The Complexity of Go

Domenic Mancuso

Introduction

Go is an ancient Chinese strategy game that is traditionally played on a 19x19 line grid. Players take turns placing one piece on the board per turn. The player chosen to make the first move uses the black pieces. The other player uses white pieces. The pieces used look like MMs. The task for either player is to capture as much territory as possible. Each piece has "liberties" - space that is able to be captured if the four orthogonal directions around a piece are surrounded by the opposing team's pieces. Either team is able to form walls or a larger mass using the eight directions adjacent to them. Once a mass has been formed, it must be completely surrounded by the opposing team's mass to be captured. Here are some examples of about to be or already captured objectives.

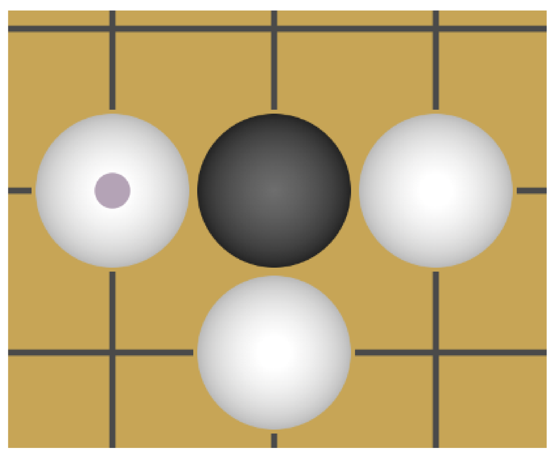

Figure 1: This black piece is at risk of immediate capture from UP/above.

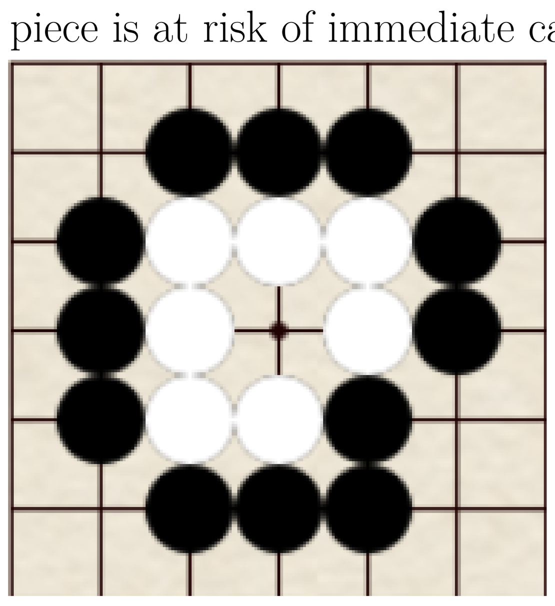

Figure 2: The white pieces are forming an 'eye'. This is vulnerable to being surrounded and captured once black fills the inner eye.

Go's most powerful formation

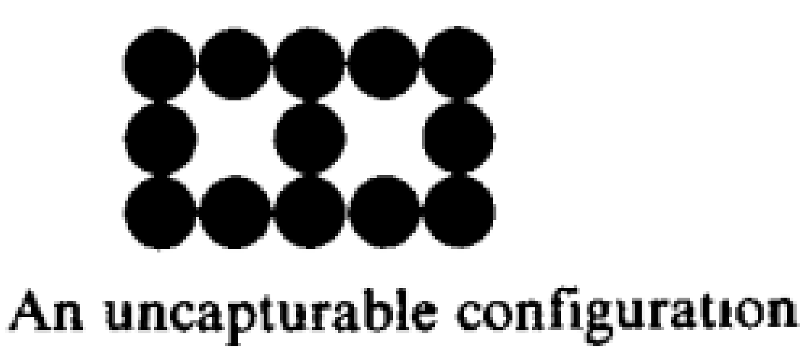

Figure 3: The double eye is important for explaining our reduction near the end of this presentation. This group cannot be captured because both eyes cannot be captured at once. If a white piece is placed in either eye, it will be captured instead.

Complexity Class of Go

Go is classified in PSPACE, which means that it is of Polynomial Space difficulty. Specifically, this means that the amount of computer memory required to derive the outcome of this game is expressed as a polynomial function which depends on the "input" size, which is the size of the game board. Since Go is not bound to a 19x19 planar, the polynomial function can vary depending on the size of the game board you choose.

Is the Complexity X 'HARD', or 'COMPLETE'?

Per the paper published in 1978 that this poster is based on, Go is classified as at least an NP-HARD problem. This means that Go is at least as computationally complex to solve as any other problem of Polynomial Space complexity.

What Game, Mode, or Puzzle is is Go reduced from?

Go is reduced from a sequence of two combinatorial games. First of, which is the True Quantified Boolean Forumula, also known as TQBF. The formula utilizes a quantifier for each player's turn, which is then evaluated using another Boolean function F. F evaluates the trueness of each combined term, which returns a Boolean value True or False. In regard to the reduction of Go, if F returns True, this signifies that the first player is in a winning position or has won, and False signifies the opposite, that the opposing player is winning or has won, considering the history of any given Go game. Here is the format for the TQBF formula. -

$$TQBF = (Q_1 v_1 Q_2 v_2 ... Q_n v_n) F(v_l, v_2 ..... v_n)$$

The reduction of Go into the TQBF can be further reduced to the Geography game. Geography simplifies this reduction because a TQBF function mimics Geography and it can also be displayed on a planar grid like Go. In Go, Geography, and TQBF, as long as both players are playing perfectly, the game continues at a stalemate. Just like in Geography, one wrong move (a player is unable to name a town or city,) from a Go or TQBF player can result in a loss if the opposing player plays perfectly. Figure 3 displays an example of TQBF and how it relates to our reduction.

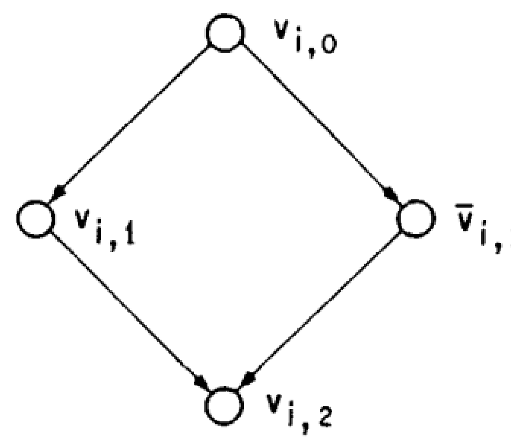

Figure 4: The planar demonstration of TQBF, which can hold in a game of Go and Geography. This is because vi,0, (i represents Boolean True or False, and v will be written as vn from here on,) which in Go would be black's first move, is inevitable - as is the first town or city that is named in a Geography game. v1 is evaluated as True or False depending on whose side the player's move favors - whose outcome then affects v2, and so on.

Gadget Types Required

The WIRE and SPLIT gadget are the only gadgets applicable to analyzing and reducing Go. Figures 5 and 6 will help us understand how WIRE and SPLIT work in the context of our reduction. Assume that players making a mutually perfect set of decisions results in a straight wire. Critical sections and general decisions made during the game will result in a crossroad - a SPLIT, which is equivalent to a TRUE or FALSE decision in a TQBF game. Split also pertains to Geography because the name of the town you choose will affect the next player's decisions.

Pipes are a perfect example of WIRE, and will be demonstrated in the following figures.

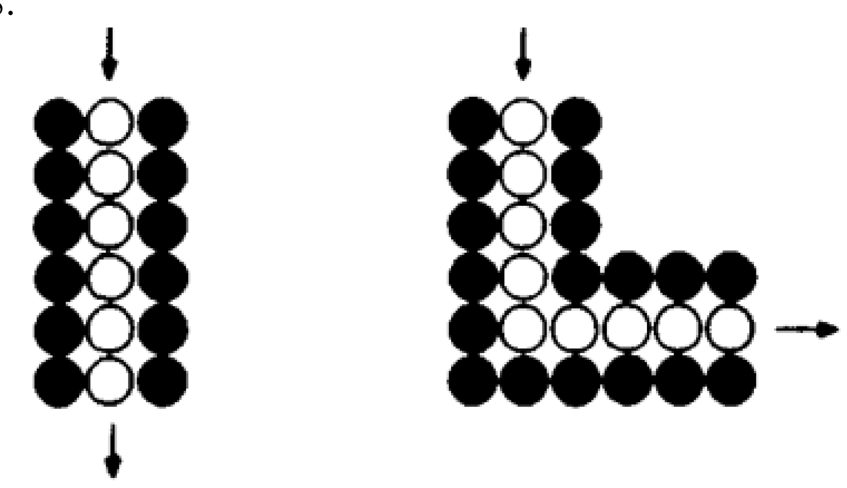

Figure 5: Two examples of pipes. Notice that players will enter and exit according to the direction of the included arrows.

Before viewing the following figure, please understand that symmetry applies to this display. As this next Go configuration is symmetric, the propositions explained can be mirrored in play if enacted in symmetry.

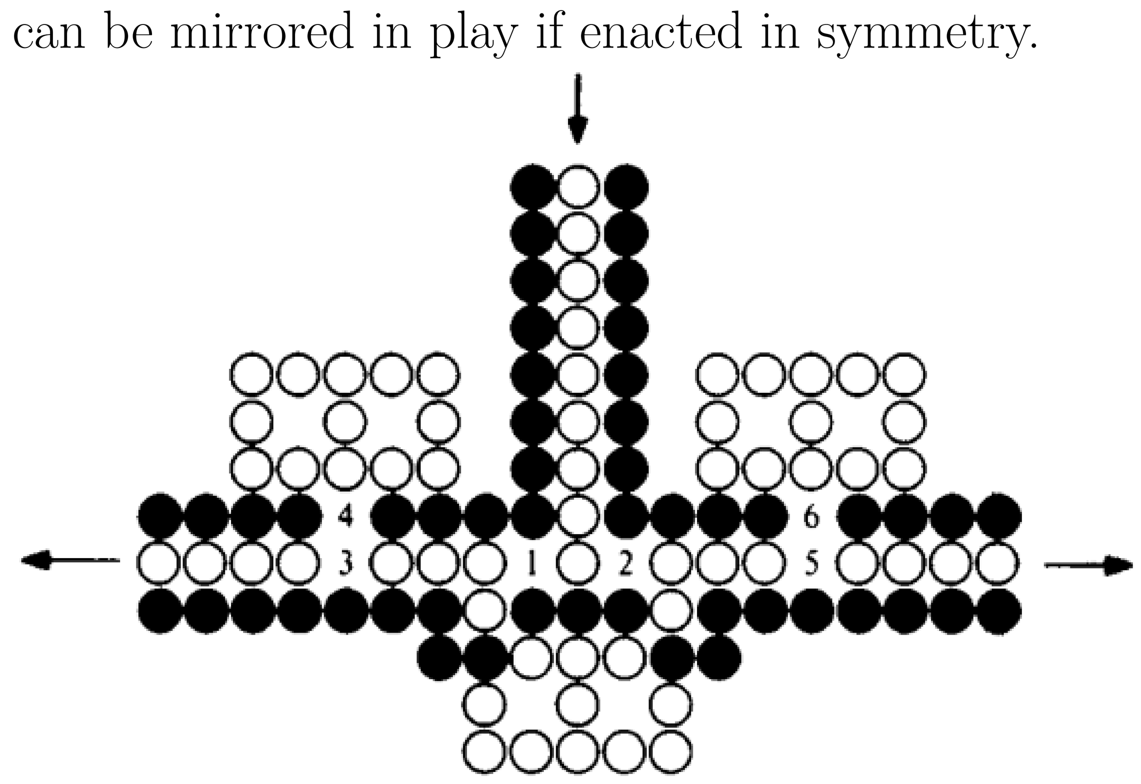

Figure 6: If both players play perfectly and White goes first and chooses to travel down the left-hand pipe, the players will choose White-l; Black-2; White-3; Black-4.

If both players follow the 'perfect' steps in the previous figure, the perfect continuation of this game is for White to travel down the left hand pipe and Black to follow, which mimics a game of Geography, (WIRE) per our reduction.

References

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COSUMI, Go vs CPU
Special thanks to the local games store for selling me a copy of Go. It is surprisingly difficult to find.