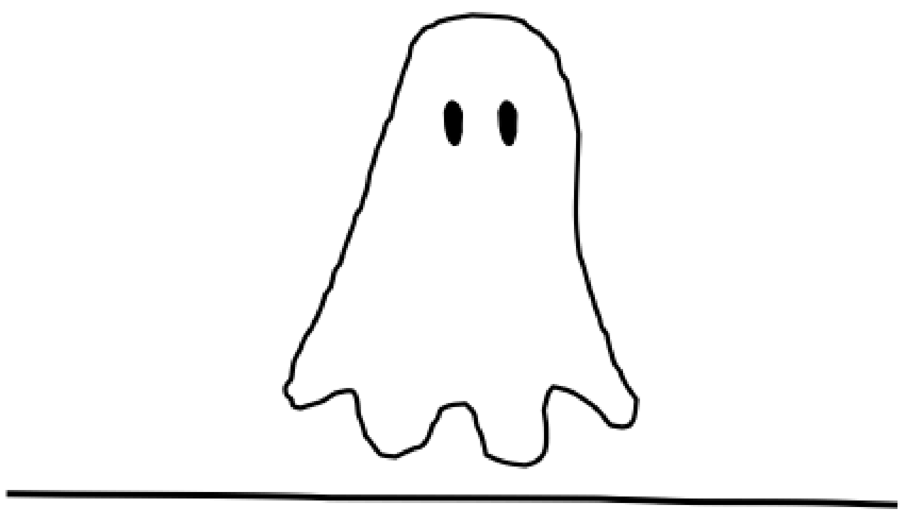
Inferential Statistics Part 2 - MATHEMATICS MADE EASY

Normal Distribution



Paranormal Distribution

1) Formulas with Simple Explanations

Z-Test Formula: Z-TEST LOOKUP TABLE

When sample size is large (n > 30), and population variance is known:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- **Z**: Z-score (how far the sample mean is from the population mean)
- $\bar{X}$: Sample mean (the average from the sample)
- μ: Population mean (the average you're comparing to)
- σ: Population standard deviation
- **n**: Sample size (how many data points in the sample)

z	.00	.01	.02	.03	.04
−3.4	.0003	.0003	.0003	.0003	.0003
−3.3	.0005	.0005	.0005	.0004	.0004
−3.2	.0007	.0007	.0006	.0006	.0006
−3.1	.0010	.0009	.0009	.0009	.0008
−3.0	.0013	.0013	.0013	.0012	.0012
−2.9	.0019	.0018	.0018	.0017	.0016
−2.8	.0026	.0025	.0024	.0023	.0023
−2.7	.0035	.0034	.0033	.0032	.0031
−2.6	.0047	.0045	.0044	.0043	.0041
−2.5	.0062	.0060	.0059	.0057	.0055
−2.4	.0082	.0080	.0078	.0075	.0073
−2.3	.0107	.0104	.0102	.0099	.0096

T-Test Formula: LOOK UP TABLE

When sample size is small (n < 30) and population variance is unknown:

One sample T-test

$$t = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$$

Where:

- $\bar{X}$: Sample mean
- μ: Population mean
- s: Sample standard deviation
- n: Sample size

Two-Sample T-Test

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\bar{x}_1$ = observed mean of 1st sample
$\bar{x}_2$ = observed mean of 2nd sample
s_1 = standard deviation of 1st sample
s_2 = standard deviation of 2nd sample
n_1 = sample size of 1st sample
n_2 = sample size of 2nd sample

PAIRED T TEST (same group before and after)

$$t = \frac{\bar{D}}{\frac{s_D}{\sqrt{n}}}$$

- n: Sample size
- $\bar{D}$: Mean difference between paired data points
- s_D: Standard deviation of the differences

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**
df								
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539

Chi-Square Test Formula:

Used for categorical data:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Where:

- χ^2: Chi-square statistic
- **O**: Observed frequency (what you actually observe)
- **E**: Expected frequency (what you expect to see)

Degrees of freedom	Significance level (α)							
(*df*)	.99	.975	.95	.9	.1	.05	.025	.01
1	-------	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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2) Small, Easy Numerical Examples

Z-Test Example:

A company claims the average salary of employees is $50,000. A sample of 40 employees shows an average salary of $52,000 with a population standard deviation of $5,000. Is the sample salary significantly different from the population mean?

- **Given:**
  - $\bar{X}$ = $52,000
  - μ = $50,000
  - σ = $5,000
  - n = 40

Z-Test Calculation:

$$Z = \frac{52,000 - 50,000}{\frac{5,000}{\sqrt{40}}} = \frac{2,000}{790.57} = 2.53$$

One-Sample T-Test Example:

A bakery claims its average cupcake weighs 120 grams. You take a sample of 15 cupcakes, and the average weight is 125 grams with a standard deviation of 10 grams. Is the sample weight significantly different?

- Given:
  - $\bar{X}$ = 125
  - μ = 120
  - s = 10
  - n = 15

T-Test Calculation:

$$t = \frac{125 - 120}{\frac{10}{\sqrt{15}}} = \frac{5}{2.58} = 1.94$$

Chi-Square Test Example:

A grocery store wants to know if customers prefer brand A or B. The store expects 50% of customers to choose each brand, but in a sample of 100 customers, 60 chose brand A and 40 chose brand B. Is the preference different from expected?

- Given:
  - Observed (O) = [60, 40]
  - Expected (E) = [50, 50]

Chi-Square Calculation:

$$\chi^2 = \frac{(60 - 50)^2}{50} + \frac{(40 - 50)^2}{50} = \frac{100}{50} + \frac{100}{50} = 2 + 2 = 4$$

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3) Problem Statements with Solutions

Problem 1: Ice Cream Sales at Two Different Stores (Two-Sample T-Test)

You own two ice cream stores. You want to know if Store A sells more ice cream than Store B. You collect data for 5 days:

- Store A Sales (in scoops): 100, 105, 98, 102, 101
- Store B Sales (in scoops): 95, 99, 97, 93, 96

Task: Use a **two-sample t-test** to compare the average sales of the two stores.

- $\bar{X}_1$ = 101.2, $\bar{X}_2$ = 96
- $s_1 = 2.8$, $s_2 = 2.24$, $n_1 = 5$, $n_2 = 5$

Solution:

$$t = \frac{101.2 - 96}{\sqrt{\frac{(2.8)^2}{5} + \frac{(2.24)^2}{5}}} = \frac{5.2}{1.58} = 3.29$$

Problem 2: Exam Scores Before and After Tutoring (Paired T-Test)

You want to know if tutoring improves student exam scores. You collect exam scores from 5 students before and after a tutoring session:

- Before: 70, 75, 68, 72, 74
- After: 78, 80, 75, 79, 81

Task: Perform a **paired t-test** to see if the tutoring improved scores.

- Differences: 8, 5, 7, 7, 7
- $\bar{D} = 6.8, s_D = 1.1, n = 5$

Solution:

$$t = \frac{6.8}{\frac{1.1}{\sqrt{5}}} = \frac{6.8}{0.49} = 13.88$$

Problem 3: Customer Satisfaction with Three Products (Chi-Square Test)

A company has three products (A, B, C). The expected satisfaction rating is that 50 customers should like A, 40 should like B, and 30 should like C. However, the observed ratings are 45 for A, 35 for B, and 40 for C.

Task: Use a **chi-square test** to see if the actual satisfaction ratings differ from the expected ratings.

- Observed (O) = [45, 35, 40]
- Expected (E) = [50, 40, 30]

Solution:

$$\chi^2 = \frac{(45-50)^2}{50} + \frac{(35-40)^2}{40} + \frac{(40-30)^2}{30} = \frac{25}{50} + \frac{25}{40} + \frac{100}{30} = 0.5 + 0.625 + 3.33 = 4.455$$

Recap:

- **Z-Test**: Large samples, known population variance.
- **T-Test**: Small samples, unknown population variance.
  - **One-sample**: Compare sample mean with population mean.
  - **Two-sample**: Compare means of two groups.

- **Paired**: Compare the same group before and after.
- **Chi-Square**: For categorical data, comparing observed with expected frequencies.

Conclusions for the Problem Statements

Problem 1: Ice Cream Sales at Two Different Stores (Two-Sample T-Test)

- **Null Hypothesis (H_0)**: There is no difference in the average ice cream sales between Store A and Store B.
- **Alternative Hypothesis (H_1)**: Store A sells more ice cream than Store B on average.

The calculated **t-value** was 3.29. This is a relatively large t-value, indicating that the difference between Store A and Store B's average sales is statistically significant.

- **Conclusion**: Store A sells significantly more ice cream than Store B, and we can reject the null hypothesis.

Problem 2: Exam Scores Before and After Tutoring (Paired T-Test)

- **Null Hypothesis (H_0)**: Tutoring does not improve student exam scores (no difference before and after tutoring).
- **Alternative Hypothesis (H_1)**: Tutoring improves exam scores (scores after tutoring are higher).

The calculated **t-value** was 13.88, which is a very high t-value, suggesting a large difference between the exam scores before and after tutoring.

- **Conclusion**: The tutoring significantly improved exam scores, and we can confidently reject the null hypothesis.

Problem 3: Customer Satisfaction with Three Products (Chi-Square Test)

- **Null Hypothesis (H_0)**: The actual customer satisfaction ratings are not different from the expected ratings.
- **Alternative Hypothesis (H_1)**: The actual customer satisfaction ratings are different from the expected ratings.

The calculated **chi-square value** was 4.455. Whether this value is significant depends on the critical value from a chi-square distribution table, which is determined by the degrees of freedom (in this case, df = 2, since there are three categories - 1). Suppose the critical value is around 5.99 for a significance level of 0.05. Since 4.455 is less than 5.99:

- **Conclusion**: The actual customer satisfaction ratings are not significantly different from the expected ratings, so we **fail to reject** the null hypothesis. There is no strong evidence to suggest a major difference between observed and expected customer satisfaction for the three products.

Summary of Conclusions:

1. **Ice Cream Sales**: Store A has significantly higher sales than Store B.
2. **Exam Scores**: Tutoring significantly improves student exam scores.
3. **Customer Satisfaction**: There is no significant difference between actual and expected customer satisfaction ratings.

In z test and one sample t test what if you remove the under root n , it will make the complexity easier ?

If you remove the square root of n (i.e., the term $\sqrt{n}$) from the denominator of the Z-Test and One-Sample T-Test formulas, it would indeed make the calculation simpler, but it would also make the test **incorrect** and **inaccurate**. Let me explain why.

Why Do We Use $\sqrt{n}$ in the Formula?

The square root of n in the denominator adjusts for the **sample size** and is crucial for calculating the **standard error.** Here's why it's necessary:

1. **Standard Error (SE):**

   - When calculating the difference between the sample mean and the population mean, we divide by the standard error to understand how much variability we expect from sample to sample.
   - The formula for standard error is:

$$SE = \frac{\sigma}{\sqrt{n}} \quad \text{(for Z-Test, when population standard deviation is known)}$$

$$SE = \frac{s}{\sqrt{n}} \quad \text{(for T-Test, when population standard deviation is unknown)}$$

   - The **larger the sample size** (n), the **smaller the standard error**, which makes the sample mean a more reliable estimate of the population mean.

2. **Impact of Removing $\sqrt{n}$:**

   - If you remove $\sqrt{n}$, you're effectively **ignoring the impact of sample size**.
   - A larger sample size gives you more accurate results because the standard error decreases as n increases. If you don't include $\sqrt{n}$, you're treating the test as if **all sample sizes are the same**, which would be incorrect.

Example Without $\sqrt{n}$:

Z-Test Example (With vs Without $\sqrt{n}$)

Suppose you want to test if the average salary in a company is $50,000. You take a sample of 40 employees, with an average salary of $52,000 and a standard deviation of $5,000.

- **With $\sqrt{n}$:**

$$Z = \frac{52,000 - 50,000}{\frac{5,000}{\sqrt{40}}} = \frac{2,000}{790.57} = 2.53$$

  The Z-score is 2.53.

- **Without $\sqrt{n}$:**

$$Z = \frac{52,000 - 50,000}{5,000} = \frac{2,000}{5,000} = 0.4$$

  The Z-score is now 0.4.

What Does This Mean?

- **With $\sqrt{n}$:** A Z-score of 2.53 means the sample mean is quite far from the population mean, which may suggest that there **is** a significant difference between the sample and population means.
- **Without $\sqrt{n}$:** A Z-score of 0.4 suggests the sample mean is very close to the population mean, which might falsely lead you to conclude that there is **no significant difference**.

Conclusion

- Removing $\sqrt{n}$ makes the calculations **simpler**, but it makes your results **less accurate** because you're ignoring the effect of sample size.
- The formula is designed this way to account for variability and to give results that scale appropriately with the size of your data. Larger sample sizes give more confidence in the results, which is reflected by dividing by $\sqrt{n}$.

In short, keep the $\sqrt{n}$ in the formulas! It's what makes the test reliable.

Range of T-Values and Z-Values & Their Interpretation

T-Values:

The t-value is used in **t-tests**, and it measures the size of the difference relative to the variation in your sample data.

- **Range**: T-values can range from **negative infinity to positive infinity** (-∞ to +∞).
  - **Small T-Value**: A small t-value means the difference between the sample data and the null hypothesis (no difference) is small. It implies that the difference might just be due to random variation.
  - **Large T-Value**: A large t-value means there is a big difference between the sample data and the null hypothesis, indicating the results are likely significant.
- For example:
  - A t-value near **0** means the sample mean is close to the population mean.
  - A **larger absolute t-value** (like 2, 3, or higher) indicates a significant difference.

Z-Values:

Z-values are used in **z-tests**, and they show how many standard deviations away a data point is from the mean of a population.

- **Range**: Z-values also range from **negative infinity to positive infinity** (-∞ to +∞).
  - **Small Z-Value**: A z-value close to **0** indicates the data point is very close to the population mean.
  - **Large Z-Value**: A z-value of **greater than 2 or 3** typically indicates that the data point is far from the mean, suggesting significance.

How to Decide If a T-Value or Z-Value Is Large or Small:

1. **Critical Values**:
   - For **t-tests**, critical values depend on the **degrees of freedom (df)** (which is related to the sample size).
   - For **z-tests**, critical values are fixed because they come from the standard normal distribution.
2. Typically, the critical values are:
   - For **z-tests** at a 95% confidence level, the **critical z-value** is ±1.96. If your z-value is beyond ±1.96, the result is significant.
   - For **t-tests**, the critical value changes based on degrees of freedom but is often around **±2** at a 95% confidence level.
3. **If the t-value or z-value** is greater than the critical value, the result is significant, and we reject the null hypothesis. If it's smaller, we fail to reject the null hypothesis.

Deriving the P-Value

The **p-value** helps us determine if the test statistic (like a t-value or z-value) is statistically significant. It is the probability that the observed result (or something more extreme) would occur if the null hypothesis were true.

Steps to Derive the P-Value:

1. **Calculate the T-Value or Z-Value:** After performing the t-test or z-test, you get a test statistic (like the t-value or z-value).
2. **Look Up the P-Value:** You then look up this value in a **t-distribution table** (for t-tests) or a **z-distribution table** (for z-tests) to find the p-value. Modern statistical software also computes this automatically.
   - For **z-tests**, since the z-distribution is fixed, you can directly look up the p-value for your calculated z-value.
   - For **t-tests**, you need to use the degrees of freedom (df) to find the corresponding p-value.
3. **Compare the P-Value to Your Significance Level (α):**
   - The significance level (α) is usually set at **0.05** (5%).
   - If the p-value is **less than α** ($p < 0.05$), the result is **statistically significant**, and you reject the null hypothesis.
   - If the p-value is **greater than α** ($p > 0.05$), the result is **not statistically significant**, and you fail to reject the null hypothesis.

Example:

Let's say you perform a **two-sample t-test** on two groups of data and get a **t-value of 2.5**. Using a t-distribution table with **degrees of freedom = 18**, you find that the **p-value** corresponding to this t-value is **0.02**.

- If you set your significance level at **0.05**, the p-value (0.02) is **less than 0.05**, so the result is statistically significant.
- This means you can reject the null hypothesis and conclude that the two groups are different.

Real-Life Example of P-Value:

Imagine a new drug is being tested to lower blood pressure. You want to see if the average blood pressure after taking the drug is significantly different from the average before taking the drug.

- **Null Hypothesis (H_0)**: There is no difference in blood pressure after taking the drug.
- After conducting a t-test, you get a **p-value of 0.03**.

- Since **0.03 < 0.05**, the result is significant, and you can conclude that the drug likely affects blood pressure.

Summary:

- **T-values and z-values** help us measure how far our sample is from the population mean.
- **Large t-values or z-values** indicate a significant difference, and small values indicate no significant difference.
- The **p-value** tells us the probability of seeing our data if the null hypothesis were true.
  - If **p < 0.05**, the result is significant, and we reject the null hypothesis.
  - If **p > 0.05**, we fail to reject the null hypothesis.

To determine whether the calculated chi-square value is large or small, we compare it to a critical value from the chi-square distribution table. The critical value depends on two things:

1. **Degrees of Freedom (df):** For a chi-square test, the degrees of freedom are calculated as:
   df=Number of categories−1\text{df} = \text{Number of categories} - 1df=Number of categories−1
   In the pizza example, there are 3 categories (cheese, pepperoni, veggie), so:
   df=3−1=2\text{df} = 3 - 1 = 2df=3−1=2
2. **Significance Level (α):** This is a threshold for how sure we want to be about the result. A common significance level is 0.05, which means we're okay with a 5% chance of making a mistake.

Step-by-Step for the Pizza Example:

1. Calculate the chi-square value:

You already did this part in the example. The formula for chi-square is:

χ2=∑(O−E)2E\chi^2 = \sum \frac{(O - E)^2}{E}χ2=∑E(O−E)2

Where:

- OOO = observed frequency (the actual survey results)
- EEE = expected frequency (what we expect if preferences are equal)

For each pizza topping:

- Cheese: (25−20)220=2520=1.25\frac{(25 - 20)^2}{20} = \frac{25}{20} = 1.2520(25−20)2=2025=1.25

- Pepperoni: (20−20)220=0\frac{(20 - 20)^2}{20} = 020(20−20)2=0
- Veggie: (15−20)220=2520=1.25\frac{(15 - 20)^2}{20} = \frac{25}{20} = 1.2520(15−20)2=2025=1.25

So, the total chi-square value is:

χ2=1.25+0+1.25=2.5\chi^2 = 1.25 + 0 + 1.25 = 2.5χ2=1.25+0+1.25=2.5

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2. Determine the critical value from the chi-square table:

Now, we check the chi-square distribution table to find the critical value for 2 degrees of freedom and a significance level of 0.05.

- For **df = 2** and **α = 0.05**, the critical value from the chi-square table is **5.99**.

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3. Compare the calculated chi-square to the critical value:

- If the calculated chi-square value is **greater** than the critical value (5.99), we say the result is **significant** (meaning the observed preferences are significantly different from what we expected).
- If the calculated chi-square value is **less** than the critical value (5.99), we say the result is **not significant** (meaning the observed preferences are not significantly different from the expected preferences).

In this example:

χ2=2.5\chi^2 = 2.5χ2=2.5

Since 2.5 is **less** than 5.99, we conclude that the chi-square value is **small**.

Conclusion:

- **Small Chi-Square Value (like 2.5 in this case):** The observed data (preferences for pizza) is close to what we expected (equal preferences). This means there's **no strong evidence** that people prefer one topping over another.
- **Large Chi-Square Value (greater than 5.99 in this case):** The observed data would be far from what we expected, indicating that there is a **significant difference** in preferences.

In this case, since the value is small, we **fail to reject the null hypothesis** and conclude that people's pizza topping preferences are **not significantly different** from what we expected (equal preferences for all toppings).

MORE PRACTICE QUESTIONS :

1. Z-Test Example

A factory claims that the average weight of a product is 50 kg. You take a sample of 40 products and find the average weight is 52 kg, with a population standard deviation of 4 kg. Is the sample weight significantly different?

Given:

- Sample mean $\bar{X} = 52$
- Population mean $\mu = 50$
- Population standard deviation $\sigma = 4$
- Sample size $n = 40$

Z-Test Calculation:

$$Z = \frac{52 - 50}{\frac{4}{\sqrt{40}}} = \frac{2}{0.632} \approx 3.16$$

Since 3.16 is a large value, we **reject** the null hypothesis that the average weight is 50 kg.

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2. One-Sample T-Test Example

A bakery claims its average cupcake weighs 120 grams. You take a sample of 15 cupcakes, and the average weight is 125 grams, with a sample standard deviation of 10 grams. Is the sample weight significantly different?

Given:

- Sample mean $\bar{X} = 125$
- Population mean $\mu = 120$
- Sample standard deviation $s = 10$
- Sample size $n = 15$

T-Test Calculation:

$$t = \frac{125 - 120}{\frac{10}{\sqrt{15}}} = \frac{5}{2.58} \approx 1.94$$

Since 1.94 is a small value, we **fail to reject** the null hypothesis that the average weight is 120 grams.

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3. Two-Sample T-Test Example

You want to compare the average test scores of two classes. Class A has an average score of 85 with a standard deviation of 8 (sample size = 30), and Class B has an average score of 80 with a standard deviation of 6 (sample size = 35). Is there a significant difference between the two classes?

Given:

- Class A: $\bar{X}_A = 85$, $s_A = 8$, $n_A = 30$
- Class B: $\bar{X}_B = 80$, $s_B = 6$, $n_B = 35$

T-Test Calculation:

$$t = \frac{85 - 80}{\sqrt{\frac{8^2}{30} + \frac{6^2}{35}}} = \frac{5}{\sqrt{2.13 + 1.03}} = \frac{5}{\sqrt{3.16}} \approx 2.82$$

Since 2.82 is a large value, we **reject** the null hypothesis that the test scores are the same.

4. Paired T-Test Example

You want to test if a new teaching method improves student scores. You measure the scores of 10 students before and after using the method. The average score improvement is 5 points with a standard deviation of 3 points. Is the improvement significant?

Given:

- Average difference $\bar{D} = 5$
- Standard deviation of differences $s_D = 3$
- Sample size $n = 10$

T-Test Calculation:

$$t = \frac{5}{\frac{3}{\sqrt{10}}} = \frac{5}{0.95} \approx 5.26$$

Since 5.26 is a large value, we **reject** the null hypothesis that the teaching method has no effect.

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5. Chi-Square Test Example

You survey 50 people and ask if they prefer coffee or tea. The results are:

- 30 prefer coffee
- 20 prefer tea But you expect an equal preference (25 for each). Is the observed preference significantly different from what was expected?

Observed (O): Coffee = 30, Tea = 20

Expected (E): Coffee = 25, Tea = 25

Chi-Square Calculation:

$$\chi^2 = \frac{(30-25)^2}{25} + \frac{(20-25)^2}{25} = \frac{25}{25} + \frac{25}{25} = 1 + 1 = 2$$

Since 2 is a small value, we **fail to reject** the null hypothesis that the preferences are equally distributed.

Since 2 is a small value, we **fail to reject** the null hypothesis that the preferences are equally distributed.

MORE PRACTICE QUESTIONS :

1. Z-Test Example

A shoe company claims the average size of their shoes is 9. You take a sample of 50 shoes and find the average size is 8.5, with a population standard deviation of 0.6. Is the sample significantly different from the claimed average?

Given:

- Sample mean $\bar{X} = 8.5$
- Population mean $\mu = 9$
- Population standard deviation $\sigma = 0.6$
- Sample size $n = 50$

Z-Test Calculation:

$$Z = \frac{8.5 - 9}{\frac{0.6}{\sqrt{50}}} = \frac{-0.5}{0.08485} \approx -5.89$$

Since -5.89 is a large negative value, we **reject** the null hypothesis that the average shoe size is 9.

2. One-Sample T-Test Example

A company claims their employees work 40 hours a week on average. You take a sample of 20 employees and find the average is 38 hours, with a sample standard deviation of 3 hours. Is the average working time significantly different?

Given:

- Sample mean $\bar{X} = 38$
- Population mean $\mu = 40$
- Sample standard deviation $s = 3$
- Sample size $n = 20$

T-Test Calculation:

$$t = \frac{38 - 40}{\frac{3}{\sqrt{20}}} = \frac{-2}{0.67082} \approx -2.98$$

Since -2.98 is a large negative value, we **reject** the null hypothesis that the average working time is 40 hours.

3. Two-Sample T-Test Example

You want to see if two brands of cereal have different average sugar content. Brand A has an average of 10 grams of sugar with a standard deviation of 2 grams (sample size = 25), and Brand B has an average of 12 grams of sugar with a standard deviation of 3 grams (sample size = 30). Is there a significant difference in sugar content?

Given:

- Brand A: $\bar{X}_A = 10$, $s_A = 2$, $n_A = 25$
- Brand B: $\bar{X}_B = 12$, $s_B = 3$, $n_B = 30$

T-Test Calculation:

$$t = \frac{10 - 12}{\sqrt{\frac{2^2}{25} + \frac{3^2}{30}}} = \frac{-2}{\sqrt{0.16 + 0.30}} = \frac{-2}{0.67268} \approx -2.97$$

Since -2.97 is a large negative value, we **reject** the null hypothesis that the sugar content is the same in both brands.

4. Paired T-Test Example

You want to test if a new workout plan improves fitness scores. You record the fitness scores of 8 people before and after the workout plan. The average improvement in their scores is 7 points, with a standard deviation of 2 points. Is the improvement significant?

Given:

- Average difference $\bar{D} = 7$
- Standard deviation of differences $s_D = 2$
- Sample size $n = 8$

T-Test Calculation:

$$t = \frac{7}{\frac{2}{\sqrt{8}}} = \frac{7}{0.707} \approx 9.9$$

Since 9.9 is a large value, we **reject** the null hypothesis that the workout plan has no effect on fitness scores.

5. Chi-Square Test Example

You conduct a survey to see if there is a preference between three types of pizza toppings (cheese, pepperoni, and veggie) among 60 people. The results are:

- 25 prefer cheese
- 20 prefer pepperoni
- 15 prefer veggie You expect equal preferences (20 for each). Is the observed preference significantly different?

Observed (O): Cheese = 25, Pepperoni = 20, Veggie = 15
Expected (E): Cheese = 20, Pepperoni = 20, Veggie = 20

Chi-Square Calculation:

$$\chi^2 = \frac{(25-20)^2}{20} + \frac{(20-20)^2}{20} + \frac{(15-20)^2}{20} = \frac{25}{20} + \frac{0}{20} + \frac{25}{20} = 1.25 + 0 + 1.25 = 2.5$$

Since 2.5 is a small value, we **fail to reject** the null hypothesis that the preferences are equally distributed.