ST517 Additional Practice Problems for Exam 2—Solutions

1.

a. $H_0: \mu = 0 \;\; vs. \;\; H_A: \mu > 0$, where μ represents the population mean difference (female-male) in price paid for a car (in this county).

b. α: 1% of hypothesis tests with these specifications* would <u>incorrectly</u> conclude that women do pay more, on average, for the same car.

β: 66% of hypothesis tests with these specifications* would <u>incorrectly</u> conclude that there is no difference, on average, in the price men and women pay for the same car.

Power: 34% of hypothesis tests with these specifications* would <u>correctly</u> conclude that women do pay more, on average, for the same car.

* The specifications (which you do not have to list unless asked) are: a 1-sided test, where the mean under the null is believed to be 0 & the mean under the alternative is believed to be 500, the standard deviation is believed to be 600, $n = 8$, and $\alpha = 0.01$. Finally, remember that $\beta = 1 - \text{power}$.

c. The power would be smaller, because it is more difficult to detect an effect when there is a lot of variability.

d. Since we are learning about the mean difference of paired data, we need to use the paired samples t-test.

The sample differences will be random since the men were randomly selected. Since the sample size is small, we do need to assume the population of differences is normal. [You have no way to check this based on the information provided and the sample size is too small for the CLT to apply, so you have to state this is still an assumption to continue with the test (which is what you were told to do in the problem).]

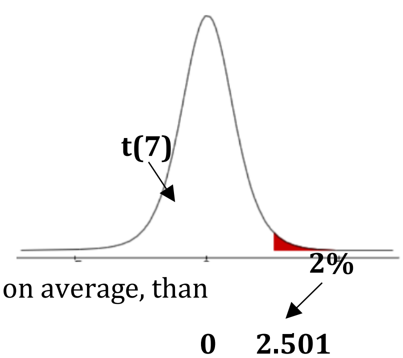
Assuming the conditions hold, we can calculate the test statistic: $t = \frac{527.20 - 0}{596/\sqrt{8}} = 2.501$

This tells us that the sample mean difference was 2.501 standard errors above what we would expect if the population mean difference were zero.

The probability of observing a test statistic of 2.501 or higher (under the t(7) distribution, assuming the prices were the same on average, is 0.02.

Since the p-value is not less than 0.01, we fail to reject H_0.

There is not enough evidence that car dealers charge women more, on average, than men for the same car (in this county).



e.

$$527.20 \pm 2.365 \frac{596}{\sqrt{8}} \Rightarrow 527.20 \pm 498.35 \Rightarrow (28.85, 1025.55)$$

f. We are 95% confident that women pay between \$28.85 and \$1025.55 more, on average, than men for the same car.

g. If we were to take many random samples of 8 pairs from this population, 95% of the resulting 95% confidence intervals we could produce would contain the true mean difference in price.

ST517 Additional Practice Problems for Exam 2—Solutions

2.

a. The research question of interest is: "Does the image of eyes watching employees make them more likely to pay for coffee.

The population that the results can generalize to is all employees at this company. [You could also say that the target population was "all employees" or "all employees represented by this sample."]

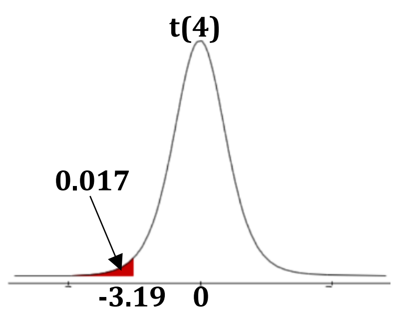
The parameter is the difference in population mean payment contribution per liter of milk when the picture was flowers (μ_1) and the corresponding population mean when the picture was eyes (μ_2).

$H_0: \mu_1 - \mu_2 = 0 \ \ vs. \ \ H_A: \mu_1 - \mu_2 < 0$

Since we are interested in learning about the difference in population means we should conduct the 2-sample t-test.

The samples are independent, but we do not have enough information to judge plausibility of the random sample or normal population conditions. We need to assume both samples were random and both populations were normal in order to continue with the test.

$$t = \frac{(0.181 - 0.417) - 0}{\sqrt{\frac{0.067^2}{5} + \frac{0.151^2}{5}}} = -3.19$$



If the conditions hold, this test statistic would follow a t-distribution with df=min(5-1,5-1)=4.

If there were no difference on average between the groups, we would expect to see a sample mean difference that is at least 3.19 standard errors below the null value in 1.7% of samples. [This is a combination of the interpretation of the test statistic and of the p-value. The clause "3.19 standard errors below the null value" is the interpretation of the test statistic. The clause "at least 3.19 standard errors below" addresses the "less than or equal to part of the interpretation of the p-value; the clause "If there were no difference on average between the groups" represents the "if the null were true" part.]

Since our sample results would be such an unlikely occurrence if the null hypothesis were true, we reject the null hypothesis as a plausible state of the world.

There is enough evidence to say that the average contribution (per liter of milk) is higher when the image is eyes when the image is flowers.

b. α: There is a 10% chance that this hypothesis testing procedure would incorrectly conclude that the average contribution (per liter of milk) when the picture is eyes is more than the average when the picture is flowers.

β: There is a 15% chance this hypothesis testing procedure would incorrectly conclude that the average contribution is the same for eyes and for flowers.

Power: There is an 85% that this hypothesis testing procedure would correctly conclude that the average contribution when the picture is eyes is more than the average when the picture is flowers.

c.

$$(0.181 - 0.417) \pm 2.132\sqrt{\frac{0.067^2}{5} + \frac{0.151^2}{5}} \Rightarrow -0.236 \pm 0.158 \Rightarrow (-0.394, -0.078)$$

d. We believe that the true average contribution (per liter of milk) when the picture is flowers is somewhere between $0.08 and $0.39 [rounding the values in the CI] less than the true average contribution when the picture is eyes.

e. The procedure we followed would produce a 90% confidence interval that contains the true difference in mean contribution 90% of the time.

f. While the hypothesis test was statistically significant, indicating that the sample results are not consistent with natural variability, the values in the confidence interval are all small given the context. It does not appear that there is a meaningful increase the contribution (per liter of milk) when the employees feel like they are being watched.

3.

a.

$$\chi^2 = \frac{(7 - 11.7)^2}{11.7} + \frac{(29 - 24.3)^2}{24.3} + \frac{(31 - 26.3)^2}{26.3} + \frac{(50 - 54.7)^2}{54.7}$$

= 1.888 + 0.909 + 0.840 + 0.403 = 4.04

Note: To calculate this, you need to calculate the expected counts in each cell of the table:

$E_{11} = {}^{(36)(38)}/_{117} = 11.7$ $\qquad$ $E_{12} = {}^{(36)(79)}/_{117} = 24.3$

$E_{21} = {}^{(81)(38)}/_{117} = 26.3$ $\qquad$ $E_{22} = {}^{(81)(79)}/_{117} = 54.7$

b. Chi-square with (2-1)(2-1) = 1 degrees of freedom

c. 1 [This is the mean of the null distribution. The null distribution is the distribution of the test statistic if the null hypothesis were true; for a Chi-square test, this is the Chi-square distribution, so the expected value of the test statistic is thus the expected value of the Chi-square distribution, which is it's degrees of freedom.]

d. It would be appropriate to conduct a follow-up multiple comparisons procedure since at a 5% level [the default significance level] we would reject the null and conclude that there is evidence of an association (or relationship) between lingering fright symptoms and sex for the population of students represented by this sample. Looking at the cell contributions to the Chi-square statistic, the largest discrepancy between observed and expected counts occurred in the first cell—fewer men than expected experienced ongoing fright symptoms. Looking across all cells of the table, it appears that men were less likely to experience ongoing fright symptoms than women.