Can the mystery of Λ be solved by considering inhomogeneities?
Introducing timescape cosmology.

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Abstract. In this paper, the main features of the standard cosmological model and the underlying theoretical issues related to the cosmological constant Λ are reviewed. Thus, David Wiltshire's proposal of a timescape cosmology is explored as a possible alternative to resolve said issues. By revising some conceptual assumptions of the General theory of Relativity, the accelerated expansion of the universe is radically questioned: it is re-interpreted as an apparent effect resulting from gravitational energy gradients between bound and unbound systems. As a consequence, the assumption of exotic dark energy with negative pressure is no longer required. This new formulation of cosmology has striking implications: cosmic clocks tick at different rates and the universe itself has a different age, depending on whether the observer is measuring from within a galaxy or from a void.

1. Introduction

In 1998, two papers published by two independent groups of researchers [1, 2] provided evidence for the accelerated expansion of the universe. The evidence was coming from the observation of a particular type of exploding white dwarf called Type Ia supernovae (SNe Ia) that had been proved to be standard candles a couple of years earlier by Phillips in [3]. The importance of this discovery resulted in the Nobel Prize for Physics being awarded to the two groups in 2011. The accelerated expansion of the universe is arguably one of the most relevant developments of recent years and it fundamentally challenges our understanding of cosmology. In fact, since Hubble's discovery in 1929 [4], the universe was expected to be slowing down as a result of gravitational pull and cosmologists were eagerly looking for the deceleration parameter q_0. Therefore the evidence of accelerated expansion raised the obvious question, what is causing the universe's expansion to accelerate?

Here is where the cosmological constant term Λ comes into play. Introduced by Einstein in his field equations of General Relativity, this constant term had been widely debated throughout the twentieth century and its role in the cosmic landscape was unclear [5]. With the discovery of cosmic acceleration, however, Λ found its way into standard cosmology: in fact, the cosmological constant term has the same effect on Friedmann's equations as a smooth fluidlike component with negative pressure and repulsive gravity. This component would counteract the gravitational pull and cause the universe to accelerate. This is how dark energy is defined. This interpretation is currently accepted by the standard cosmological model, the so-called Lambda Cold Dark Matter (ΛCDM) model. Within this framework, dark energy is identified as the cosmological constant Λ. Since SNe Ia observations, ΛCDM has passed several tests and has been validated by multiple independent observational probes. Despite the success of this model, the physical interpretation of the cosmological constant Λ remains an issue in modern cosmology and there does not seem to be an easy answer. In 1968, Zel' Dovich showed that Λ could be interpreted as the zero-point energy of quantum fluctuations [6]. However, this causes an incredibly large discrepancy between the theoretically predicted value and the measured value. It is known as the cosmological constant problem [7]. Moreover, accepting Λ in the standard model of cosmology also leads to the so-called coincidence problem.

In this paper, I review these theoretical issues and explore David L. Wiltshire's timescape cosmology proposal as a viable substitute for the current standard framework. In this recently developed theory, he claims that General Relativity is yet to be completely understood. By refining some of its assumptions, Wiltshire reaches quite astonishing conclusions: the universe does not have a unique clock, but rather time flows at different rates depending on whether it is measured from voids or galaxies. According to this interpretation, cosmic acceleration would only be an apparent effect caused by gravitational energy gradients that are in turn a result of clocks ticking at different rates.

Sections 2, 3 and 4 concern the current status of cosmology: in Section 2 I introduce the theoretical background of ΛCDM and provide an explanation of the canonical interpretation of the cosmological constant within this context. Section 3 exposes the main observational evidence for the accelerated expansion of the universe. Finally, Section 4 deals with the theoretical issues related to the cosmological constant Λ. The second part of this review focuses on understanding Wiltshire's model and why it could solve some of the open questions of present cosmology. Section 5 tries to answer the question: is the universe homogeneous? To this purpose, I show a recent survey of the large scale structure as well as presenting how inhomogeneities are dealt with in cosmology. This section provides fundamental background for understanding the theoretical claims on which timescape cosmology is based. In Section 6 I talk about the theory, its implications and its current status. Finally, I provide a summary in Section 7, concluding that although the problems with the standard ΛCDM model are still unsolved, the timescape framework could at the very least provide a new and insightful approach to the way these problems are dealt with.

2. Theoretical framework

This section provides the fundamental mathematical tools needed to understand the ΛCDM model and the role of the cosmological constant within it. Note that it is not a complete description of the mathematical underpinnings of the model. Rather, this should be regarded as an overview of the main ideas of current cosmology from a more rigorous perspective. This will hopefully help the reader to better understand why some of them are challenged. It is worth mentioning that natural units are used throughout this paper: the speed of light is set to $c = 1$.

2.1. Einstein's field equations

General Relativity (GR) was developed by Einstein and first made public in 1915. It concerns the interplay between 4-dimensional spacetime and matter: as a theory of gravity, it constitutes the framework of cosmology. Its essence is nicely summarised by John Wheeler's famous quote: "Spacetime tells matter how to move, matter tells spacetime how to curve" [8]. GR fundamental equations are the so called Einstein field equations and are written as:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1}$$

with

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor, $g_{\mu\nu}$ is the spacetime metric and $T_{\mu\nu}$ is the energy-momentum tensor. In Eq. 2 $R_{\mu\nu}$ is the Ricci tensor whereas R is the Ricci scalar curvature.

This set of differential equations relate the curvature of spacetime represented by the Einstein tensor with the energy and momentum content of the universe, identified with the $T_{\mu\nu}$ tensor. Notice that Λ makes its first appearance in these equations as well. It is interesting to consider that the Λ term can be interpreted in two different ways depending on whether it is placed on the LHS or the RHS of Eq. 1 [9]: as written above, this additional term is just a contribution to the curvature of spacetime. However, if it is moved to the RHS, it can be interpreted as a contribution to the source term of the energy-momentum tensor. This latter interpretation of the cosmological constant suggests the existence of a new component in the universe which will turn out to be, at least according to the current standard interpretation, dark energy. I will discuss more on this later, in section 4.

2.2. Friedmann-Lemaître-Roberson-Walker cosmology

In 1917, when Einstein applied GR to the large scale structure of the universe [10], he made the assumption that the universe was homogeneous and isotropic: this is known as the cosmological principle. It greatly simplifies the field equations and it has been a fundamental assumption of cosmology ever since. Considering a 4-dimensional manifold with labels $\{x^0, x^1, x^2, x^3\}$ the interval between arbitrary neighbouring events in spacetime is given by:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{3}$$

Indices μ and ν can take values $\{0, 1, 2, 3\}$. Assuming an homogeneous and isotropic universe then allows to divide up spacetime geometry so that each hypersurface of constant coordinate, conventionally x^0, will meet the following requirements:

(i) it is spherically symmetric

(ii) it has the same value of Ricci scalar curvature R as any other hypersurface

The first constraint arises from the assumption that the universe needs to be isotropic at every point, whereas the second accounts for homogeneity [11]. Without going into the details of the derivation[1], it is then possible to conclude that the most general metric with these properties is the so called Friedmann-Lemaître-Roberson-Walker (FLRW) metric which can be written as:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right], \tag{4}$$

where (r, θ, ϕ) are the spherical polar coordinates, k is the curvature of the universe and $a(t)$ is the scale factor. The curvature constant k can take different values depending on whether the universe is flat ($k = 0$), negatively curved (saddle shape, $k < 1$) or with positive curvature (3-dimensional hypersphere, $k > 1$). It shall be noticed that the universe has been proved to be flat, as a result of evidence coming from the cosmic microwave background (CMB) radiation [12]. Furthermore, the scale factor $a(t)$ is a dimensionless parameter that relates the proper distance between two objects at time t and their co-moving distance. Both k and $a(t)$ are fundamental parameters of standard cosmology. Now, applying the FLRW metric to the field equations we obtain the two main results of the ΛCDM model, the so called Friedmann's equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \tag{6}$$

[1] The keen reader can find a nice and full derivation on Chapter 12 of *A First course in General Relativity*, B. Schutz, Second edition (2009).

where H is the Hubble constant, with a present day value estimated to be $H_0 = 100h$ km s^{-1} Mpc^{-1}, h being the dimensionless Hubble parameter with a value of ~ 0.7 [13]. Moreover, ρ is the energy density of the universe and p its pressure. From Eq. 6 it is clear that a fluid with negative pressure $p_\Lambda < -\frac{1}{3}\rho$ will mimic the effects of the constant Λ term and counteract the negative contribution coming from ordinary matter (ρ_m and $p_m > 0$), hence causing cosmic acceleration. Therefore, for a cosmological constant the equation of state is set to $\rho_\Lambda = -p_\Lambda$. Defining the equation of state parameter w:

$$w = \frac{p}{\rho}. \tag{7}$$

We conclude that the equation of state parameter for the cosmological constant $w_\Lambda = -1$. For completeness, it is also worth noting that though cosmic acceleration is currently considered to be caused by a Λ component with $w_\Lambda = -1$, it could also arise from a more general form of dark energy with negative pressure [14].

Another fundamental feature of a flat Universe, such as the one described by standard cosmology, is that its total energy density parameter $\Omega_{\text{tot}} = 1$ [5]. Given the content of the universe it is therefore possible to write:

$$\Omega_{\text{tot}} = \Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k, \tag{8}$$

where Ω_m is the energy density parameter for matter (both ordinary and dark matter), Ω_r for radiation, Ω_k for the curvature of the universe and finally Ω_Λ is the energy density parameter for the cosmological constant. An important aspect of these parameters is that their evolution in time can reveal the composition of the universe throughout its own history. Eq. 5 can be rewritten in terms of energy density parameters to give:

$$H(z)^2 = H_0^2\Big[\Omega_{m0}(z+1)^3 + \Omega_{k0}(z+1)^2 + \Omega_{r0}(z+1)^4 + \Omega_{\Lambda 0}\Big] \equiv H_0^2 E(z), \tag{9}$$

where the zero subscripts indicate the present day value of the parameters. It is clear that they scale differently with respect to the redshift z. At the present epoch, the radiation energy density Ω_{r0} is negligible and the value of $\Omega_{k0} \sim 0$ since $k \sim 0$. Consequently, Eq. 8 is greatly simplified and the remaining parameters have values $\Omega_{m0} \sim 0.3$ and $\Omega_{\Lambda 0} \sim 0.7$. The energy density parameter Ω_{m0} is further split between ordinary matter and dark matter, another fundamental component of the ΛCDM model which will not be discussed in detail here. However, for a complete picture, Fig. 1 provides a representation of the components of the universe at present. This makes the search for the meaning to the cosmological constant all the more important. Not only it is responsible for the accelerated expansion of the universe, but at present epoch it seems to be making up around 70% of the energy budget of the universe. Looking at the glass half-full, the last two decades of discoveries and the establishment of the ΛCDM model as a standard cosmology have brought about great progress in our understanding of the universe. However, according to this very interpretation, most of the universe is made up of something we yet do not understand.

3. Observational evidence

Before moving on to the theoretical issues that fundamentally challenge the ΛCDM model, in this section I focus on the observational evidence of the accelerated expansion of the universe. To talk about this, it is first necessary to address the measurement problem on cosmic scales, which is to answer the question: how do we know how far something is? Amongst the many ways of probing distances, standard candles turn out to be quite relevant. These objects are powerful tools in cosmology because they have fixed absolute magnitude M and it is known

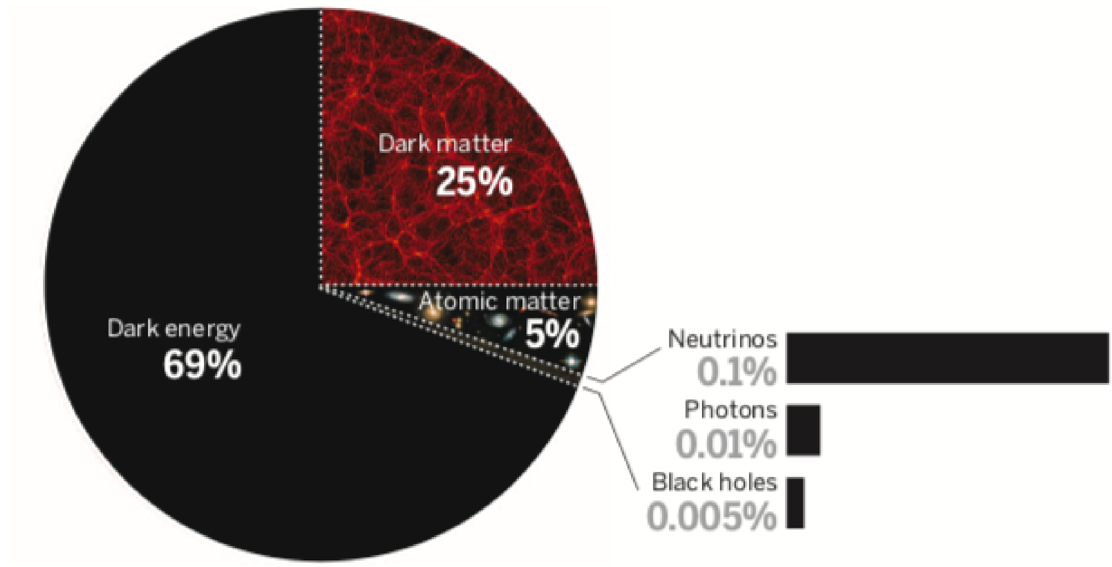


Figure 1: This graph shows the components that are thought to make up the universe at the present epoch. The vast majority of the universe is made up of dark energy, whereas matter (both dark and ordinary) comprise 30% of the total energy budget. A very small percentage is made up of more exotic components, such as neutrinos, photons and black holes. This picture is taken from [15].

that the brightness of an object is related to its distance by the inverse square law. Hence the luminosity distance d_L, which can be thought of as the apparent brightness of an object as a function of redshift, is defined as:

$$d_L = \sqrt{\frac{L}{4\pi F}} = (1+z)r(z) \tag{10}$$

where L is the intrinsic luminosity, F the energy flux and $r(z)$ is the comoving distance at a redshift z. For a flat universe with $k = 0$ and a constant equation of state parameter w it is possible to specify [5],

$$r(z) = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_{m0}(1+z')^3 + (1-\Omega_{m0})(1+z')^{3(1-w)} + \Omega_{r0}(1+z')^4}}. \tag{11}$$

Further, by considering the apparent and absolute magnitudes, respectively m and M, the luminosity distance d_L can be related to the distance modulus $\mu \equiv m - M$ as follows:

$$\mu = 5\log_{10}\left(\frac{d_L}{10\text{pc}}\right) = 5\log_{10}\left[\frac{(1+z)r(z)}{\text{pc}}\right] - 5. \tag{12}$$

Once again, this is in no way a comprehensive overview of the observational tools available to astrophysicists to probe the cosmos, but rather a basic introduction to the physical quantities employed when measuring SNe Ia, which are possibly one of the most relevant cosmological probes to investigate the expansion of the universe.

3.1. Type Ia Supernovae

Since after Hubble's discovery of the expanding universe, SNe Ia had been identified as promising candidates for the measurement of cosmic expansion, see Baade [16]. By the 90's of the 20th

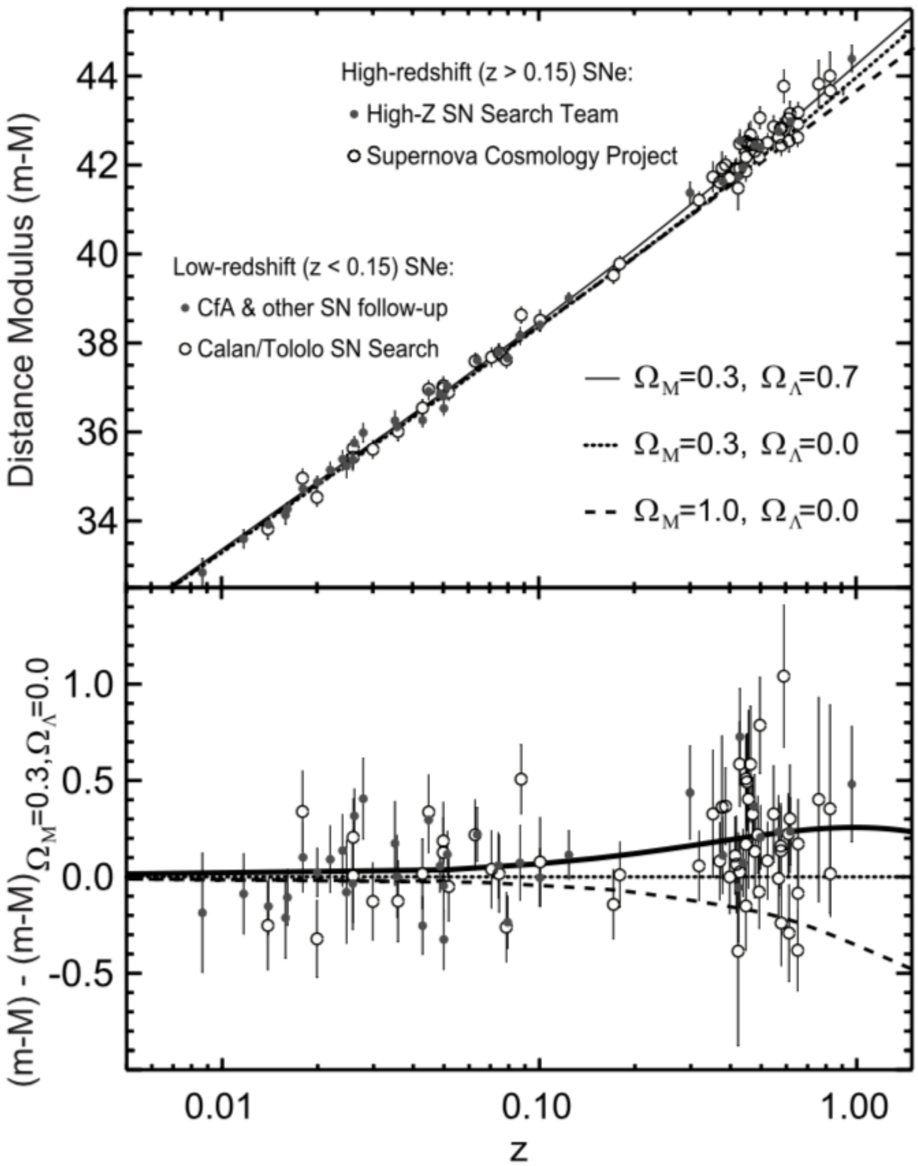


Figure 2: *Upper:* the distance modulus μ is plotted against the redshift z, the results are fitted with different constraints for the Ω_m and Ω_Λ density parameters. *Lower:* The residual plot relative to a $\Omega_m = 0.3$ model. From these plots it is clear that a universe without a cosmological constant is not an appropriate fit for the data. In fact, to the surprise of both groups at the time, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ represents the best fit to the observations in this dataset. This plot is taken from [20].

century, it had been proved that SNe Ia could be used as standard candles and technology had developed enough to make these measurements possible [17]. As a consequence, two different projects, the Supernova Cosmological Project (SCP) and the High-Z Supernova Search (HZSNS) started working to obtain data from SNe Ia. The HZSNS group led by Brian Schmidt published their findings in 1998 [1] and was soon followed by Saul Perlmutter's SCP team that published their paper in 1999 [2]. The observations made by both groups for $z > 0.15$ supernovae are showed in Fig. 2, as well as observations of lower redshift supernovae that had been previously measured by the Calan/Tololo Supernova Search (CTSS) in [18, 19].

In both samples the light coming from SNe Ia appears fainter than expected, meaning that the universe is unexpectedly accelerating. Having collected these measurements, it became clear that an Einstein-de Sitter universe with $\Omega_m = 1$ was not a good fit to the data. Not only that, but also an alternative non-Λ model with $\Omega_m \approx 0.2$ and $\Omega_\Lambda = 0$ was ruled out. As a solution and best fit model, a Λ universe was proposed with a density parameter related to the cosmological constant $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. I have already explained the implications of this discovery and the awe of the scientific community when this was announced. However, since 1998, the evidence of the accelerated expansion of the universe has only been validated by more and more observations. In fact, the number of SNe Ia detected in the night sky has been consistently growing in the last two decades, allowing more precise estimates. In addition to that, the concordance model of cosmology has been proved by a number of different independent probes such as the CMB, the baryonic acoustic oscillation (BAO) and weak gravitational lensing [21]. Measurements coming from different probes serve as constraints on the current cosmology and have been very useful to increase the accuracy to which we estimate the parameters of the model. Fig. 3 is an example of how different probes can constrain parameters. In this case, the 329 DES-NS3YR measurements from the first 3 years of the Dark Energy Survey Supernova

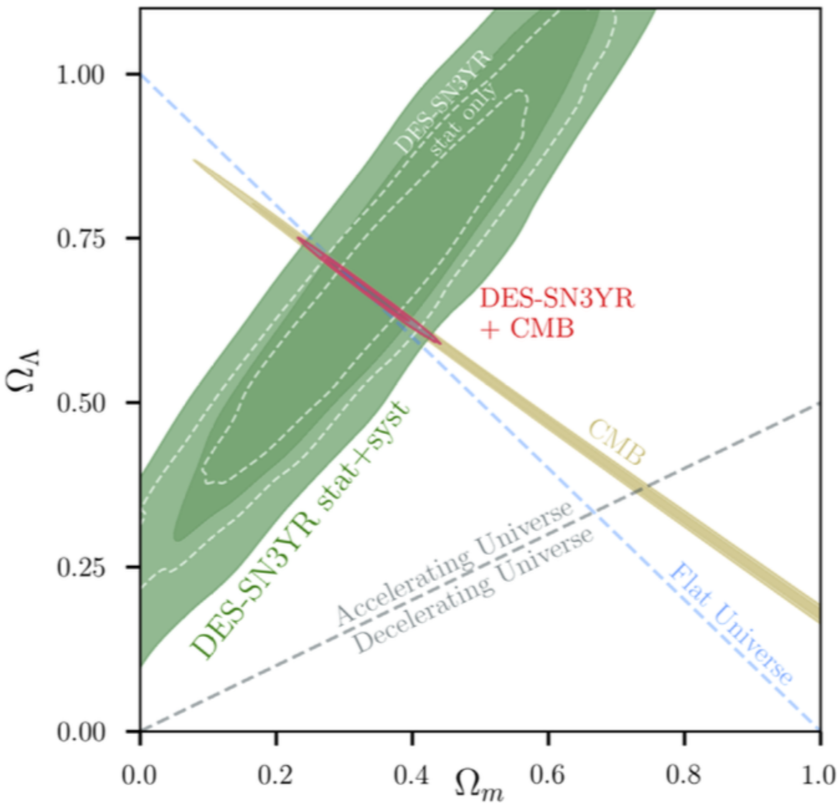


Figure 3: The plot shows the values of the density parameters Ω_m and Ω_Λ constrained by different cosmic measurements, namely the DES-SN3YR sample and the CMB data coming from the Planck collaboration. It is worth noting that different contours are displayed for DES-SN3YR, indicating different levels of confidence: the inner contour only includes statistical errors, whereas the outer one also accounts for the systematic errors. By looking at the grey dashed line it is clear that the universe is currently accelerating. In red, we find the estimates for a universe constrained by the measurements of DES-SN3YR + CMB. Finally, the dashed blue line further restricts the model requiring a flat universe. This image is taken from [22].

Program (DES-SN) are combined with CMB analysis from the Planck collaboration [23]. As a result of these measurements, the values of the density parameters have been estimated to be $\Omega_m = 0.335 \pm 0.042$ and $\Omega_\Lambda = 0.670 \pm 0.032$ [22]. Despite the incredible success of standard cosmological model in the last 20 years, some physicists have a problem accepting that most of the universe is made up of a smooth dark energy which is yet to be comprehended from a theoretical standpoint. In fact, although dark energy works very well with observations, its theoretical meaning opens up more questions than it answers and so far it has only led to problematic predictions. These problems and their possible solutions will be discussed in the following sections.

4. Issues with the cosmological constant Λ

Dark energy might appear to be "just what theorists ordered", as written by Turner in 2003 [24]. Whenever the accelerated expansion of the universe was discovered, everything fell into place and accepting a cosmological constant in the model was a natural consequence of the observations. However, fine-tuning is required to explain the size of this component of the universe, as well as the time at which it became relevant. Both these fine-tuning problems constitute serious theoretical issues.

4.1. The cosmological constant problem

The cosmological constant has a history that long predates the discovery of the accelerating universe. It was first introduced by Einstein in the attempt to obtain a static solution to his equations of GR when applied to the cosmic scale. This solution was however soon abandoned when Hubble showed that the universe was not static, but was expanding instead. After that, the role of Λ became ambiguous until in the 1960's, it was realised that contributions to the energy density of vacuum behave exactly like a cosmological constant [7]. Namely, in vacuum the energy-momentum tensor:

$$\langle T_{\mu\nu} \rangle = -\langle \rho \rangle g_{\mu\nu}. \tag{13}$$

If this is plugged into Eq. 1 the result has the same effect as an additional constant term $8\pi G \langle \rho \rangle$ to the cosmological constant. Defining an "effective cosmological constant" [7] as $\tilde{\Lambda} = \Lambda + 8\pi G \langle \rho \rangle$, the field equations look like:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \tilde{\Lambda} g_{\mu\nu} = 0. \tag{14}$$

Similarly, the cosmological constant can be regarded as a contribution to the energy density of vacuum. Defining by analogy an "effective vacuum energy" [7] $\rho_V = \langle \rho \rangle + \Lambda / 8\pi G$, Eq. 1 takes the form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 8\pi G \rho_V g_{\mu\nu} = 0. \tag{15}$$

This leads to the identification of the cosmological constant with the vacuum energy density of the universe. In 1968, Zel'Dovich attempted to explain this in terms of quantum fluctuations [6]. In fact, from quantum mechanical considerations it is known that the zero-point energy of vacuum does not equal zero but rather $\hbar\omega/2$. Hence, if each quantum field contributes to the total energy density, ρ_V can be calculated as

$$\rho_V = \frac{1}{2} \sum_{\text{fields}} g_i \int_0^\infty \sqrt{k^2 + m^2} \frac{d^3 k}{(2\pi)^3} \approx \sum_{\text{fields}} \frac{g_i k_{\text{max}}^4}{16\pi^2}, \tag{16}$$

where g_i is a weight that depends on the degrees of freedom of each quantum field and $k_{\max}$ is a momentum cutoff to stop the integral from diverging [5]. According to this calculation the value of $\rho_V \approx 5.156 \times 10^{96}$ kg m^{-3}. Though theoretically this looks like a promising candidate for interpreting the meaning of dark energy, if the value for ρ_Λ[2] is calculated from cosmological constraints, its estimated value is $\rho_\Lambda \approx 6.4 \times 10^{-27}$ kg m^{-3} [13].
This dramatically large discrepancy between the two estimates has come to be known as the cosmological constant problem and it is one of the most inaccurate predictions in the history of physics [25].

4.2. The coincidence problem

A less discussed but rather puzzling issue with the cosmological constant is the so called coincidence problem. As shown in Eq. 9 the density parameters evolve with redshift, meaning that the energy contribution of the components of the universe changes with time. This can be nicely visualised with the aid of Fig. 4 where radiation, matter and Λ energy densities are plotted against redshift. At earlier times (higher z), matter and radiation were dominant in the universe. At later epochs (lower z), radiation density becomes so small that is generally neglected and dark energy density is increasingly important. In the future, according to the predictions of the ΛCDM model, the cosmological constant term will begin to dominate over all the other parameters: as the universe expands, matter will be increasingly spread across the universe therefore becoming less relevant. As matter energy density ρ_m approaches zero, the universe will tend to a de Sitter model ($\rho_m = k = 0$, which in turn gives $H^2 = \Lambda/3$, from Eq. 5): in this model, the scale factor $a(t)$ varies exponentially with time. As a consequence, at some point in the future the universe will be expanding so fast that it will be impossible to observe any object outside of our own galaxy, which is going to be held together by gravitational pull. In this scenario, the present day coincidentally constitutes a particular time in the cosmic evolution with Ω_m and Ω_Λ having the same order of magnitude: this implies some sort of fine-tuning between these parameters and leads to the natural question: why now? How is it possible that we are living exactly in this moment of the universe's history?
Furthermore, a minority of cosmologists sees an additional problem with the cosmic coincidence. As discussed above, the cosmological constant Λ became relevant only at later epochs and it is intimately related to the beginning of the accelerated expansion. At the same time in cosmic history, the structure of the universe entered the non linear regime: the matter that was previously homogeneously distributed began to clump forming clusters, filaments and voids, eventually resulting in the non-homogeneous universe that we observe today [26]. At this point, the reader might wonder why this coincidence would constitute a problem and here is the reason: it has been claimed by a number of cosmologists that the effect of large scale inhomogeneities could be the cause of the apparent acceleration that we observe from SNe Ia. As a consequence, the currently accepted hypothesis of exotic dark energy would be wrong and rather an Einstein-de Sitter universe would be required. In other words, accepting this alternative view, the universe is not filled by a dark fluid with negative pressure, but only by matter. However, this claim comes with a challenge: if the universe does not have an homogeneous and isotropic FLRW geometry, we need to find a new way of describing its lumpy and inhomogeneous structure.
This second coincidence is central to David Wiltshire's work, one that he tries to address and solve through timescape cosmology, as I will discuss in Section 6.

[2] In this context ρ_V and ρ_Λ represent the same physical quantity. However, I use the subscript "V" to stress that it refers to vacuum energy density, whereas "Λ" emphasises that the contribution of vacuum energy density acts exactly like a cosmological constant.

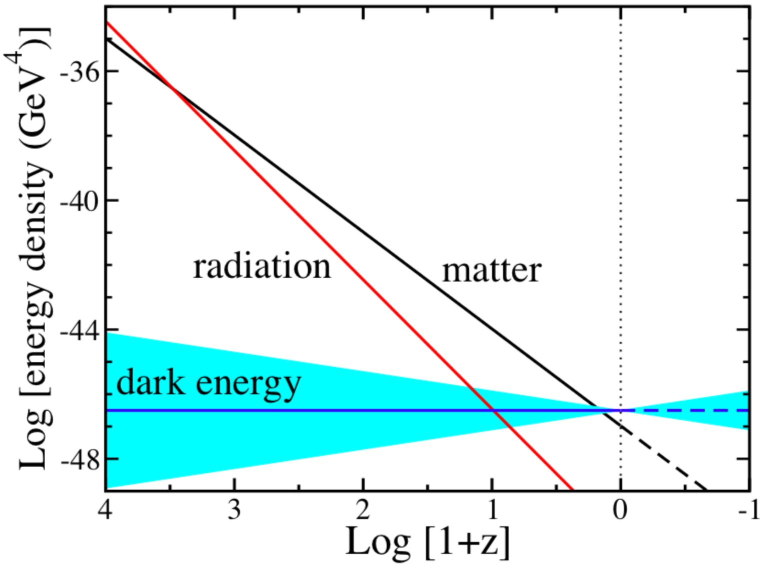


Figure 4: Logarithmic plot showing the evolution of the energy densities with respect to redshift (matter in black, radiation in red and the Λ or dark energy contribution in blue). Since dark energy density does not depend on z, ρ_Λ is constant and becomes more relevant at later times (the dashed vertical line indicates the present day). The light-blue band represents the equation of state parameter $w = -1 \pm 0.2$. Taken from [5].

5. Inhomogeneous universe: what is the real geometry of spacetime?

As argued in Section 2.2, standard cosmology assumes the universe to be homogeneous and isotropic at large scales, therefore requiring a FLRW metric. However, the cosmological principle is not based on tested observations but rather on the philosophical assumption of the Copernican principle, which dictates that there should be no privileged observer in the universe. In other words, all observers should describe the same universe regardless of where they are within it [27]. Contrarily, recent data suggest that the universe presents complex structure even on large scales and that some areas are over dense while some other are under dense. At the present day, observations show that voids of characteristic scale of $30h^{-1}$ Mpc constitute $40 - 50\%$ of the volume of the universe [28]. As an example, Fig. 5 displays voids of different sizes surrounded by filaments and clusters of galaxies.

Clearly, the present distribution of matter is inhomogeneous and its structure complex. Within this scenario, it is impossible to find an exact solution to Einstein's field equations. The challenge is therefore to find an appropriate spacetime metric $g_{\mu\nu}$ that accounts for the lumpiness of the universe or, more generally, to find a way to match the average geometrical structure of the universe, with a FLRW metric which provides an exact solution to the field equations [28]. This is the so called *fitting problem* and it was first brought up by George Ellis in [30]. There are a number of different ways in which the problem can be tackled. An attractive option is to attempt to solve the field equations exactly, for an inhomogeneous metric. Possibly, the most well-known amongst the models that use this approach is the Lemaître-Tolman-Bondi (LTB) model. The problem with this, however, is that to find exact solutions the metric is required to be highly symmetric. These assumed symmetries just do not seem to appear in the "real" universe: these models are therefore generally regarded as mathematically interesting but not realistically useful to describe the geometry of the large scale structure. Another possibility is to consider backreaction effects. Backreaction is defined as the effect of inhomogeneities on the

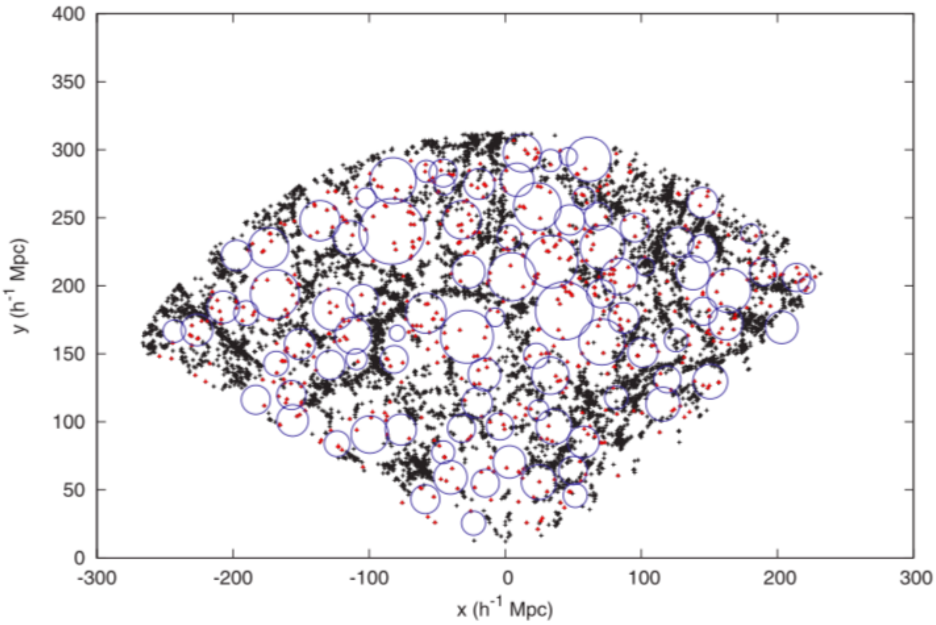


Figure 5: This plot shows data from the Sloan Digital Sky Survey (SDSS) Data Release 7 in 2012. This is at present one of the most complete surveys available. Voids are circled in blue here and their size varies from a radius r of $\sim 10h^{-1}$Mpc to the largest voids measuring $\sim 30h^{-1}$Mpc [29].

properties of an averaged homogeneous background. In other words, it is the deviation from an ideal FLRW metric caused by the non-homogeneous structure of the universe. In general, accounting for the effects of backreaction on an averaged spacetime metric $\bar{g}_{\mu\nu}$ can be simply written as:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \tag{17}$$

where the inhomogeneous metric $g_{\mu\nu}$ is the "real" metric of the lumpy universe and it is a result of the sum of the background $\bar{g}_{\mu\nu}$ with a "backreaction metric" $\delta g_{\mu\nu}$. Whether backreaction effects are relevant in cosmology is key to the argument here, but it is to the present day a vastly discussed topic and there is not a conclusive answer yet (see Buchert *et al* [31] for arguments supporting the relevant effects of backreaction, or Green and Wald [32, 33] for arguments against it). Within this debate, standard cosmology assumes that the averaged metric $\bar{g}_{\mu\nu}$ must be a homogeneous FLRW one, as supported by the evidence from the CMB and the observational success of this *a priori* assumption. According to this view, any backreaction effect must be small and it can therefore be treated as a perturbative correction [26]. However, an increasing number of cosmologists feels that this naive approximation might be missing some fundamental aspects of the geometry and dynamics of spacetime.
Finally, a third option is to find an averaging scheme in order to deal with the locality and non-linearity of the metric tensor in the field equations. In general according to this approach, if an appropriate averaging scheme is assumed for an inhomogeneous spacetime metric, Eq. 1 can be rewritten as:

$$\langle G^{\mu}_{\nu} \rangle = \langle g^{\mu\lambda} R_{\lambda\nu} \rangle - \frac{1}{2} \delta^{\mu}_{\nu} \langle g^{\lambda\rho} R_{\lambda\rho} \rangle = 8\pi G \langle T^{\mu}_{\nu} \rangle. \tag{18}$$

The terms in this equation have the same meaning as in Eq. 1, with the crucial difference that they are now averaged [26]. It is interesting to note that the Ricci scalar term has now been substituted by $\delta^{\mu}_{\nu} \langle g^{\lambda\rho} R_{\lambda\rho} \rangle$ where δ^{μ}_{ν} is the Kronecker delta operator. The reason is that it is impossible to define a constant scalar curvature for an inhomogeneous universe. Hence, it is substituted by set of parameters that describe the curvature of the spacetime metric in each direction. While many such schemes have been proposed, only the average scheme proposed by Buchert in the early 2000s will be covered here, as it is employed as a framework in the context of timescape cosmology.

5.1. Buchert's averaging scheme

Within this scheme, only scalar quantities are averaged, assuming an irrotational dust universe. For any scalar Ψ, averaging and time evolution are not commutative, a property that can be expressed as:

$$\partial_t\langle\Psi\rangle - \langle\partial_t\Psi\rangle = \langle\Psi\theta\rangle - \langle\theta\rangle\langle\Psi\rangle, \tag{19}$$

where θ is called the volume average expansion scalar and it is defined as $\langle\theta\rangle = 3(\dot{\bar{a}}/\bar{a})$. This non-commutative relation quite neatly highlights an important feature of this scheme: the RHS of Eq. 19 is non-zero, indicating the presence of backreaction. Conversely, in a FLRW scenario averaging and time evolution would commute resulting in the equality $\partial_t\langle\Psi\rangle = \langle\partial_t\Psi\rangle$. This is conceptually crucial to understand the difficulties of averaging a non-homogeneous background. For the rigorous mathematical derivation see [34, 35]. Buchert's scheme yields the equations:

$$3\frac{\dot{\bar{a}}^2}{\bar{a}^2} = 8\pi G\langle\rho\rangle - \frac{1}{2}\langle\mathcal{R}\rangle - \frac{1}{2}\mathcal{Q}, \tag{20}$$

$$3\frac{\ddot{\bar{a}}}{\bar{a}} = -4\pi G\langle\rho\rangle + \mathcal{Q}, \tag{21}$$

$$\partial_t\langle\rho\rangle + 3\frac{\dot{\bar{a}}}{\bar{a}}\langle\rho\rangle = 0, \tag{22}$$

with $\langle\mathcal{R}\rangle$ being the average spatial curvature and $\mathcal{Q}$ the kinematic backreaction term [36]. It is also worth noticing that $\bar{a}(t) \equiv [\mathcal{V}(t)/\mathcal{V}_i]^{\frac{1}{3}}$ where $\mathcal{V}$ is the spatial volume: the scale factor is defined in terms of the average volume, rather than being a unique value depending on the geometry. Eq. 20 and Eq. 21 are clearly reminescent of the two Friedmann's equations whereas Eq. 22 can be regarded as a continuity equation for the averaged density ρ. The clear analogy of these results with the standard Friedmann equations is one of the reasons why this scheme has been chosen by Wiltshire for the development of timescape cosmology.

6. Timescape cosmology

Having outlined the context of the cosmological debate that motivates timescape cosmology, as well as the averaging approach that was employed in its development, the theory is presented here. I will not go into the technical details of the phenomenological model, but rather explore the main ideas of the theory, its interpretations and the consequences that can be inferred from it. Wiltshire's starting point is to claim that some subtle aspects of GR have been overlooked when applying the theory to cosmic scales. In particular, the role of gravitational energy which is inherently difficult to define, as space itself carries energy.

6.1. Foundational aspects

The Strong Equivalence Principle (SEP) is a fundamental concept of GR. This principle claims that for any event in spacetime a local inertial frame of reference can be chosen in order to eliminate the effects of gravity within a certain spacetime neighbourhood [37]. SEP implies that gravity is frame dependent: this comes from requiring that the laws of physics as formulated by

Special Relativity (SR) must hold in any frame. As a consequence, gravity is concluded to be a property of spacetime itself. Not only that, but by Eq. 1, gravity is also intimately intertwined with the matter and energy content of the universe. Einstein himself stated,"in a consistent theory of relativity there can be no inertia relatively to space, but only an inertia of masses relatively to one another" [10]. In this formulation, time and space are a unified physical entity that does not merely serve as a background to other physical phenomena, but rather it is shaped by their interaction.

However, as required by SEP, one can only infer about a local inertial frame of reference. In other words, the relation between the geometry of spacetime and the energy content of the universe can only be known for such frames: this means that GR is only formulated locally. Therefore when it is applied to large scales, it is natural to ask how it is possible to infer anything about the global structure of the universe. In standard cosmology, i.e., within the framework of the ΛCDM model, this problem has generally been overlooked by assuming the cosmological principle: if the universe is homogeneous, all the local inertial frames will show the same properties, meaning that they are practically indistinguishable. There are no gradients in the spacetime curvature and although GR is, in principle, only defined locally, it is possible to infer about the global nature of spacetime. In other words, it is possible to define a unique cosmic clock and a unique cosmic rod that can be used to measure distances unambiguously. It also follows that, conveniently, the clocks and rods we deduce from within our galaxy will be appropriate ones to time and measure the whole universe.

Clearly, this situation cannot be applied for an inhomogeneous universe that shows complex foam-like structure: in this situation, it is impossible to deduce anything about the global geometry of the universe, due to the local nature of GR. Even assuming that on an arbitrarily large scale the cosmological principle and Copernican assumption hold, which is in fact what we see, it would not be correct to apply a FLRW metric. In [36] this subtle argument is supported by an example that I will now try to explain: let us imagine a universe filled with equally distanced black holes with same mass and electric charge. Let us also assume that the gravitational pull that they exert on each other is balanced by Coulomb repulsion, therefore creating a static universe. This universe can clearly be thought of as homogeneous in an average sense, since matter is overall equally distributed. Nonetheless, the observations made by an observer inside the gravitational well of a black hole are going to be very different from the ones of an observer that sits in a region at an equal distance from all the surrounding black holes. This can be intuitively understood as a consequence of GR: the clocks of the two observers will not be ticking at the same rate, as much as their rods will not be measuring the same distances. This example differs from the "real" universe in quite a few ways, most importantly because in reality the universe is not static but it is expanding instead. Nonetheless, it explains the core of the subtle claim that is put forward by timescape cosmology. With this "charged-black hole universe" example, the possibility of employing a FRLW-like metric in a universe with a complex foam-like structure has been discarded. How is it then possible to solve the problem of locality? Here is where Buchert's averaging scheme comes into play. It is in fact through an appropriate averaging of the field equations that the locality problem is solved.

6.2. Averaging in timescape: interpretation and implications

In general, what is done by an averaging scheme is to "stitch" together different local regions for which the field equations can be solved and therefore find an averaged solution similar to Eq. 18. Hence, we ought to find a domain within which such regions can be suitably defined. To do so, a fundamental characteristic of the current non-homogeneous universe needs to be considered: though it is expanding, such expansion is not happening within virialized systems, which are systems where masses are held together by mutual gravitational pull. This means that there is a crucial difference between over-dense bound systems and unbound under-dense systems which

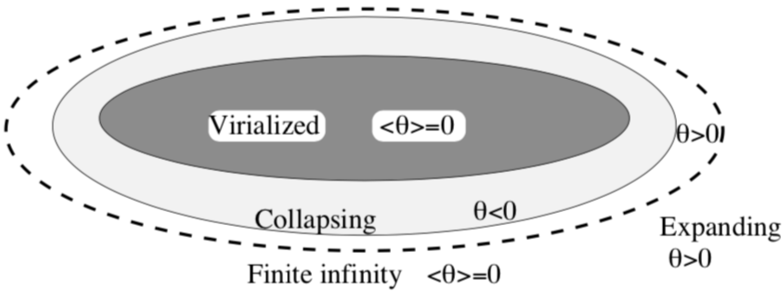
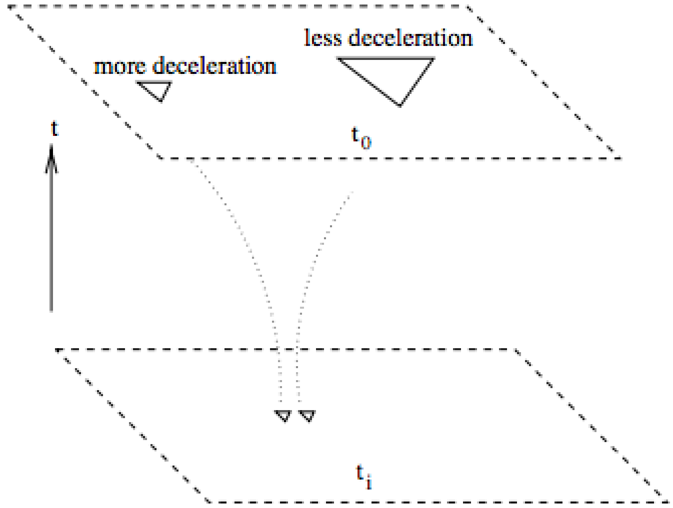


Figure 6: A representation of *finite infinity*, provided in [36]. The bound system is not expanding, meaning that its volume average expansion (as it was defined in Section 5.1) $\langle\theta\rangle = 0$. Conversely, the unbound region beyond it, which can be thought of as an under-dense void region, is expanding. Therefore for such region, $\langle\theta\rangle > 0$. Finally, there is a "collapsing" region which feels the gravitational pull of the bound system: this area will clearly have $\langle\theta\rangle < 0$. Finite infinity demarcates the boundary between bound and unbound systems, such that a region defined within those boundaries has volume average expansion $\langle\theta\rangle = 0$.

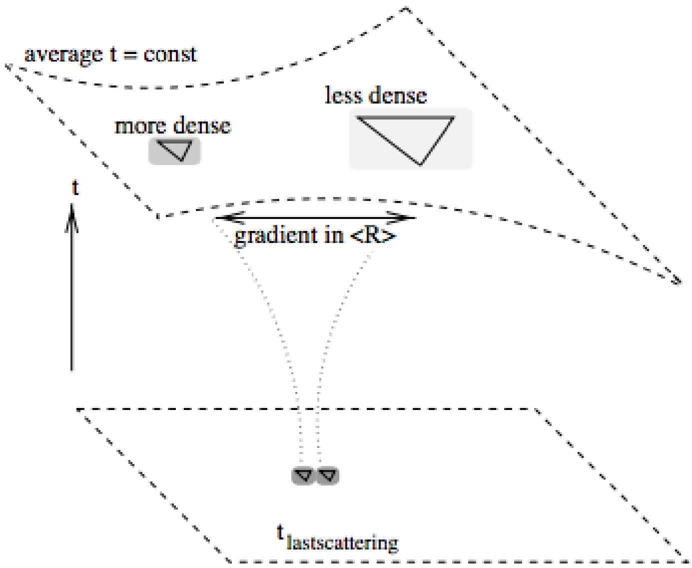
are expanding, such as voids. It is possible to imagine to enclose both systems within a region such that the average volume expansion $\langle\theta\rangle = 0$, as showed in Fig. 6. This serves to define the concept of *finite infinity* (referred to as fi) [36], a fundamental idea in the context of timescape. Having defined *finite infinity*, Wiltshire postulates a new equivalence principle that can be applied to such regions. The Cosmological Equivalence Principle (CEP) is here reported as written in [38]:
"At any event, always and everywhere, it is possible to choose a suitably defined spacetime neighbourhood, the cosmological inertial frame (CIF), in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation,"

$$ds^2_{CIF} = a^2(\eta)[-d\eta^2 + dr^2 + r^2 d\Omega^2], \tag{23}$$

where $d\Omega^2$ is the solid angle. Without further examining the details of this mathematically rigorous definition, which was inserted for sake of completeness, I now explore the implications of having set this operational framework by defining both *fi* and the CEP.
It is crucial to understand that a region defined within a *finite infinity*, i.e. a cosmological inertial region (CIR), has a FLRW geometry by the definition of the Cosmological Equivalence Principle [37]. This however, does not mean that the universe can be treated in a FLRW fashion on a global scale: regional frames ought to be "stitched" together and appropriately averaged, which in timescape cosmology means to be averaged employing Buchert's scheme. The issue to be addressed now is to understand conceptually what it means to relate two such frames. To understand this, imagine a lattice of observers isotropically expanding with respect to 3-dimensional spatial coordinates. If at some point such system started to isotropically decelerate, it would be possible to define energy, but the net force applied on the lattice would be overall zero, meaning that the system would still be an inertial frame. Now if it is assumed that two such lattices, originally expanding at the same rate, then decelerate with different rates, say $\alpha_1 > \alpha_2$, then as a result their proper times would differ. Namely, $\tau_1 < \tau_2$: in the lattice of observers that has decelerated more, time flows more slowly.
This is illustrated in *Panel (a)* of Fig. 7. Fundamentally, the fact that the clocks of the two frames are not synchronised causes a gradient in gravitational energy. By CEP, this scenario is exactly equivalent to what is observed in the dynamical evolution of the universe, *Panel (b)* of



(a)



(b)

Figure 7: *Panel (a):* At some moment in the past, say t_i, two distinct lattices of observers (represented by the two small triangles in the lower plane) are homogeneously expanding. As time progresses the two frames start decelerating at different rates. At some later time, namely the present day t_0 (upper plane), the two frames will have different proper times τ since they have been decelerating at different rates: more precisely the leftmost frame, that has decelerated more, will have aged less than the rightmost frame that had smaller deceleration. In the context of timescape cosmology, this scenario is exactly equivalent to the one depicted in *Panel (b):* at time of last scattering, the universe is homogeneous and it is therefore possible to identify any two frames with equal matter distribution (lower plane). As the universe evolves, structure is created and the two frames originally homogeneous have now different densities and therefore different proper times: the leftmost, more dense reference frame will have aged less than the rightmost one which is also less dense. The clocks within the two frames are ticking at different rates. Although it is still possible to define a constant time on average, there will be a gradient in spatial curvature due to the difference in the densities of the frames which needs to be taken into consideration. This picture is taken from [37].

Fig. 7: two frames with equal density at time of last scattering evolve to have different densities at some later point in time. This in turn causes gradients in the average scalar curvature $\langle \mathcal{R} \rangle$. Here we reach a conclusive point: by equating these two situations, which is made possible by the CEP, we are able to link gradients in the average spacetime curvature with gravitational energy gradients [26]. The framework of timescape is now conceptually complete.
From this it is concluded that the measured accelerated expansion of the universe is only an apparent effect due to those gravitational energy gradients [39]: we measure it because our measurements are done from within a bound system (a galaxy). Within this subtle formulation however, the Copernican principle is still valid: we are typical observers in a bound system. Nonetheless, an observer measuring from an unbound system (a void), will not measure an accelerated expansion. Therefore, in this model the cosmological constant $\Lambda = 0$, implying that there is no such thing as dark energy in our universe. Finally, the cosmic age is not the same everywhere, as generally assumed by standard cosmology. It varies going from values of ~ 14.7 Gyr as measured from a galaxy, to values of ~ 18.6 Gyr as measured from a volume-average region [37, 40]. Therefore, timescape solves the theoretical issues presented in section 4. There is no longer a need to justify the cosmological constant Λ as vacuum energy since the hypotesis of dark energy is discarded. Furthermore, the coincidence problem is naturally solved by understanding that the accelerated expansion started when the universe entered the non linear regime and structure began to appear.

6.3. Testing the theory: current state of observations
Timescape has an indubitable appeal: although still preserving the mathematical structure of GR it accounts for the apparent accelerated expansion of the universe without the need of invoking exotic dark energy. The only price to pay is a close revision to the application of the current theory of gravity to the large scales of the universe. However there is a reason why the scientific community is so fond of the ΛCDM model: its incredible success at matching observation. In this section the timescape approach is tested.
The biggest challenge to test timescape is to understand the biases that arise from probing cosmology in a bound system. Measurements taken by observers in a galaxy and observers in voids will in fact differ. The first step towards this goal is therefore to reinterpret observational quantities according to this new non-homogeneous framework [40]. To do so, the fractal bubble (FB) model is generally employed: this model identifies two relevant scales, one representing voids and the other associated with the galaxies around them. Within the bubbles that contain galaxies the geometry of spacetime is flat, which is instead not true for void regions. The re-interpretation of cosmological observables is a complex process that I will not discuss in detail: the operational procedures adopted to achieve this can be found in [41]. Moreover, intimately related to the this challenge is the issue of analysing the big data sets coming from different missions and programs currently available. The numerical codes that have been developed over the years support a standard FLRW cosmology and are not suitable for testing timescape. This is an ongoing limitation of the theory that is expected be implemented in the next years.
At the present day however, the theory has already passed three major tests and they have all proved it to be consistent with cosmological probing. First of all, it is consistent with Type Ia Supernovae luminosity distances, which are quite obviously a fundamental testing ground: validation from this probe means that the hypothesis of an apparent accelerated expansion due to curvature and energy gradients in the inhomogeneous universe is a viable candidate. Moreover, timescape has also been showed consistent with the temperature anisotropies of the CMB as well as fitting the observations from baryon acoustic oscillation [42]. The resulting fits of these three independent probes are showed in Fig. 8.
Another important validation for the timescape approach as come in recent years from the work of a Hungarian and Hawaiian collaboration [43]: by assuming as initial conditions an Einstein-

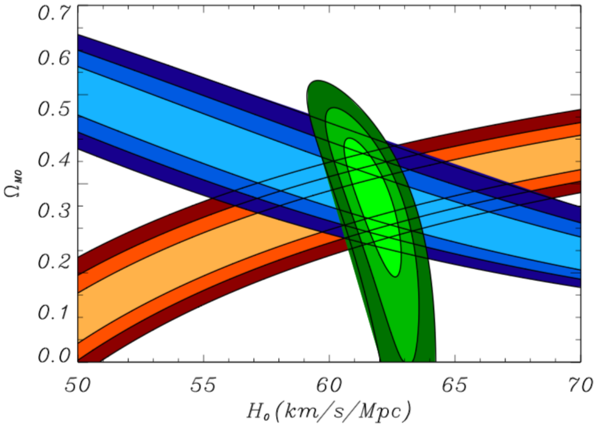


Figure 8: The contour plot shows the present-day mass density parameter Ω_{m0} and Hubble constant H_0 constrained by three different fits. The green contour is the best fit of Sne Ia data. The three contour lines represent 1σ, 2σ and 3σ confidence levels. The blue and red contours are instead fits of the CMB observations and the baryon acoustic oscillations data respectively. For both these latter contours, the confidence lines represent the 2%, 4% and 6% limits. Note that the fits are consistent with the results of standard cosmology. Plot taken from [40].

de Sitter universe with $\Omega_m = 1$ and $\Lambda = 0$ they evolve the model numerically by using a timescape-like scheme [44], the average expansion rate approximation (AvERA) approach. As a result, the time evolution of this non-homogeneous universe results incredibly close to the evolution of the standard ΛCDM model. Namely, in Fig. 9 the scale factor $a(t)$, the Hubble constant $H(t)$, the redshift $z(t)$ and the density $\rho(t)$ are plotted against time. At late times, it is evident that a standard Einstein-de Sitter model diverges from the predictions of standard cosmology. Conversely, the timescape-like evolution appears to match quite closely the standard interpretation both at early and late times. Finally, in the same paper the best fit to Type Ia Supernovae is plotted as showed in Fig. 10: remarkably, the fit from the AvERA simulation matches the observations even better than ΛCDM. Although it is still early to claim anything conclusive, the results from this first numerical simulation provide impressive clues of the validity of this approach. In summary, timescape cosmology has already passed a number of tests and the results appear quite promising: the fact that they are indistinguishable from ΛCDM is an hint that the theory is heading in the right direction, due to the incredible success of the standard model at matching obervations. At the same time though, it needs to be kept in mind that this does not prove the validity of timescape over standard cosmology. In the coming years, the re-interpretation of cosmological parameters in the context of timescape will be completed and it is hoped that upcoming programs, such as the Euclid mission and the CODEX experiment will provide further validation to support the model [44].

7. Conclusions

The establishment of a concordance cosmological model is an enormous scientific achievement but has also brought about many challenges that shake our understanding of the universe to its very core. But physics always flourishes in crisis: the inexplicable nature of dark energy and the astonishing realisation that this mysterious entity is responsible for the ultimate fate of the universe has originated a rich landscape of theories and conjectures, as well as providing a direct link to the quantum realm. What is going to be required to unravel this cosmic puzzle? This we do not know yet. Maybe a super-symmetric explanation of particle physics, maybe a theory of

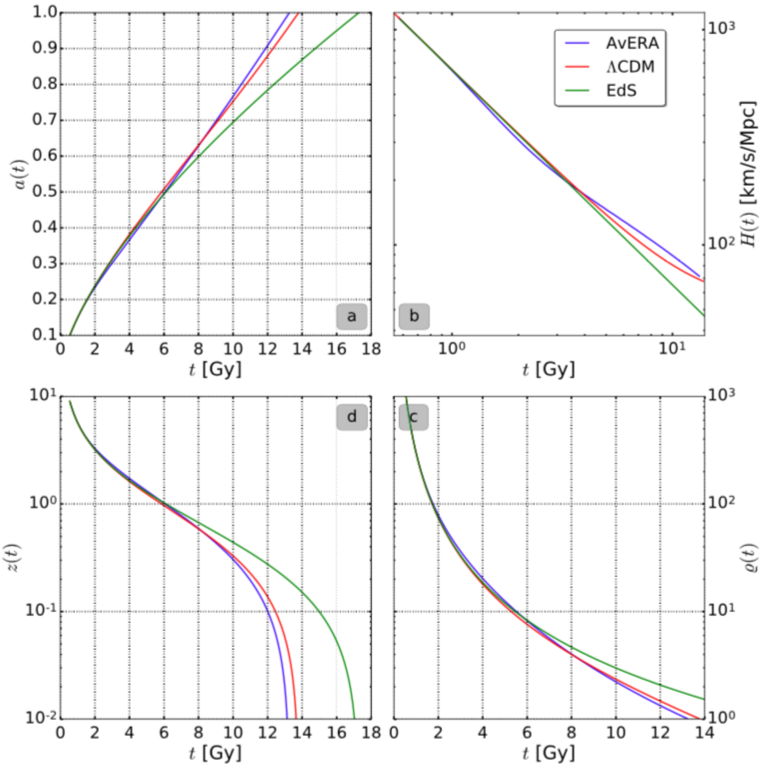


Figure 9: Fundamental cosmological parameters are shown with respect to time. From top left going clockwise: *(a)* the scale factor, *(b)* the Hubble constant, *(c)* the matter density and *(d)* the cosmic redshift, on the bottom left. The evolution in time of these parameters is plotted for three distinct models of the universe: in green a $\Omega_m = 1$ Einstein-de Sitter model, in red the standard ΛCDM and in blue the numerical simulation performed by the authors of the paper. This simulation is based on an AvERA approach and it is timescape-like in nature. Plot taken from [43].

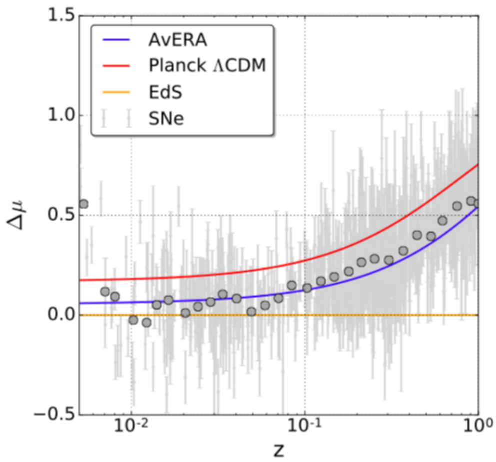


Figure 10: The distance modulus $\Delta\mu = \mu - \mu_{EdS}$ of SNe Ia from the SuperCal compilation is plotted against redshift (the error bars are plotted as grey lines whereas the binned values are displayed as grey spots). In the graph three different models are fit to the measurements. In yellow an Einstein-de Sitter model; more relevant are the blue and the red lines. The red line represents the standard cosmology ΛCDM, whereas the timescape-like model is displayed in blue. The AvERA approach seem to fit observations better than the standard model. Graph taken from [43].

quantum gravity that would finally unify the two pillars of modern physics. Within this scenario, timescape cosmology provides an elegant yet extremely subtle answer that does not require the assumption of some dark fluid, as much as no aether turned out to be required to explain the constant speed of electromagnetic waves, more than one hundred years ago. Wiltshire's proposal is not conservative, as some cosmologists have claimed: although it does not require new physics to explain the current observations, it provides a deeper interpretation of our theory of gravity, attempting to understand its profound meaning as well as the complex structure of the universe we live in. In this paper, I have outlined its main features as well as its position within the wider ongoing cosmological discussion. As a recently developed theory, it is too early to rush into any conclusive statements. However, its claims and implications should certainly be taken into consideration in the present debate: if not as true claims, at least as a valuable attempt of a longed-for paradigm shift that cosmology has been needing to make the next leap forward.

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