of possible models using objective criteria like the model evidence.
- Recent developments in this area include approaches towards “network discovery” involving the identification of graph models for effective connectivity that best explain empirical data.

Integration of integration



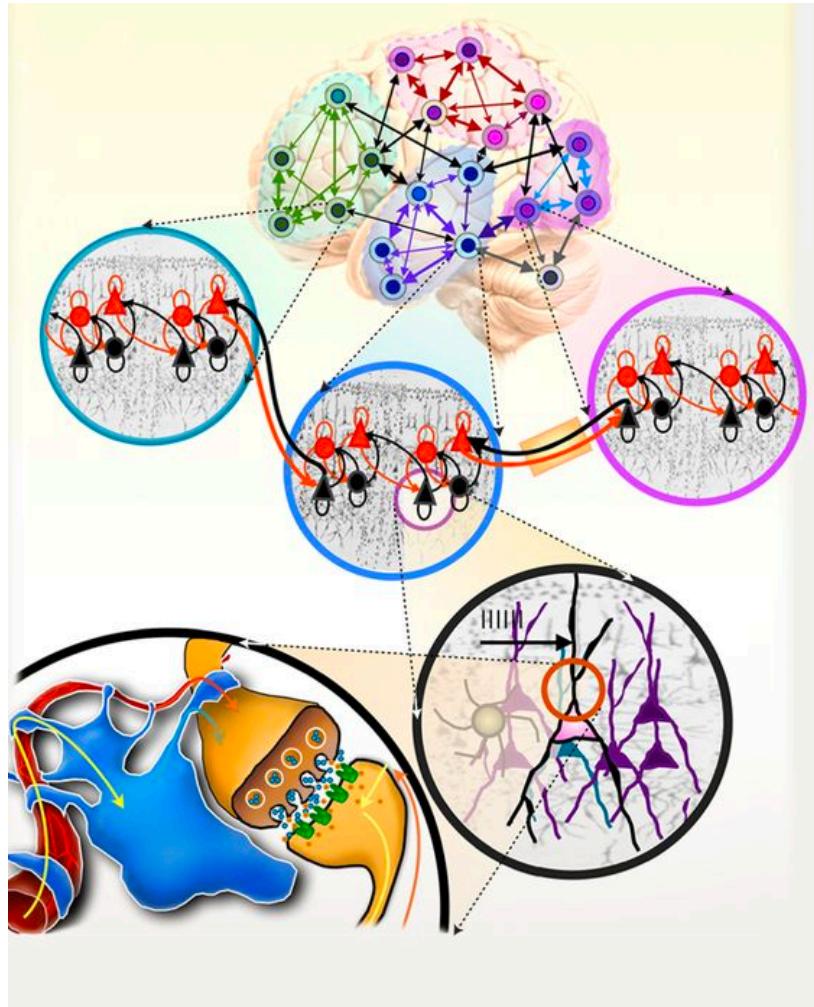
- Schematic of the multiscale hierarchical organization of brain networks: from neurons and macrocolumns to macroscopic brain areas.
- A network is composed of nodes and their links, called edges. A node, defined as an interacting unit of a network, is itself a network composed of smaller nodes interacting at a lower hierarchical level

Integration of integration



- In this structural hierarchy, the function of a module is to integrate and contextualize the more specialized functions of its submodules.
- **Example:** *visual perception in the primary visual cortex assimilates the diverse orientations detected by a multitude of ocular columns, which themselves integrate more basic neuronal operations, within each macrocolumn.*

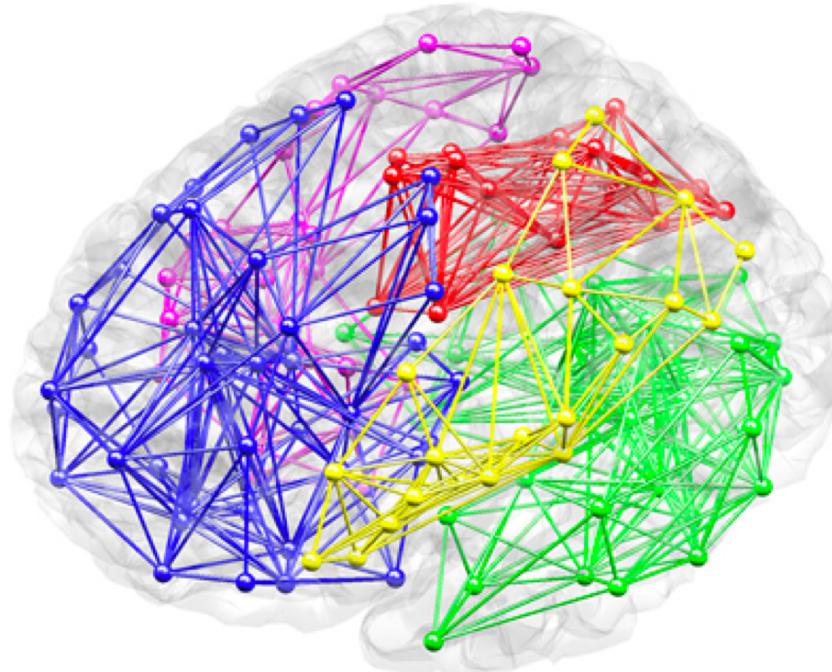
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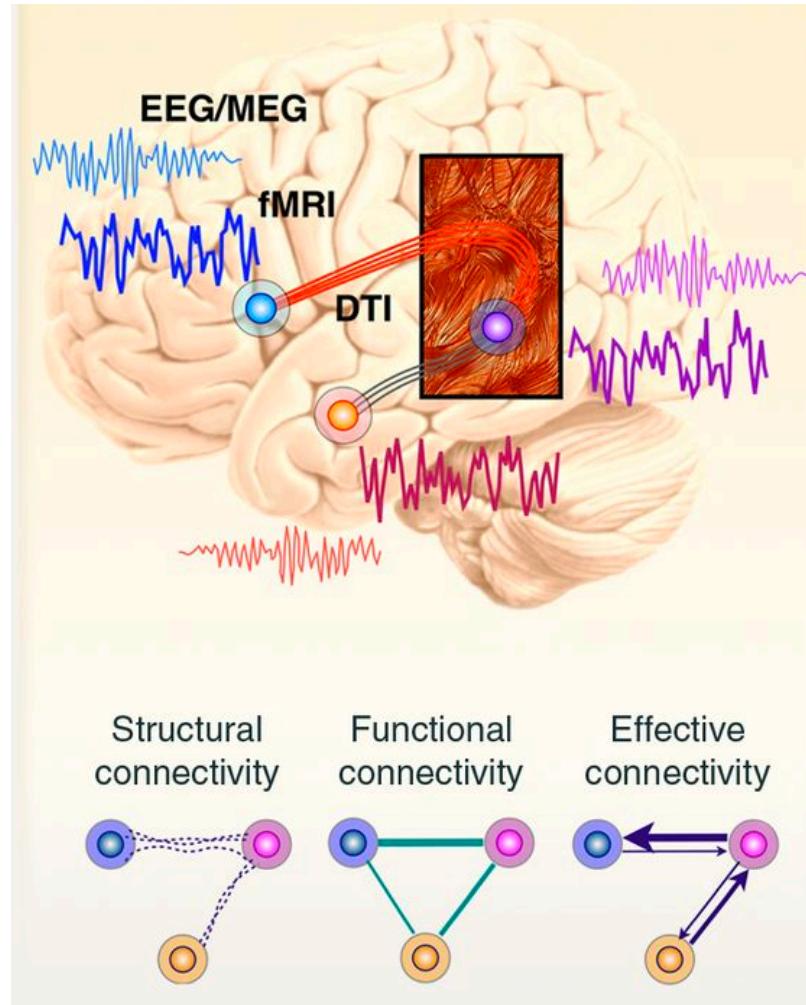
- Brain function or cognition can be described as global integration of local integrators.
- Local integration entails specialized functional processing mediated by short-range connections, intrinsic to a module at any scale.
- Global integration subserves higher cognition, facilitated by long-range connections, such as extrinsic corticocortical connections.

Introduction in brain network science

- The functional brain network may be described mathematically as a graph: a collection of nodes interconnected by a set of edges
- Network analysis for both structure and function in the brain starts with the identification of nodes as interacting units and their interconnections, called edges

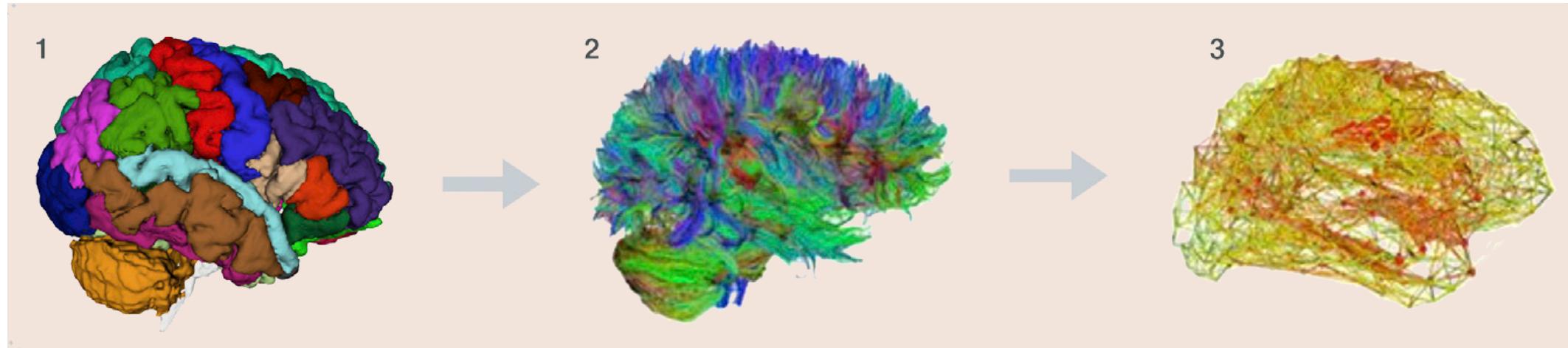


Introduction in brain network science



- Depictions of edges in a brain network, as defined by three types of connectivity: structural, functional, and effective. Structural connectivity refers to anatomical connections. Functional and effective connectivity are generally inferred from the activity of remote nodes.
- Functional connectivity, defined by the correlation or coherence between nodes, does not provide directionality or causality.
- Effective connectivity is estimated by using a model of neuronal interactions, it can evaluate directionality.

Reconstruction of brain network

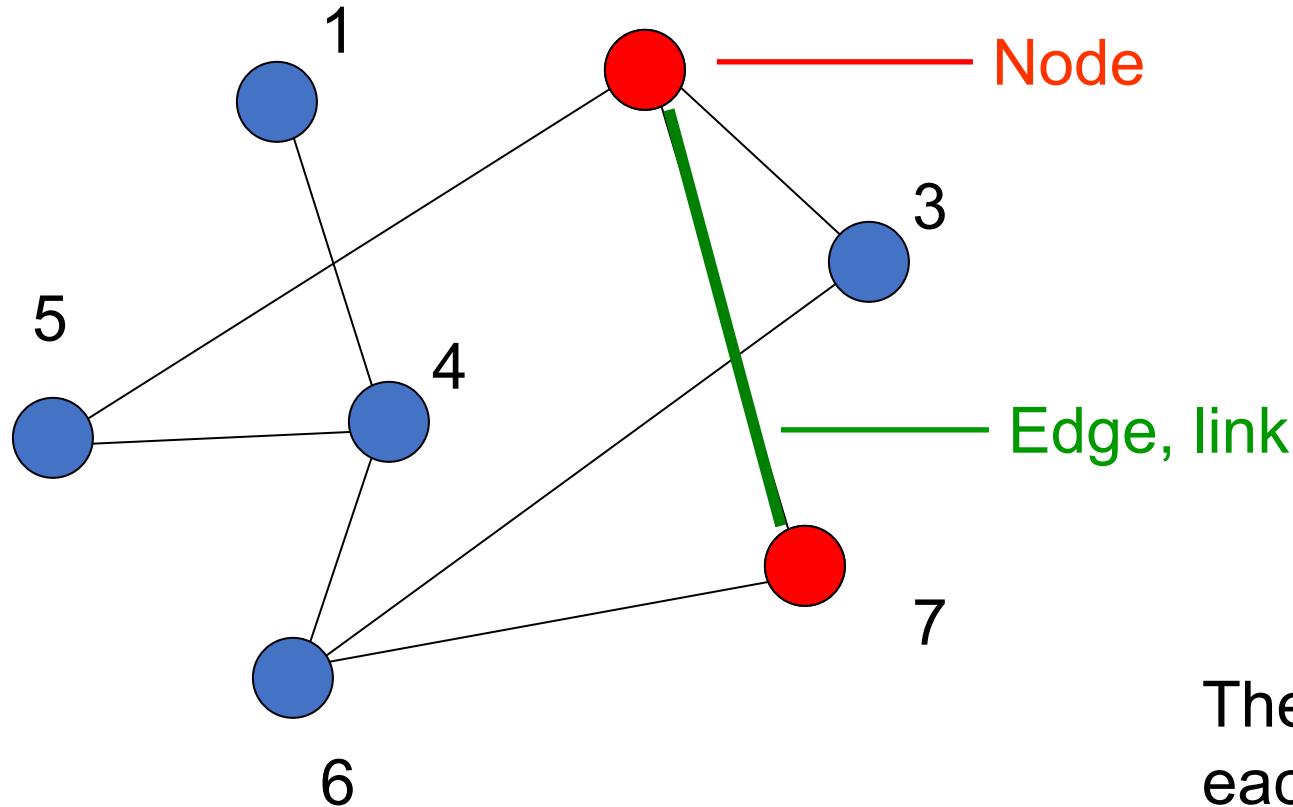


The examination of neuroimaging-derived brain networks involves parcellation of the cortex or whole brain into coherent brain regions

The association between the nodes is quantified in terms of structural or functional connectivity

These pairwise associations are aggregated into a connection matrix representing a graph or network

Network definitions and measures



adjacency matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The nodes connected to each other (for example, 2 and 7) are called adjacent.

Characteristics of network nodes: degree

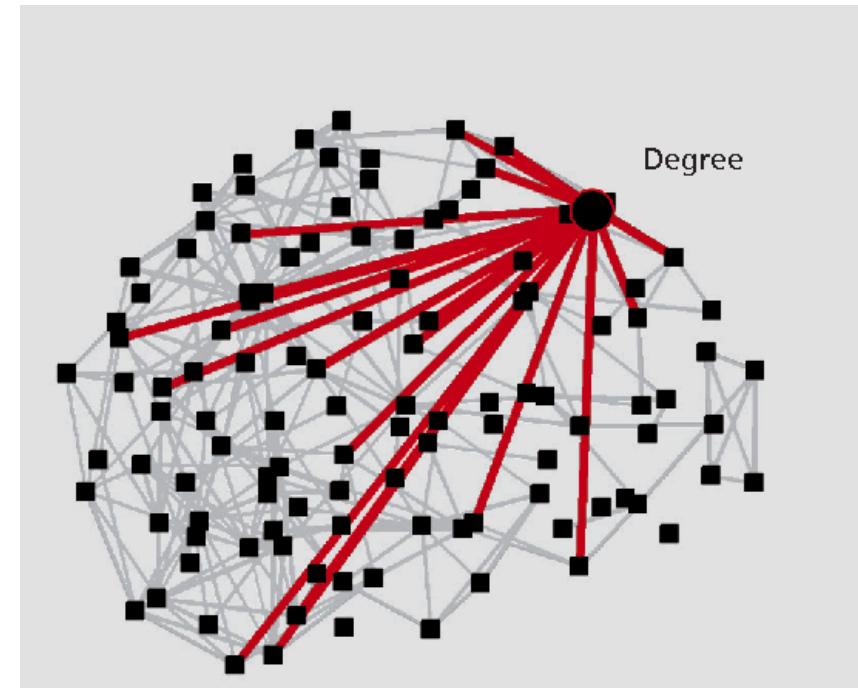
- The degree of node i is the number of links for this node :

$$k_i = \sum_{j \in N} a_{ij}$$

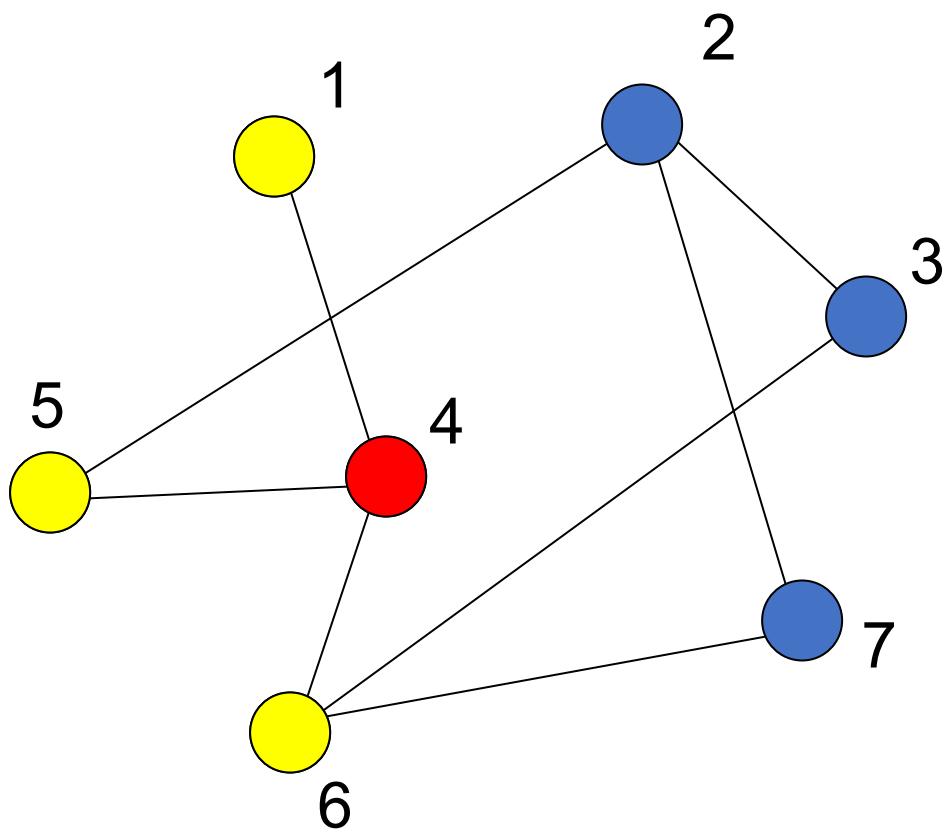
- The average degree of the nearest neighboring elements

$$k_{nn,i} = \frac{1}{k_i} \sum_{j \in N_i} k_j = \frac{1}{k_i} \sum_{j=1}^N a_{ij} k_j$$

- The distribution of the degrees of nodes $P(k)$



Characteristics of network nodes

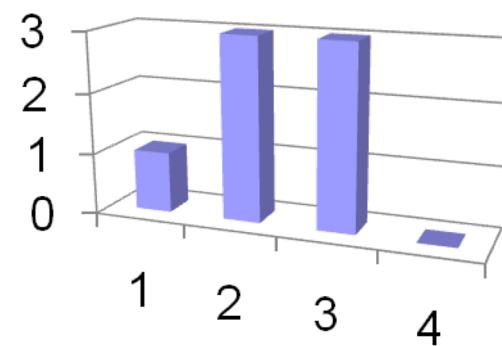


$$k_4=3$$

$$k_{nn,4} = \frac{1}{3}(1 + 2 + 3) = 2$$

Node	<i>N</i>
1	1
2	3
3	2
4	3
5	2
6	3
7	2

$P(k)$



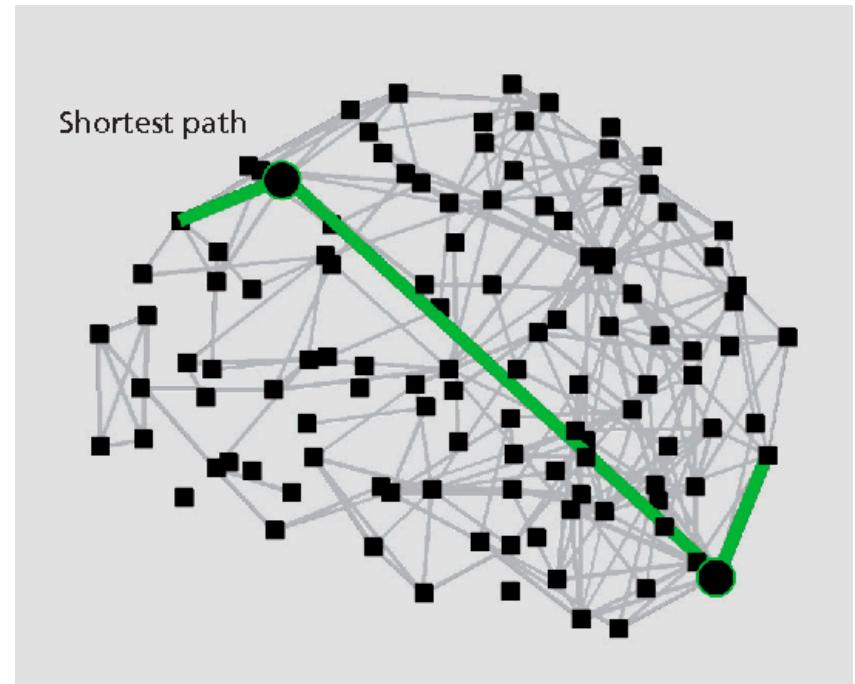
Characteristics of connections: shortest path

- Shortest path, d_{ij}
- Mean shortest path
(characteristic path length)

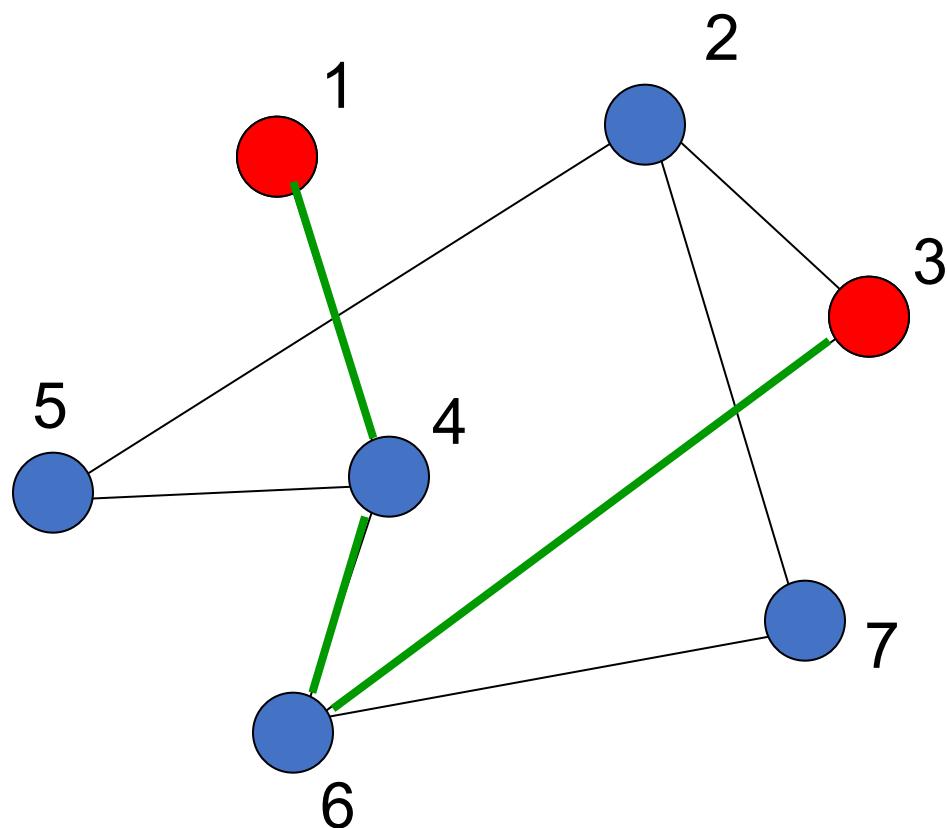
$$L = \frac{1}{N(N-1)} \sum_{i,j \in N; i \neq j} d_{ij}$$

- Efficiency

$$E = \frac{1}{N(N-1)} \sum_{i,j \in N; i \neq j} \frac{1}{d_{ij}}$$



Characteristics of connections: shortest path



$$d_{13}=3$$

Coupling matrix

$$D = \begin{pmatrix} 0 & 3 & 3 & 1 & 2 & 2 & 3 \\ 3 & 0 & 1 & 2 & 1 & 2 & 1 \\ 3 & 1 & 0 & 2 & 2 & 1 & 2 \\ 1 & 2 & 2 & 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 1 & 0 & 2 & 3 \\ 2 & 2 & 1 & 1 & 2 & 0 & 1 \\ 3 & 3 & 2 & 2 & 3 & 1 & 0 \end{pmatrix}$$

$$L=1.86$$

Characteristics of network nodes: centrality

- **Centrality identifies the most important nodes within a network**
- Betweenness centrality quantifies the number of times a node acts as a bridge along the shortest path between two other nodes.

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

σ_{st} is the total number of shortest paths from node s to node t, and $\sigma_{st}(v)$ is the number of those paths that pass through v

- For each node its degree (the number of links of the node) is calculated and degree centrality is proportional to the degree of the node

$$C_D(v) = \deg(v)$$

Characteristics of network nodes: centrality

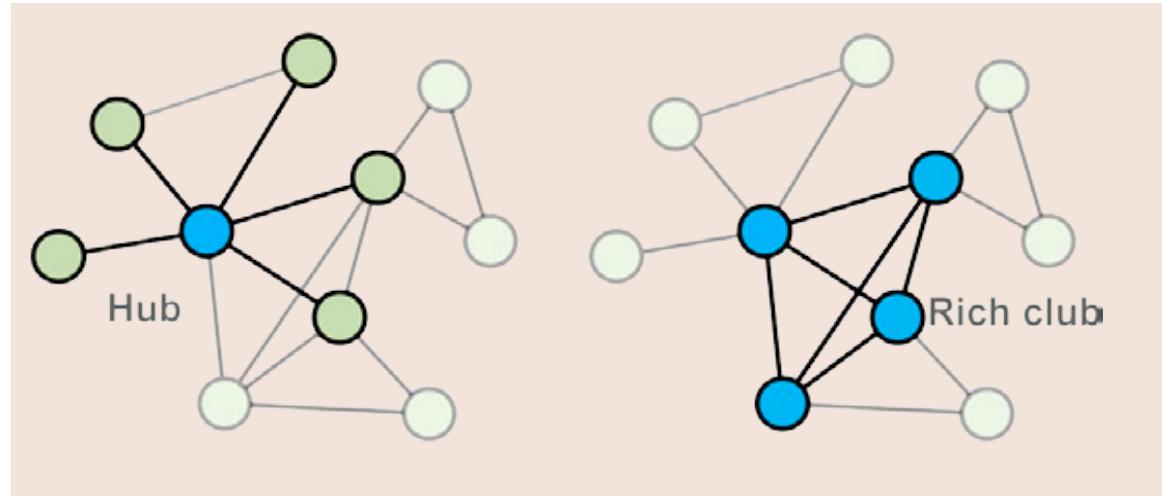
- The eigenvector centrality is a measure of the influence of a node in the network.
- This method assigns relative scores to all nodes in the network based on the concept that node connections with “high impact” make a greater contribution to node centrality than links connecting nodes with low impact.
- The eigenvector centralities of nodes can be found from eigenvector equation

$$\mathbf{Ax} = \lambda \mathbf{x}$$

- In general, there will be many different eigenvalues for which a non-zero eigenvector solution exists, but since the entries in the adjacency matrix are non-negative, there is a unique largest eigenvalue, which is real and positive. The i -th component of this eigenvector gives the relative centrality of the i -th node in the network.

Brain network integration

- Measures of integration reflect the efficiency of communication among all nodes in the overall network.
- Two commonly used and inversely related measures are path length and global efficiency.
 - Path length indicates the average number of steps it takes to traverse the network.
 - Global efficiency reflects the efficiency with which information can be distributed throughout the network.

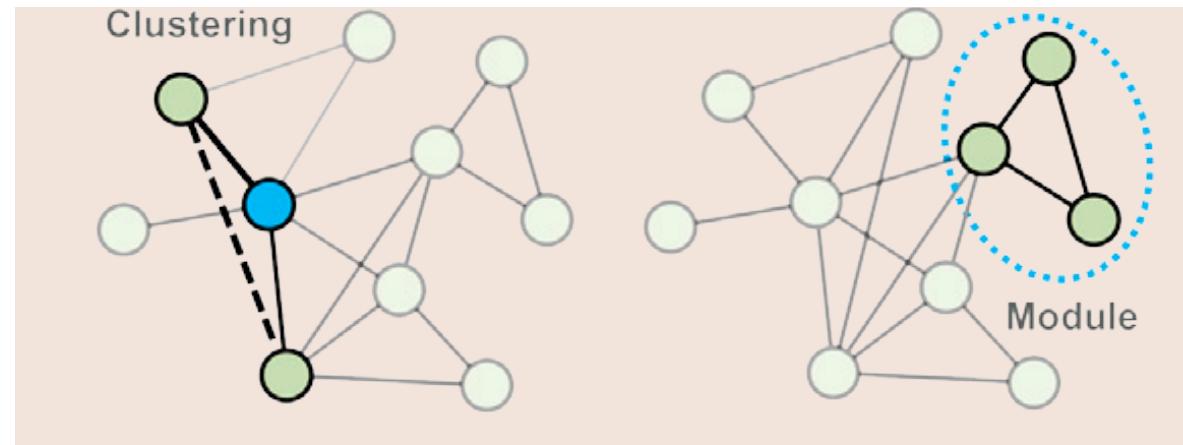


Hubs are nodes with a high degree of centrality.

Rich club organization refers to the tendency of high-degree nodes (ie, hubs) to be more densely connected mutually than to nodes of lower degree.

Brain network segregation

- Measures of segregation reflect the degree to which the network can be subdivided into local communities of clusters or modules that are strongly interconnected, with relatively sparse connectivity to the rest of the network.

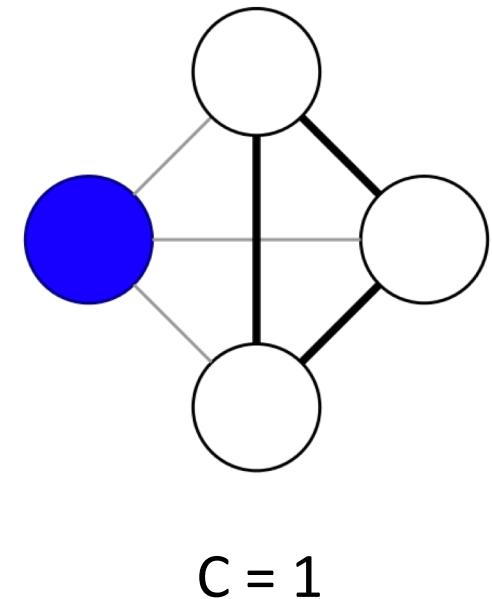


Network clustering can be quantified using the clustering coefficient, indicating the extent to which the neighbors of a node are also mutually connected

Modularity reflects the extent to which the network as a whole can be decomposed into modules

Brain network clustering

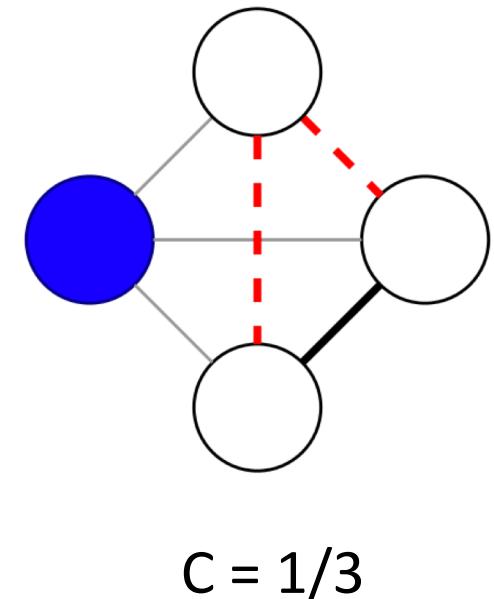
- The clustering coefficient is based on triplets of nodes. A triplet is three nodes that are connected by either two (open triplet) or three (closed triplet) undirected ties.
- The global clustering coefficient is the number of closed triplets (or $3 \times$ triangles) over the total number of triplets (both open and closed).



$$C = \frac{\text{number of closed triplets}}{\text{number of all triplets as closed and open}}$$

Brain network clustering

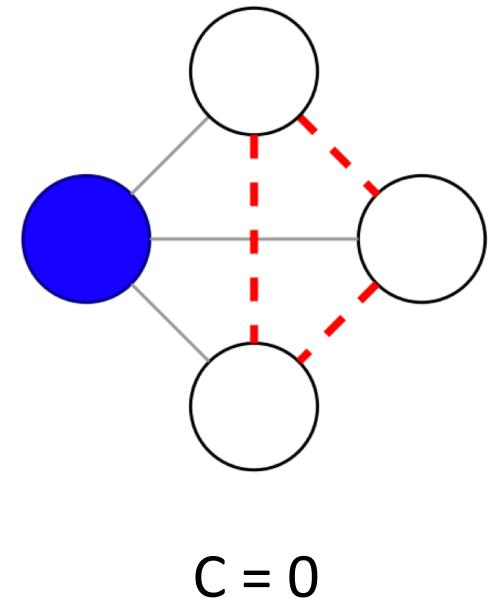
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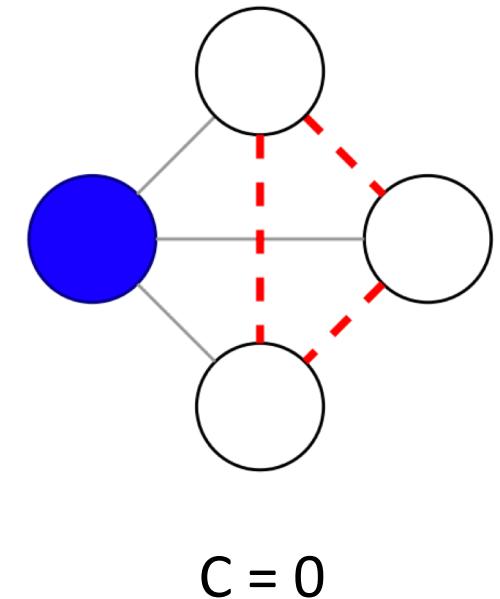
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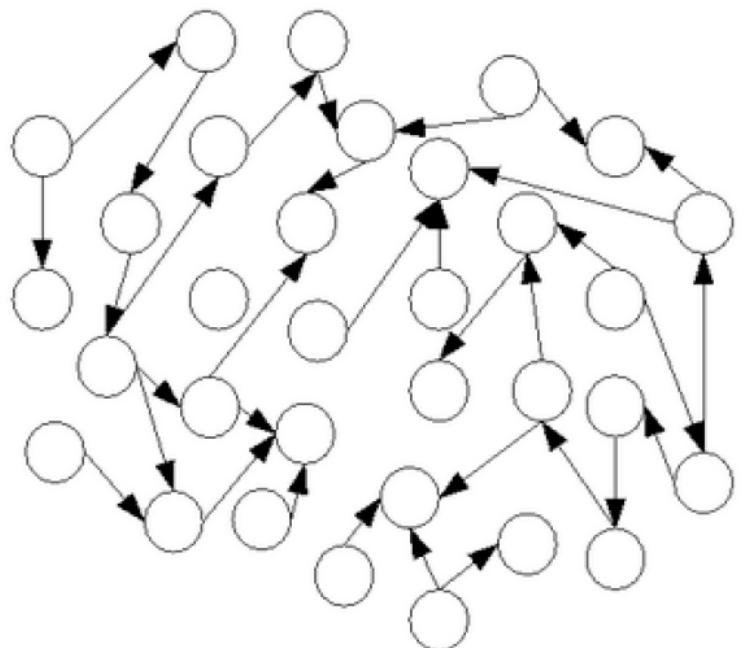
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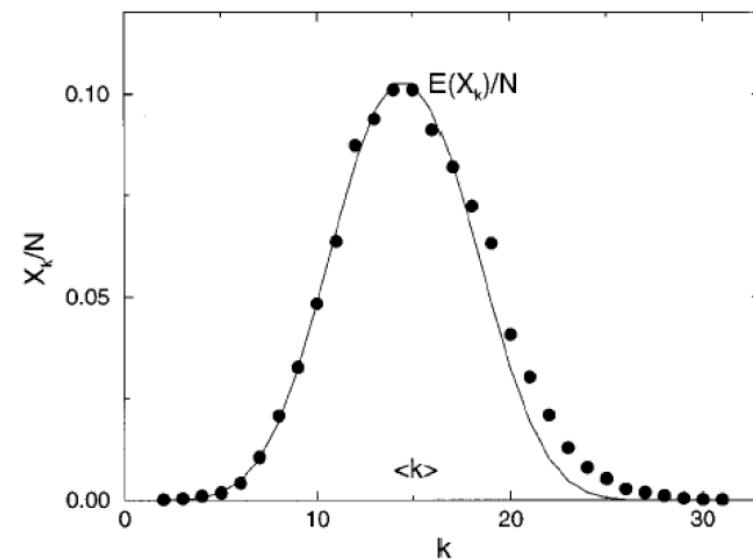
Random network

Random graphs may be described by a random process which generates them

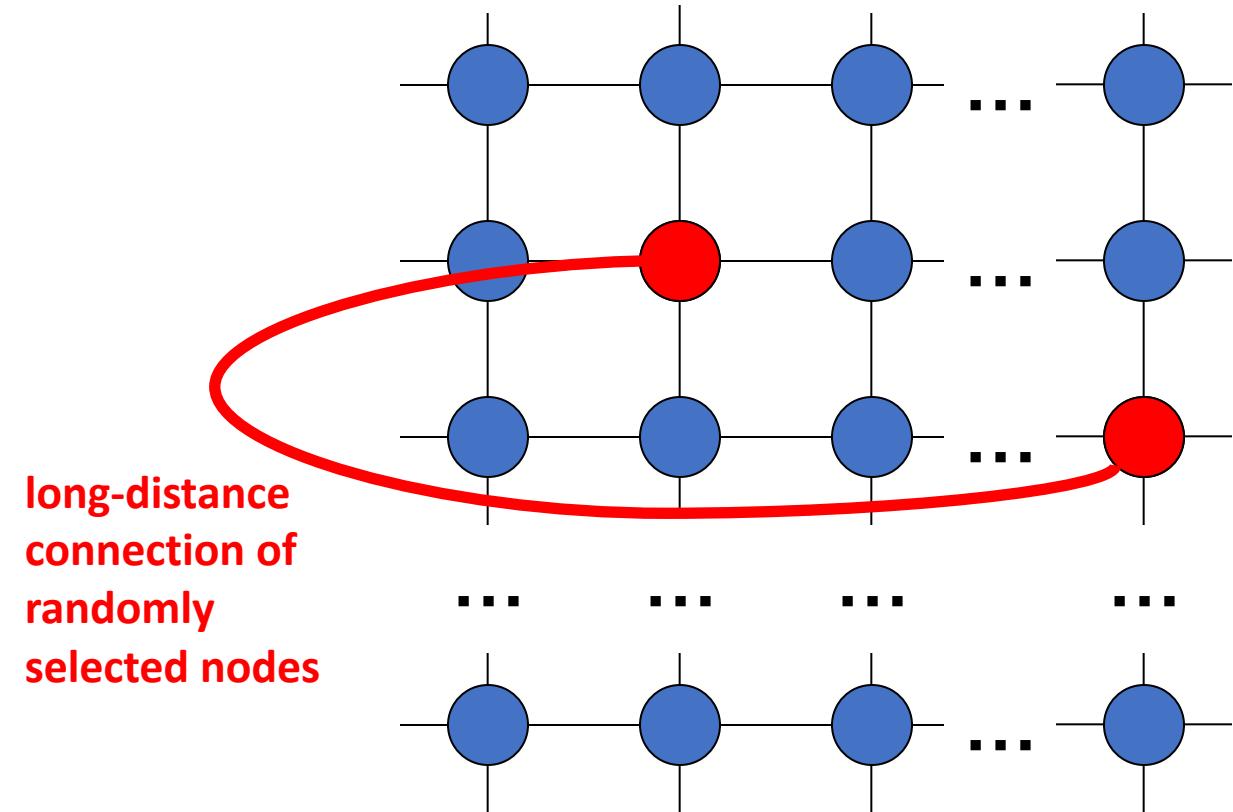


The distribution of the degrees of the nodes follows the Poisson distribution

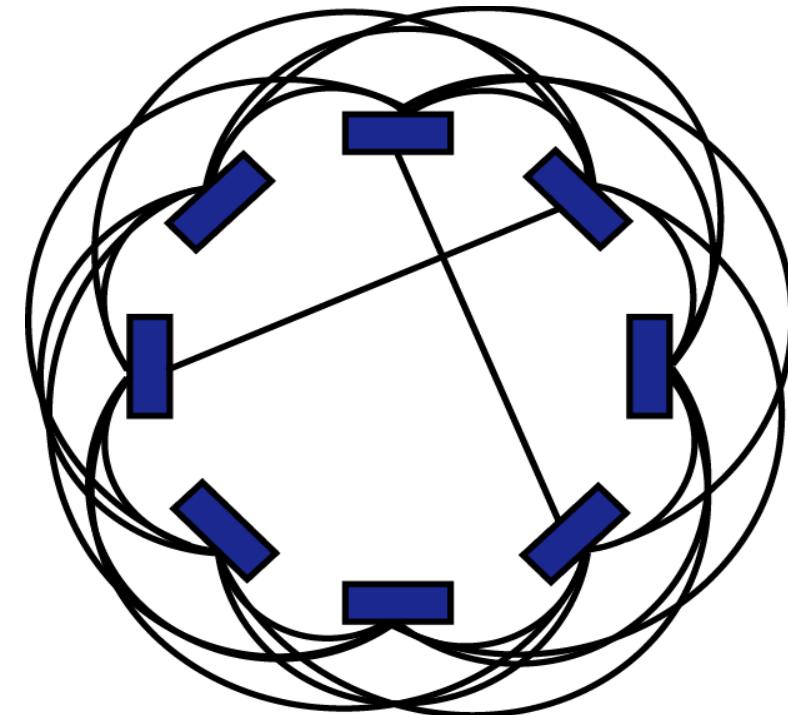
$$P(k) = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$



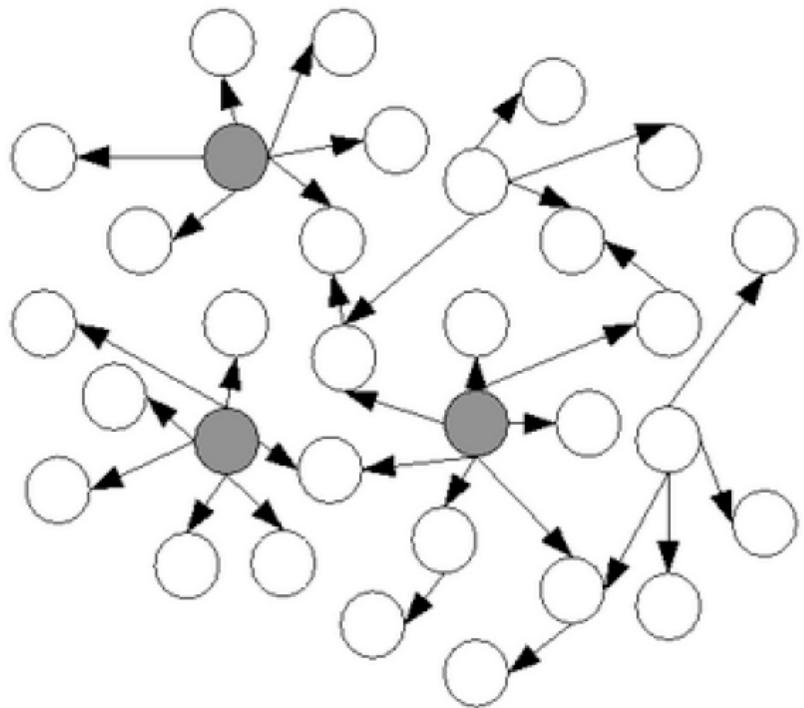
Small-world network



$$d_{ij} \sim \sqrt[D]{N} \quad D \text{ is dimension}$$

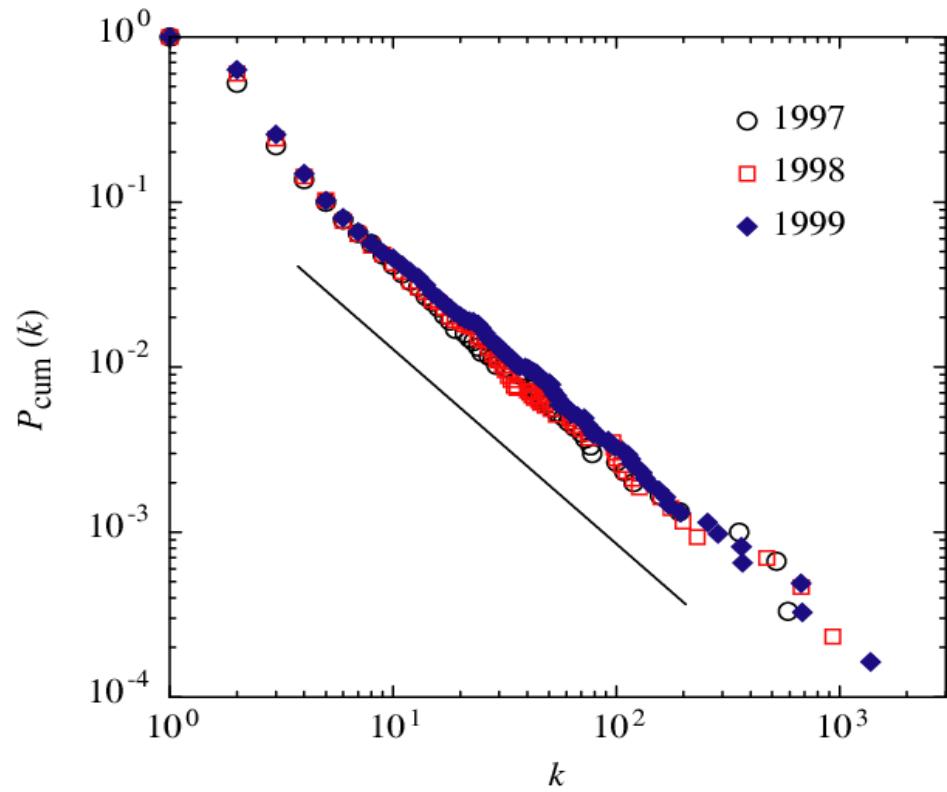


Scale-free network



$$P(k) \sim k^{-\gamma}$$
$$2 < \gamma < 3$$

The distribution of the degrees of the nodes follows the power distribution



Conclusion

