# PARALLEL ROLES FOR NONFORMAL REASONING IN EXPERT SCIENTIFIC MODEL CONSTRUCTION AND CLASSROOM DISCUSSIONS IN SCIENCE<sup>1</sup>

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#### **Abstract**

We describe model construction processes in scientifically trained experts in Part I and identify similarities to important learning and teaching processes in science class discussions in Part II. Our work with science experts has analyzed data from videotaped protocols of experts thinking aloud about unfamiliar explanation problems. These studies document the value of nonformal heuristic reasoning processes such as analogies, identification of a new variable, Gedanken experiments, and the construction and running of visualizable explanatory models. At a larger time scale, some subjects went through model evolution cycles of model generation, evaluation, and modification that utilized the heuristic reasoning processes above. In addition, the prevalence of imagistic simulation as an underlying foundation in these episodes suggests that it may be important to pay greater attention to this process in the analysis of nonformal thinking than is commonly done. To our knowledge these three levels of processes have not been emphasized in the past. They complement empirical processes of discovery, experimentation, and evaluative argumentation documented by others and provide a perspective on the nature of scientific thinking that includes the idea that model formation can involve creativity through non-empirical processes such as analogy, "running" a mental model, and Gedanken experiments. Diagrams of how the above processes interact may give us some new ways to picture the roles of nonformal reasoning and learning processes during qualitative model construction. These can be contrasted with more procedural and traditional reasoning processes of formal deduction and induction by enumeration or statistical inference. The nonformal learning and reasoning processes discussed here may be less procedural and carry less certainty than those traditional forms of reasoning, but they can be powerful engines for discovery if used within a self-correcting cycle of evaluation and modification.

These are compared to reasoning/learning processes that exemplary teachers foster in whole class discussions in physical science in Part II and some important similarities are noted. Their classes also went through cycles involved in model generation, evaluation, and modification at a macro-time-scale level. At a micro level, similar heuristic reasoning strategies as seen in the experts were used. This description at two hierarchical levels of processes helps to organize and clarify the purpose of specific, cognitively targeted teaching strategies. Techniques for diagramming discussions help to illuminate these processes as well as the role of teacher scaffolding and co-construction of the models being learned. Our rationale for the study is that comparisons to expert reasoning can sharpen our ways of describing the reasoning of students and teachers during discussions, and help in describing important teaching strategies.

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### INTRODUCTION

This study focuses on several questions related to the nature of scientific thinking: What processes are used by experts during creative model construction? Are nonformal as well as formal thought processes used? Are these learning processes similar to those fostered by exemplary teachers in conceptual classroom discussions? Studies in history of science have paved the way for a model based learning approach to understanding science, e.g., Giere (1988) and Nersessian (2008). The first part of the present study attempts to complement their approach by analyzing data from video-taped protocols of experts. Nonformal expert reasoning processes are identified at several levels in Part I. These are compared to learning processes that exemplary teachers attempt to foster in whole class discussions in physical science in Part II and some important similarities are noted.

Work in science education also contributes to a model based theoretical framework, such as Glynn & Duit (1995), Duschl & Osborne (2002), Gilbert (2004), Quintana, Reiser, Davis, Krajcik, et al (2004), Clement (2008), Osborne, Erduran, & Simon (2004) and others. In this framework, thinking about processes of model construction helps to organize and clarify the purpose of narrower, more specifically targeted reasoning strategies. Our rationale for the study is the hope that comparisons to expert reasoning can sharpen our ways of describing the reasoning of students and teachers during discussions, and help in describing important teaching strategies.

### **Overall Method**

The overall method consists of:

- (1) an exploratory qualitative case study of expert reasoning to identify major nonformal reasoning components at different levels.
- (2) an exploratory qualitative case study of teacher student interactions exhibiting some of the same reasoning components and levels.

The resulting study is qualitative, generative and descriptive and is not intended to project frequencies of strategies to a population. It is intended to help us define new constructs to look for in model construction behavior of both experts and classrooms that have their initial grounding in video tape case study data; and the episodes analyzed also provide initial existence demonstrations for key phenomena.

# PART I: ROLES FOR NONFORMAL REASONING IN EXPERT SCIENTIFIC MODEL CONSTRUCTION

### **Background**

Previous studies have examined elements of the problem of how processes like analogy, imagery, and model construction can be used in science. Nersessian (1992), Trickett and Trafton (2002), Dunbar (1999), and Clement (1989, 1994) have described processes by which experts utilize analogies to construct models for conceptually difficult problems. Nersessian (2008) has suggested that mental simulation may be a central process scientists use to construct and run scientific models. Finke (1990) has shown how lay subjects can combine images in novel ways to produce new images with new interpretations. Barsalou (1999) has described a theory of perceptual symbols which represent schematic elements of perceptual experience and that can be integrated to produce simulations. But too little research exists on the collective relationships between model construction, heuristic reasoning, and the use of imagery or mental simulation. This first study explores that domain by analyzing transcript episodes of experts as they use nonformal reasoning processes.

### Method for Part I

Eleven experts were asked to think aloud while working on unfamiliar explanation problems. Experts were professors or doctoral students who had passed comprehensives in technical fields. The interviewer used only minimal probing for clarification. Protocol analysis was conducted via a constant comparison method (Strauss and Corbin, 1998), used here to develop new observational and theoretical constructs for describing reasoning and learning processes. Extended individual case studies of problem solutions/explanations examined how several reasoning processes can be combined together to support each other.

I will present examples of each process from transcripts to provide initial documentation of their use. Subjects were recorded while thinking aloud about the following problem, illustrated in Figure 1; I will call this the target problem or target case.

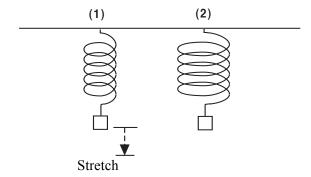


Figure 1: Spring Problem: A weight is hung on a spring. The original spring is replaced with a spring made of the same kind of wire, with the same number of coils, but with coils that are twice as wide in diameter. Will the spring stretch from its natural length more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible.) Why do you think so?

## Overview of Findings: Nonformal Reasoning Strategies Used

Although observation and experiment are extremely important in science, this expert study focused on the less studied rationalistic side of scientific thinking, and materials for experimental observation were excluded from the interviews. Nevertheless subjects used many rationalistic reasoning strategies to generate predictions and explanations successfully, including Gedanken (thought) experiments, which are extremely interesting because they *feel* empirical to the scientist. The case studies of solutions document the presence and import of *nonformal heuristic reasoning processes* such as *analogies*, *concept identification or differentiation*, *extreme cases*, *the constructing and running of visualizable explanatory models*, and *Gedanken experiments*. The transcripts also provide existence demonstrations of many examples of *imagistic simulation* occurring in conjunction with the heuristic reasoning processes above (Clement, 2009). Imagistic simulations were evidenced by subjects making a prediction about a system's behavior accompanied by one or more imagery indicators, such as spontaneous imagery reports and/or depictive gestures.

# **Case Study**

### Using an Analogous Case: Long and Short Bending Rods

In this paper due to space limitations I will focus on the solution of a single subject S2. Although the subjects were experts in technical fields, none were mechanical engineers, and they were working at the frontier of their own personal knowledge on an unfamiliar problem. From this it is plausible that their methods have some overlap with those used on the frontier of science. For the spring problem, S2 first generated an **analogous case** in which he predicted that a long horizontal rod fixed at one end would bend more than a short one (with the same weight attached to the other end of each rod), inferring that segments of the wider spring would bend more and therefore stretch more (It is true that the wide spring stretches more.) He says:

(1) "I have one good idea to start with; it occurs to me that a spring is nothing but a rod wound up uh, and therefore maybe I could answer the question for a rod."

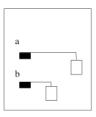


Figure 2: Bending Rods Analogy

and later:

(2) "I have a strong intuition--<u>a physical imagistic intuition</u>--that this [points to longer rod] will bend a lot more than that [shorter rod] will."

A subject working on a target problem uses an analogy when they generate or recall a case that is significantly different from the target case, but that also may have structural similarities to the target, so that findings from it maybe applicable to the target case. In this instance, the analogous case of the bending rods is anchored in a physical intuition that appears to involve imagery. I will use underscored type to identify observations that provide some evidence for imagery (both kinesthetic and visual) use, such as the <u>spontaneous imagery report</u> in excerpt 2 above. Here the analogy gives S2 the correct prediction for the spring, but he still has doubts about his understanding of the system.

# Running a Model and Checking it against Known Constraints: Bending Seen As Inconsistent

Once S2 began to take seriously the idea that bending could actually be occurring in the spring wire, we say that he begins to use bending as an *explanatory model* for understanding how the spring is stretching, not just a playful, expedient analogy for getting a prediction. An explanatory model is a (usually hidden) mechanism that explains why the system behaves the way it does, by explicating the structure or dynamics of the system. However, S2 quickly became concerned about the appropriateness

of bending as an explanatory model because of the apparent lack of a match between bending producing an increasing slope in the rod on the one hand, and a lack of increasing slope in the wire in a stretched spring on the other. One can visualize this discrepancy here by thinking of the increasing slope a bug would experience walking down a bending rod and the constant slope the bug would experience walking down the helix of a stretched spring. (This is my own descriptive analogy for purposes of clarity- not the subject's.) (Another way for the reader to see this problem is to note that the bending model predicts that the slope of the wire and the distance between coils will increase as one goes down the spring, as shown in Figure 3. Yet this does not happen in real springs). The full transcript is quite long; therefore verbatim excerpts are presented here. (Brackets in transcript indicate my comments.) This discrepancy led him to question whether the bending rod was an adequate explanatory model for the spring.

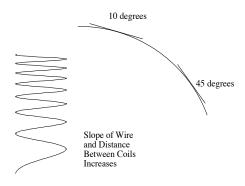


Figure 3: asymmetric spring

(3)"But then it occurs to me that there's something clearly wrong with that [bending rod] metaphor, because ..it would (raises hands together in front of face) droop (moves r. hand to the right in a downward curve) like that, its slope (retraces curved path in air with l. hand) would steadily increase, whereas in a spring, the slope of the spiral is constant...

#### Later he says:

..... "You get a spring which stretches more and more at the bottom. The loops are wider apart there. But that isn't the case...they're uniform all the way around."

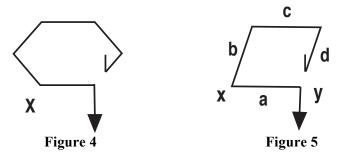
This appears to be a case where he imagines dynamically or "runs" the idea of bending taking place in the spring as it stretches, as shown in Fig. 3 above. That is, he examines the consequences of **running a model**—the "bending model"—in consecutive segments of the spring. In examining whether the bending idea agrees with known constraints about springs, he decides that there is a conflict with the property that an ideal (massless) spring stretches uniformly. We say that a subject runs an explanatory model when they use imagistic simulation to animate the model and make a prediction for an outcome of the model. Evidence of this would be relevant imagery indicators such as imagery reports or depictive gestures occurring near a prediction that comes from an expressed explanatory model. Examples of such gestures are underlined in episode 3. (There is not space for a review here, but an increasing variety of studies of depictive gestures suggest that they are expressions of core meanings or reasoning strategies and not simply translations of speech. Others indicate that the same brain areas are active during real actions and corresponding imagined actions.)

This anomaly or mismatch appears to bother him considerably and drives further work on the problem. Certainly an important positive feature of the above section is the subject's ability to criticize his own

initial model. Several other subjects who thought of the bending rod model did not make this interesting criticism of it.

# Torsion Insight: Identifying a New Variable

After a (half hour) period of frustration in trying to make the bending model work, this subject finally produces an extremely productive **analogy** when he generates the idea of the hexagonally shaped coil in Figure 4 and moves from there to the idea of the square shaped coil in Figure 5.



(4) "Aha! Now this is interesting. I imagined; ...the square is sort of like a circle and I wonder....what if I start with a rod and bend it once (places hands at each end of rod in Figure 2 and motions as if bending a wire) and then I bend it again.

What if I produce a series of successive approximations to... the circle by producing a series of polygons! Maybe that would clarify because maybe that, that's constructing a continuous bridge, or sort of a continuous bridge, between the two cases [the rod and the coil]. Clearly there can't be a hell of a lot of difference between the circle and say, a hexagon..."

These analogies lead him to a major breakthrough in the solution, which corresponds to the way engineering specialists view springs, as follows:

(5) "Now that's interesting. Just looking at this [hexagon] it occurs to me that when force is applied here, you not only get a bend on this segment, but because there's a pivot here (points to x in Figure 4), you get a torsion effect...

Aha! Maybe the behavior of the spring has something to do with twist (<u>moves hands as if twisting an object</u>) forces as well as bend forces (<u>moves hands as if bending an object</u>). That's a real interesting idea. That might be the key difference between this [bending rod] which involves no torsion forces, and this [hexagon]. Let me accentuate the torsion force by making a square where there's a right angle.

(6) Now [in Figure 5]...I have two forces introducing a stretch. I have the force that bends this...segment [a] and in addition I have a torsion force which twists [segment b] at vertex, um, X... (makes motion like turning a door knob with one hand)"

Here he appears to imagine the situation in Figure 5 as if side 'a' were a wrench acting at x to twist the end of side 'b' through an angle, while 'c' keeps the other end of 'b' from turning, resulting in a twisting deformation of the metal in 'b'. That is, pulling down at 'y' twists the metal throughout side 'b' like twisting a piece of toffee; except that unlike toffee, side 'b' is made of resilient metal so that it would spring back and untwist if one were to remove the downward force at point 'y'. (The same would be true for all other adjacent rod pairs.)

Twisting of the wire and the resulting torsional strain is in fact the most important source of stretching and restoring force, in the analysis of spring behavior as understood by engineers. Its discovery here represents a scientific insight in **identifying a new variable** and causal mechanism for stretching. (See the appendix for a primer on the concepts of torsion and toque.) Later the subject draws Figure 6 to explain how a downward force F would produce torsion and twisting in segment 'w'.

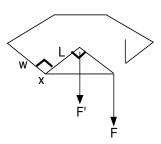


Figure 6: Torsion in w produced by torque from an adjacent segment in hexagonal coil

## Using an Imagistic Simulation to Examine an Analogous Case

The subject still needs to determine what torsion in the square spring would predict for the answer to the original problem. The simpler analogous case he uses to consider this hypothesis is to compare a long and a short rod, each of which is twisted with the same torque (twisting force).

- (7) "Now making the sides longer certainly would make the [square] spring stretch more....
  the longer the segment (moves hands apart) the more the bendability (moves hands as if bending an object)...
- (8) Now the same thing would happen to the torsion I think, because if I have a longer rod (moves hands apart), and I put a twist on it (moves hands as if twisting something in Figure 7), it seems to meagain physical intuition--that it will twist more, hush (looks to side and pauses 4 sec.) I'm- I think I trust that intuition... I'm (raises hands in same position as before and holds them there continuously) imagining holding something that has a certain twistyness to it, a-and twisting it...."





**Figure 7: Evidence for imagistic simulation:** Expert making spontaneous depictive gestures as he makes a prediction. "If I have a longer rod (moves hands apart), and I put a twist on it (<u>moves hands as if twisting a rod</u>), it seems to me--again, physical intuition--that it will twist more."

This is an example of making a prediction from an **imagistic simulation**, evidenced by the subject making a prediction about a system's behavior accompanied by one or more imagery indicators, such as

spontaneous imagery reports and/or depictive gestures. We have this kind of evidence within each major kind of heuristic reasoning in episodes 2, 3, 5, 7, and 8 above. The twisting rod episode just above illustrates how the nonformal thinking processes we are identifying can be *nested*. The first step was to generate a productive *analogy* in the form of the square coil. The second step was to *partition* the square coil by considering the effects on a single side of the coil. The third step was to use *imagistic simulation* within that piece of the analogous square coil case to make a prediction for it.

(Note: S2 also is encouraged by seeing that a spring made of square coils will stretch with an equal distance between the coils, unlike the false situation he imagined in Figure 3, a spring with increasing slope and an increasing distance between the coils toward the bottom. That is, when he "runs" the square coil being stretched, there may be bending in each side, but because bending and slope "start over from zero" at each corner, the slope from the bends does not accumulate by adding. The same would be true for torsion effects. The square coil is a new case in which the increasing slope difficulty does not occur, suggesting it is a way to resolve his previous anomaly.)

# Summary of Nonformal Heuristic Reasoning Processes Identified so Far

The above episodes include examples of the following heuristic reasoning processes: **Using an Analogy** (e.g. the square coil), **Running a Model** (e.g. the bending model is run within the spring in transcript episode 3), and **Identifying a New Variable** in episode 5. There is also evidence within these cases for the additional involvement of an **Imagistic Simulation** process as a subprocess operating within the other three processes above.

Overall pattern of model evolution. These processes can be seen as contributing to an overall pattern of model evolution as shown in Figure 8, where time runs from left to right. So far we have evidence for only the first two models shown in the second row from the top, starting with an initial model where the spring wire is thought of as bending during stretching. This model is eventually modified to form a second model where the spring wire is both bending and twisting. Above that, the first row shows analogous cases generated by the subject that contributed to the models by suggesting elements of the mechanism in the spring. The third row shows results from real or thought experiments that support or conflict with the model as it evolves (both of them in this case study are thought experiments).

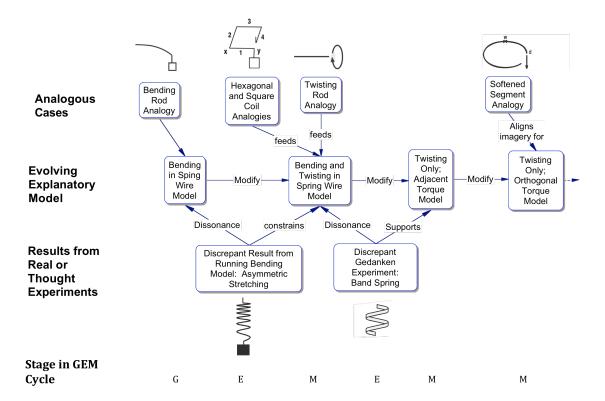


Figure 8: Model evolution sequence for S2 involving a GEM cycle, influenced by analogies and experiments

Overall, we can see the beginning of a pattern of a **Model Generation**, **Evaluation**, **and Modification** cycle (or GEM cycle). The initial bending model was generated, then evaluated negatively, then modified to include twisting. These steps are labeled G, E, M in the bottom row, and so far we have only seen evidence for those first three steps. I will account for the other pieces of the diagram in a later section.

# **Imagistic Simulation**

How can one explain S2's ability to make predictions from cases? The imagery related observations underlined above that appear near a prediction can be explained via what Clement (1994) called **imagistic simulations** wherein: (1) the subject has activated a somewhat general and permanent (perceptual motor) action schema that can control the action of twisting real objects; (2) the schema assimilates an image of two rods of different lengths that is more specific and temporary; (3) the action schema "runs through its program" vicariously without touching real objects, generating a simulation of twisting the two rods, and the subject compares the effort required for each. Such a simulation may draw out implicit knowledge in the schema that the subject has not attended to before and thereby produce newly heeded expectations about behavior in a subsequent dynamic image, or simulation (Clement, 1994; 2003). It is related to what Schmidt (1982) calls motor schemas, Hegarty (2002) calls mental animation, Schwartz and Heiser (2006) call mental simulation, Trickett and Trafton (2002) call movies in the mind, Nersessian (2008) calls simulative reasoning, and Barsalou (1999) calls simulation.

Imagistic simulation is a process that may help us to explain the enigmatic ability of scientists to perform thought experiments, defined here in the *broad sense*, as the act of generating or considering an untested, concrete, system (the "experiment") and attempting to predict its behavior (Clement, 2002). Aspects of the experiment must be new and untested in the sense that the subject is not informed about their behavior

from direct observation or from an authority. 'Concrete situation' here means a situation potentially perceivable via the senses or via instruments; i.e. the experiment is one that would yield empirical observations if it could actually be performed. Each of the cases pictured in Figure 8 can be considered to involve a untested thought experiment in this broad sense of the term (with the possible exception of the bending rods--because he may have witnessed this effect in real life). A narrower related term is defined in the next section.

# **Evaluative Gedanken Experiment: The Band spring**

A closely related concept of 'thought experiment' in a *narrower sense* is what I call an **evaluative Gedanken experiment** (Clement, 2002, 2008, 2009). There is no consensus on a precise definition for this term but I use it here to mean a thought experiment especially designed or selected by the subject to help *evaluate* a concept, model or theory. An example is S2's case of a spring made of a vertically oriented band of material shown in Figure 9. (the reader might imagine the thin metal strip unwound from a coffee can, reshaped to make a spring, say, 3" wide.) This invented case allows him to test whether bending is as necessary as twisting as the primary mechanism at work in a spring.



Figure 9: Gedanken experiment: A band spring that can twist but not bend

(9) "How about a spring made of something that can't bend. And if you showed that it still behaved like a spring you would be showing that the bend isn't the most important part. Or isn't particularly relevant at all maybe somehow...How could I imagine such a structure?... I'm thinking of something that's made of a band... we're trying to imagine configurations that wouldn't bend. Since it's cross section is like that (see Figure 9) ... it can't bend in the up-down (indicates up/down directions with hands) direction like that because it's too tall. But it can easily twist (motions as if twisting an object)."

Given the imagery report here, I interpret this to mean that the subject imagined that such a spring would still be quite stretchable even though the band "cannot bend in the up-down direction," challenging the necessity of bending as not "particularly relevant at all" to stretching. In this type of evaluatory Gedanken experiment he designs a special case where the bending model yields a prediction, (predicts no stretch) but where he also has some other independent source of information that can evaluate that prediction (physical intuition predicts that it will stretch). This is an evaluative Gedanken experiment because it is designed by him to help him test a model.

At this point S2 appears to be shifting from a model of the spring wire both bending and twisting to a model where it is undergoing twisting alone. This is shown in Figure 8 as the third model in the evolution sequence in the second row from the top. This is the result of an additional part of the GEM cycle of evaluation and modification in the figure, as shown in the bottom row. The general form of a GEM cycle is shown in Figure 10.

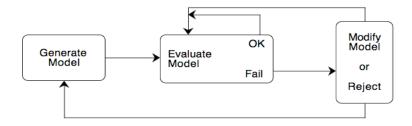


Figure 10: GEM cycle of model generation, evaluation, and modification

# A Refined Analogy for Alignment: Twist in Quarter Coil

Although S2 has made a huge breakthrough in identifying torsion and torque as a new causal variable in the mechanism of a spring, the torque arm 'L' acting on an adjacent segments in Figure 6 drawn by the subject is actually still misaligned geometrically from an engineering point of view. (Engineering theory would work with the more usual symmetrical springs where the point from which the spring is hung is at the center of the top coil and attached to the spring by a crossbar as in Figure 12. But the real problem is that if one were to integrate for a polygon by passing to an infinite number of sides and calculating the stretch produced by such a spring using the torques applied between adjacent segments approach, as shown in Figure 6, the torques would shrink to zero in the limit and stretch would come out to zero!!) Eventually S2 is able to use a more subtle analogy, shown in Figure 11, to reach a revised model, where he able to actually envision forces causing twisting in a circular spring coil. By drawing and imagining part of a circle with two points 90 degrees apart in Figure 11, he is able to imagine a downward force at 'b' exerting a twisting force on a softened segment at point 'a', so that the twisting deformation in 'a' allows point 'b' to drop as a contribution to stretch. This "orthogonal leverage" view of torque is much closer to the way a mechanical engineer would view torque in a spring than the "torque from an adjacent segment" view in the hexagonal coil in Figure 6.

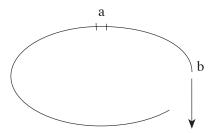


Figure 11: Twist in the quarter coil: An imagistic alignment analogy

(10) "The actual round spring is just a case of that [twisting] happening infinitesimally uh, all the way around the spring and all your distance down is gained by the kind (<u>makes twisting motion with r. hand</u>) of twist effect. ....Let's ... consider this (marks segment (a) in circular coil in Figure 11) an infinitesimal place where twists can occur...And the pull that you could think of as twisting that is ..., uh, 90° around [at b]. (<u>makes twisting motion with r. hand</u>) .... as putting a (<u>makes twisting motion with l. hand over drawn increment (a) in Figure 11</u>) torsion on that increment of

the spring."

This is the first occasion, late in the protocol, where S2 has actually been able to envision a hidden mechanism for twisting occurring in the spring that is aligned visually so that it begins to capture how forces are actually causing the twisting in a circular spring coil. Here the subject transforms the system by "softening" part of it at w (and "rigidifying" all other parts) to form a close analogous case which makes the possibility and source of twisting effects obvious. This and his depictive gestures over the drawing are consistent with using imagistic simulation to 'run' the new analogous case. Here the torque arm acts orthogonally across the coil from a distant point. This new orthogonal torque model is shown in Figure 8 as an important final modification to S2's model.

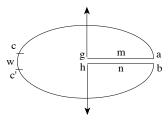


Figure 12 Symmetric coil with arms to center

(Note: Even though considering deformations in a small element with other portions of the system rigid is a standard technique in physics/engineering, I interpret the fundamental process behind this technique as being analogy. Transformations are being applied to the original target system that, although they are small, make this a different case from the original, with the faith, under certain assumptions, that it will be analogous to the original problem. It is only a small step from here to the view shown in Figure 12, which is the somewhat more symmetric diagram of the kind used in engineering texts.)

Later the subject distinguishes between confidence in the *answer* to the spring problem, which has been quite high, and confidence in his *understanding* of it, and estimates that his torsion analysis has increased his understanding of the system from "way, way down" up to "like, 80%".

## Overall Pattern of Model Evolution Cycles and Levels of Processing

In summary, the expert examined in this section appeared to use the following nonformal thinking processes:

- III. An overarching Modeling (GEM) Cycle process of Model Generation, Evaluation, and Modification at a Macro level, as shown in Figure 13.
- II. Nonformal, Heuristic Reasoning Processes at a Micro level: analogy, running a model, identifying a new variable, and conducting a Gedanken experiment
- I. An underlying process of Imagistic Simulation evidenced within all of the above processes

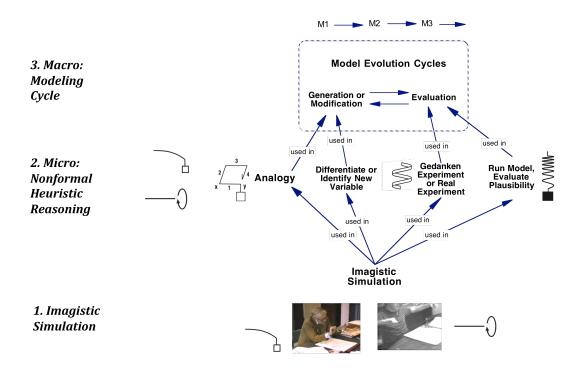


Figure 13: Hierarchical view of three levels of nonformal thinking processes and subprocesses

The above three categories of processes appear to be arranged in order from longer to shorter, that is from larger time scale processes to smaller time scale processes. It also makes sense from the protocol to conceptualize imagistic simulation occurring as a *subprocess* for analogy, running a model, or running a Gedanken experiment. And in turn the analyzed sequence shown in Figure 8 provides some evidence that the processes at Level II appear to serve one of the processes at Level III in a nested way. This motivates the hierarchical subprocess relationships pictured in Figure 13, showing these as three different levels of processes. Processes at a lower level are seen as subprocesses serving the level above it. The imagery indicators underlined within the heuristic reasoning episodes in this case study provide some evidence that imagistic simulation at Level 1 in Figure 13 is a subprocess used within the Level 2 processes above it. And the levels appear to correspond to different time scales in the protocol. Once a case for imagistic simulation is formulated and imagined, the simulation process is seen as happening quickly, within seconds, whereas Level II processes may take longer, and those at Level III even longer. This gives us a way to organize these processes hierarchically.

# Limitations and Conclusion, Part I

The prevalence of imagistic simulation as an underlying foundation in these episodes, as seen in Figure 13, suggests that it may be important to pay greater attention to this process in the analysis of nonformal learning than is commonly done. I do not claim that it is always involved as a subprocess for the processes at Level 2, but have presented some evidence based on case study data, providing an existence demonstration, that it can be a very important subprocess. Similarly I do not claim that analogy is always used as a subprocess for generating a model, but we have seen some evidence that it can be quite powerful in that role.

Figures 8 and 13 give us new ways to picture the roles of nonformal reasoning and learning processes during qualitative model construction. These can be contrasted with more procedural and traditional

reasoning processes of formal deduction and induction by enumeration or statistical inference. The nonformal processes discussed here may be less procedural and carry less certainty than those traditional forms of reasoning, but they can be powerful engines for discovery if used within a self-correcting cycle of evaluation and modification. There are surely other processes (such as extreme cases and partitioning, (Polya (1954; 1957); Clement (2009)) that I have not had space to deal with here, but these diagrams give us a starting framework that can be expanded. One interesting feature seen in Figure 8 depicting model evolution is that it did not matter that S2 began with a faulty model that was later rejected. Through the process of model generation, evaluation, and modification, he was able to use his initially faulty model as a useful starting point to engage in an ultimately productive and successful process of model construction.

# PART II: ROLES FOR NONFORMAL REASONING IN CLASSROOM DISCUSSIONS IN SCIENCE

# Purpose, Theoretical Framework, and Rationale

In Part II we compare the nonformal reasoning/learning processes identified in experts to learning processes that exemplary teachers attempt to foster in whole class discussions in physical science. Our rationale is the hope that comparisons to expert reasoning can sharpen our ways of describing the reasoning of students and teachers during discussions, and help in describing important teaching strategies. Here we focus on two exemplary science teachers leading whole class discussions of electric circuits in high school physics classes using the CASTLE curriculum. We are finding that such teachers encourage many of the same nonformal learning processes seen in experts.

### **Previous Research**

Previous literature describes a variety of styles for conducting whole-class discussions (Hammer, 1995; van Zee & Minstrell, 1997; Duschl & Osborne, 2002). Other important research has focused on listing questioning strategies that teachers utilize at the dialogical level to engage students (Chin, 2007; McNeill & Krajcik, 2008). While it is believed that discussions addressing mental models can support higher levels of students' thinking about scientific phenomena (Hestenes, 1996; Hogan, et al, 2000; Vosniadou, 2002), very little research has been done on identifying and describing the specific types of cognitively-focused teacher moves that can be used to support students in these model building processes. Rare exceptions are that in the important studies done by Chin (2007) and Hogan, et al (2000) one or two out of dozens of strategies they identified refer specifically to processes involved in basic model construction. Discrepant events and analogies are most commonly cited in the conceptual change literature, but examination of tapes of skilled teachers shows that these are only two of many strategies. We will build on previous studies by our group addressing model based whole-class teaching strategies in the life sciences (Nunez-Oviedo, et al., 2008).

# **Quantitative Study**

In order to indicate whether the CASTLE teachers (and in particular the two most experienced teachers) in the study produced a significant amount of learning worth studying, we also implemented a quantitative research design as follows. An investigation was conducted with: (1) a non-model-based group comprised of 262 high school students who were following traditional instructional approaches based primarily on teacher lecture and extensive use of quantitative problem solving with a traditional circuits-based lab component; (2) a model based discussion group of 282 students who were frequently

engaged in whole-class discussions in which they co-constructed explanatory models with the teacher; they used the CASTLE curriculum which also has a circuits-based lab component. Before instruction, all students completed a 20 item pre-test designed to measure qualitative electric circuit concepts and reasoning skills on both simple and complex unfamiliar problems. Subjects also indicated a confidence level for each answer. After completing their 6 – 8 week instructional units, both groups completed an identical post-test. All teachers were blind to the test contents. Teachers for both groups were selected based on recommendations for good instructors who had participated in development workshops, and by locating teachers who either were or were not utilizing the model-based CASTLE curriculum. All teachers were asked to teach the physics of electricity in their normal way. The pretest means of the traditional group and model based discussion group were extremely close and not significantly different. Both groups had significant pre-post gains. We found significant performance gain differences in favor of the model-based group (24.6% gain) over non-model-based group (5.9% gain) (Cohen's d effect size = 1.29). Because of the limitations of the sampling opportunity afforded to us we view this result as exploratory. However, it is well suited for its main purpose in this mixed methods study, which is to indicate whether CASTLE teachers, and in particular the two most experienced CASTLE teachers we wished to study intensively, are producing substantial amounts of learning worth examining further in qualitative case studies. Remarkably, the average gain for those two teachers was almost one standard deviation above the other model based teachers, who in turn had gains that were almost one standard deviation above the control group's. These are unusual gain differences. While we do not have tight enough controls to be sure that all of the unusual gains for the two teachers are due to pedagogy, they indicate that something special occurred in these classes. Case studies of teachers designated as exemplary on the basis of actual student learning data are rare and we felt that this gave us an important opportunity to study them closely.

## **Qualitative Study**

## **Research Questions**

Our major questions for this study were descriptive: What strategies do exemplary teachers use to guide discussions? How do they promote the learning of explanatory models? In addition to *dialogical* strategies designed to encourage students to share and elaborate on ideas, do skilled teachers use *cognitive* strategies for fostering model construction? Do strategies fall into different levels?

#### Method

In the qualitative study we collected video tapes of the two exemplary teachers as they led whole class discussions about preceding laboratory experiences. Tapes were transcribed and a constant comparison method was used to identify whole class teaching strategies being used. A second expert analyst was used to critique and increase precision in rubrics periodically in joint coding during this period. This part of the study is qualitative, generative, and descriptive and is not intended to project frequencies of strategies to a population. It is intended to generate new descriptions of teaching strategies that have their initial grounding and existence demonstrations in video tape case study data. Refinements in coding led eventually to the development of a list of strategies and rubrics for recognizing them. Four extended discussions were coded for each of the two teachers. This paper will present data from one of these discussions to form grounded hypotheses about the existence of two levels of teaching strategies.

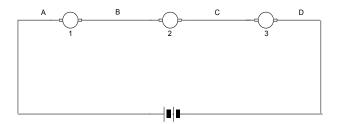


Figure 14 Bulbs in Series Circuit

# **Analysis**

Here we will examine a short example of a discussion of what is happening to charge flow in each section of a series circuit with 3 identical bulbs shown in Figure 14. The fuller study is described in Williams (2011). In a lab previous to this discussion, the students had observed that the brightness of each bulb is the same. They also had placed compasses under each wire and noted that the deflection of the needle (indicating rate of flow or current) was the same for each wire. The instructor then attempted to foster construction of a model of what was happening to the moving charge in the bulbs in this circuit. The diagram in Figure 15 exemplifies the analysis of teaching strategies used in such discussions—here there is space only for an abridged transcript of a discussion. In the dialogue shown, the teacher and students alternate speaking as time moves from left to right. Row 2 (from the top) shows the teacher's inferred view of the students' mental model ideas developing from left to right. The validity of these diagrams for representing teaching strategies and student model interpretations was confirmed by the teachers in later interviews

In this diagram students bring useful concepts of charge flow and energy to the discussion but also have typical serious challenges ahead in differentiating between charge, current, energy, and electric potential (voltage), and dealing with misconceptions about current and charge being "used up" in the bulbs. The instructor knows this will take weeks to develop fully and is therefore drawing out students' initial ideas at this point, encouraging reasoning, and guiding students to gradually sharpen their language and consider evidence in a cycle of model evaluation and revision that will go on for some time in this difficult topic area. Thus the discussions we are studying are student centered for active reasoning, but they are also teacher guided for eventual convergence.

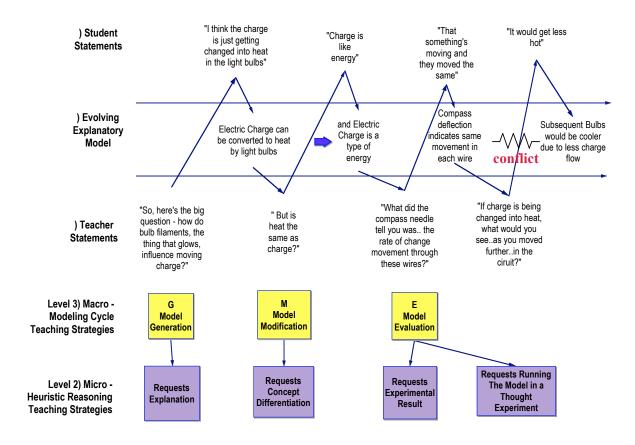


Figure 15: Two Levels of Teaching Strategies in Science Class Discussions

Initially the students exhibit conceptual difficulties by saying that some of the charge or current will be "changed into heat" or be used up in each bulb, and that it may be like 'energy'. In order to set up some dissonance with the idea of charge getting 'changed into heat', the instructor then tries to focus their attention on what the compass needle observations told them from the lab-- that current was the same on each side of a bulb, and not used up in the bulb. There he is using the strategy of Requesting an Experimental Result, shown in purple at Level 2. We coded this as also serving the larger purpose of Model Evaluation at Level 3. Next the instructor Requests Students to Run their Model in a Thought Experiment, by asking what they would see (from the compasses) if charge is being changed into heat, as stated in their model. This is intended to set up a conflict with what they actually observed. This represents only the beginning of a model evolution sequence but more extended sequences are described in Williams and Clement (2013, this proceedings) and Williams (2011).

Figure 15 shows two of the same levels of processes as those found in the expert protocols in Figure 13 (Levels 2 and 3). Level 3 in Figure 15 proposes that the teacher was engaged in fostering model evolution processes of model generation, evaluation, and modification in discussions. The analysis also indicates how strategies like promoting concept differentiation at Level (2) can be done in the service of a goal at Level (3) like model modification. Thus this teacher scaffolded many of same scientific reasoning processes that experts use, and did so at different levels. There is a need to find transparent ways of describing such processes; diagrams like Figure 15 make it possible to envision different levels of strategies being used simultaneously by a teacher.

Another Level 2 Strategy: Analogy. A process not explicitly present in the dialogue in Figure 15 is the use of analogy. The Castle curriculum bases many of its subsequent lessons on an analogy from air pressure and flow to potential and charge flow in circuits. So those analogies will be used extensively in later discussions. A more specific example occurred after the students examined two kinds of light bulbs with thick and thin filaments visible

- T: "So why would the long bulb have higher resistance? What is that like? Is there an analogy that you can think of that would explain why this (thin) filament would have higher resistance than this (thick) filament?"
- S1: "If think that it would be easier to flow through something thicker than thin. If you have, let's say, a river that is wide, it is easier for the water to flow through that than a narrow river."
- S2: "Basically if it is a one lane highway then traffic will be really bad and it will be really slow but if it is a five lane highway then the traffic will be faster."

Here the teacher used the strategy: Requesting an Analogy, at Level 2, as part of a Model Generation strategy at Level 3. These analogies do not map perfectly onto the circuit as the target, but they are an intuitive base that can be used as a starting point for generating a model of resistance.

### **Summary of Strategies Identified**

Eight discussions led by the two exemplary teachers were analyzed in Williams (2011). He identified thirty nine strategies, but we have consolidated and winnowed these to a list of common strategies that we think could be used with teachers. These are listed hierarchically below. Four teaching strategies in bold at Level 3 and 8 strategies at Level 2 are listed.

### **Model Generation**

- •G1) Teacher Requests or Provides Explanation
- •G5) Teacher Requests or Provides Analogy

### **Model Evaluation**

- •E3) Requests or Provides Result from Running a Model (May Support or Conflict w. Model)
- [•E4] Requests or Provides a Thought Experiment (May Support or Conflict w. Model)
- •E5) Requests or Provides Real Experiment or Gedanken Experiment (May Support or Conflict w. Model)

### **Model Modification**

- •M2) Requests or Provides Concept Differentiation
- •M1) Requests or Provides Model Refinement including Additions, Subtractions, or Replacement of Model Elements

## **Observation**

•O1) Requests or Provides Evidence from Observations

To give an idea of the number of teacher moves documented from the four macro strategies over six discussions, Table 1 shows this for each of the two teachers studied. This data supports initial impressions from the videotapes of Teacher B's very active involvement in the discussion compared to Teacher A's more reserved style. However, overall, students made more contributions than the teachers (see Williams, 2011 for details).

	Observation	Model Generation	Model Evaluation	Model Modification
Teacher				
A	31	58	62	10
Teacher B	51	144	146	19

**Table 1 Teacher Contributions to Model Development** 

<u>Dialogical strategies not shown here</u>. For some purposes, we can add a third level of dialogical strategies that are designed to foster active student participation in general (not shown in Figure 15). For example, in that discussion the teacher often simply repeated things the students said to reflect responsibility back to the students for elaborating further on their ideas. He also paraphrased student responses into clarifying follow up questions and summarized where the discussion had gone to improve communication. Such dialogical strategies have been emphasized in previous literature on discussion leading, e.g. in van Zee and Minstrell (1997), Hogan, et al, (2000), Chin (2007), and others. However, our emphasis here is on cognitive teaching strategies that go beyond these and complement other important studies of promoting sociological norms in the classroom. The diagrammatic analyses in Williams (2011) included both dialogical strategies and the two levels of cognitive strategies shown in figure 15. We believe such representations may be powerful aids for envisioning the teacher's process of supporting model construction in discussions at three different levels.

Imagistic simulation level. We have been considering educational parallels to Levels 2 and 3 in Figure 13 but so far we have not talked about Level 1, Imagistic Simulation processes. Experts gave evidence for using these within all of the heuristic reasoning processes at Level 2. The Williams (2011) electricity study did not deal with this level but another study of classroom discussions in high school physics did (Stephens and Clement, 2010). In case studies of two physics classes, they found that student analogies, extreme cases, and Gedankens at Level 2 were almost always accompanied by indicators of imagistic simulation at Level 1 such as gestures. Thus they provided some initial evidence that Levels 1 and 2 in Figure 13 can operate simultaneously in students in classroom discussions.

### **OVERALL CONCLUSION**

### **Expert Learning Processes**

Case studies of successful solutions in Part I suggested some experts can construct progressively deeper theories as dynamically imageable explanatory models. Analysis of these protocols led to a general description of processes for building imageable models (see Figure 13), described at several levels of processing from smaller to larger time scales: (1) individual imagistic mental simulations; (2) the heuristic reasoning processes mentioned above that utilize imagistic simulations; and (3) model evolution cycles of model generation, evaluation, and modification that in turn utilize heuristic reasoning processes. To our knowledge these three levels of processes have not been emphasized in the past. They complement empirical processes of discovery, experimentation, and evaluative argumentation documented by others and provide a perspective on the nature of scientific thinking that includes the idea that model formation can involve creativity through non-empirical processes such as analogy, "running" a mental model, and Gedanken experiments. Other unusual features are the separation into time scale levels of processing, and the emphasis on the embodied grounding of runnable models in imagistic simulations.

## **Educational Parallels and Implications**

The foremost educational contribution we are pursuing is that these analyses of discussions and comparisons to expert reasoning can sharpen our ways of describing the reasoning of students and teachers during discussions, and help in describing important teaching strategies. There were certainly differences between our experts and students: the experts generated large reasoning sequences on their own without help; and they used more sophisticated concepts and Gedanken experiments. But we found many similarities in that many of the expert reasoning/learning processes were also being successfully encouraged by teachers in classroom discussions:

- a) A pattern of progressive model evolution
- b) Three processes at a macro level for orchestrating model construction: model generation, model evaluation, and model modification. These can be seen in the similarities between the expert model evolution sequence in Figure 8 and the classroom model evolution sequence in Figure 15. (Opportunities for a fourth macro-process, observation, were not present in the expert problem solving interviews.)
- c) Eight processes at a micro level of heuristic nonformal reasoning
- d) An hierarchical, process subprocess relationship between b) and c) above

Patterns b), c) and d) can be seen in the similarities between the upper two levels of expert thinking in Figure 13 and the two levels of strategy in classroom teaching in Figure 15.

It may seem daunting to contemplate teachers learning eight plus four different cognitive strategies for discussion leading, but not all need to be learned at once to have an effect. And if teachers can learn to think about the four Macro strategies at Level 3 first, those may provide a learning framework for incorporating many of the Micro strategies at Level 2. This is something we plan to trial in the future in teacher education courses, as described in Williams and Clement (2013). We attempt to teach strategies gradually, one layer at a time, and eventually teach practitioners to learn (chunk), say 2 or 3 of the Micro strategies under a Macro strategy. An initial stepping stone is to first learn to recognize such strategies in videos of good discussion leading.

The idea that various kinds of nonformal learning processes can be valuable may not be apparent to students who are used to an educational system that places a premium on logic or mathematical knowledge. This suggests that it could be beneficial to discuss these NOS ideas with students--an important question for future research.

# Appendix Introduction to Concepts of Torque and Torsion

The concepts of torque and torsion can be introduced via Figure 14. For torque, if we ignore segment hb there and think of segment *ab* as a pipe and segment *ga* as a pipe wrench we are using to turn the pipe clockwise so that the pipe goes into a tight, threaded socket at 'b', then **torque** can be thought of roughly as the "twisting force" applied by the wrench to the end of the pipe at 'a' to turn it. The torque will be greater in proportion to the length of the wrench, r, since longer wrenches provide more leverage and more "twisting force". When F is perpendicular to r, the torque applied to the end of the pipe is equal to the force applied, F, times r.

$$T = F \times r$$

To define **torsion**, we need a different scenario. Imagine that ab is a steel rod only 1/8" thick with the end 'bh' fixed in concrete so that the far end of the rod at 'b' cannot turn. Then if we clamp a vise grip wrench 'ga' to the near end of the rod at 'a', applying the same torque will end up only twisting (deforming) the metal in every element of the entire rod 'ab' somewhat, so that the near end at 'a' turns through the angle  $\beta$  shown in Figure 14.11 (called the angular displacement, or, informally, total amount of twist in the rod) and stops. If the rod is made of resilient metal, it will be elastic, meaning that if we remove the force F, the metal in the rod will untwist and spring back to its original orientation where  $\beta$  was zero. **Torsion** refers to an action that twists a material resulting in stresses and strains that makes the rod want to spring back to its original shape. If the rod is twice as long, but r and F are the same, the angle  $\beta$  will double. That is because the torque and resulting torsion stress will be the same as before, but there will be twice as much metal to deform under that stress, producing twice the total twist. In the protocols, subjects sometimes use the word "torsion" as defined above, but also sometimes misuse the term torsion slightly to mean torque, so they must be read in context.

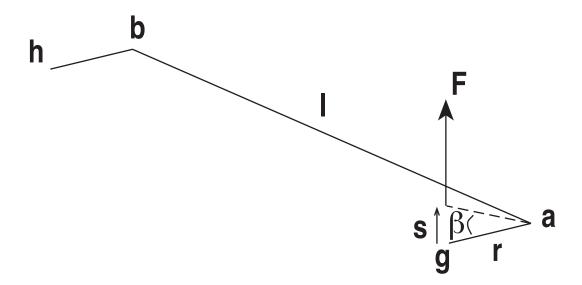


Figure 14: Torque and torsion

#### REFERENCES

- Barsalou, L.W. (1999). Perceptual symbol systems (with commentaries and author's reply). *Behavioral and Brain Sciences* (22) 577-660.
- Chin, C. (2007). Teacher questioning in science classrooms: Approaches that stimulate productive thinking. *Journal of Research in Science Teaching*, 44(6), 815-843.
- Clement, J. (1988). Observed methods for generating analogies in scientific problem solving. *Cognitive Science*, 12: 563-586.
- Clement, J. (1989). Learning via model construction and criticism: Protocol evidence on sources of creativity in science. Glover, J., Ronning, R., and Reynolds, C. (Eds.), *Handbook of creativity: Assessment, theory and research*. NY: Plenum.
- Clement, J. (1994). Use of physical intuition and imagistic simulation in expert problem solving. Tirosh, D. (Eds.), *Implicit and explicit knowledge*. Norwood, NJ: Ablex Publishing Corp.
- Clement, J. (2002). Protocol evidence on thought experiments used by experts. Gray, W., and Schunn, C. (Eds.), *Proceedings of the Twenty-Fourth Annual Conference of the Cognitive Science Society*. Mahwah, NJ: Erlbaum.
- Clement, J. (2003). Imagistic simulation in scientific model construction. *Proceedings of the Twenty-Fifth Annual Conference of the Cognitive Science Society*, 25. Mahwah, NJ: Erlbaum.
- Clement, J., (2008). Creative model construction in scientists and students: The role of imagery, analogy, and mental simulation. Dordrecht: Springer.
- Clement, J. (2009). The role of imagistic simulation in scientific thought experiments. *TOPICS in Cognitive Science*, 1: 686–710.
- Clement, J., Zietsman, A., Monaghan, J. (2005). Imagery in science learning in students and experts. Gilbert, J. (Ed.) *Visualization in Science Education*. Dordrecht, The Netherlands: Springer.
- Dunbar, K. (1999). The scientist in vivo: How scientists think and reason in the laboratory. In L. Magnani, 'N. Nersessian, &P. Thagard (Eds.), *Model-based reasoning in scientific discovery*. New York: Plenum Press.
- Duschl, R. A. & Osborne, J. (2002). Supporting and promoting argumentation discourse in science education. *Studies in Science Education*, *38*, 39-72.
- Finke, R. (1990). Creative imagery: Discoveries and inventions in visualizations. Hillsdale, New Jersey: Lawrence Erlbaum Associates.
- Giere, R. (1988). Explaining science: A cognitive approach. Chicago: Chicago University Press.
- Gilbert, J. (Ed.), (2005). Visualization in science education. Dordrecht. The Netherlands: Springer.
- Glynn, S. M., & Duit, R. (1995). Learning science meaningfully: Constructing conceptual models. In S. M. Glynn & R. Duit (Eds.), *Learning science in the schools: Research reforming practice* (pp. 3-33). Mahwah. NJ: Erlbaum.
- Gooding, D. (1992). The procedural turn: or, Why do thought experiments work? In Giere, R. (Ed.) *Cognitive models of science*. Minneapolis: U. of Minnesota Press.
- Hegarty, M. (2002). Mental visualizations and external visualizations. In Wayne Gray and Christian Schunn, Eds., *Proceedings of the Twenty-Fourth Annual Conference of the Cognitive Science Society* 22, 40. Mahwah, NJ: Erlbaum.
- Hogan, K., Nastasi, B.K. & Pressley, M. (2000). Discourse patterns and collaborative scientific reasoning in peer and teacher-guided discussions. *Cognition and Instruction*, 17(4), 379-432.
- Kosslyn, S. (1980). *Image and mind*. Cambridge, MA: Harvard University Press.
- McNeill, K. and Krajcik, J. (2008). Scientific explanations: Characterizing and evaluating the effects of teachers' instructional practices on student learning. *Journal of Research in Science Teaching*, 45 (1), 53-78.
- Nersessian, N. (2008). Creating scientific concepts. Cambridge: MIT Press.
- Nunez-Oviedo, J., Clement, J. and Ramirez, M. (2008). Developing complex mental models in biology through model evolution, in *Model based learning and instruction in science*, edited by J. Clement and M. A. Ramirez. Dordrecht: Springer, p.173-194.
- Osborne, J. F., Erduran, S. & Simon, S. (2004). Enhancing the quality of argumentation in school science. *Journal of Research in Science Teaching*, 41(10), 994-1020.
- Polya, G. (1954). Mathematics and plausible reasoning. Trenton, NJ: Princeton University Press.

- Polya, G. (1957). How to Solve It, 2nd ed., Princeton: Princeton University Press.
- Quintana, C., Reiser, B., Davis, E. A., Krajcik, J., Fretz, E., Golan, R., et al. (2004). A scaffolding design framework for software to support science inquiry. *Journal of the Learning Sciences*, 13(3), 337-386.
- Schmidt, R. A. (1982). Motor control and learning. Champaign, IL: Human Kinetics Publishers.
- Schwartz, D. L., & Heiser, J. (2006). Spatial representations and imagery in learning. *The Cambridge handbook of the learning sciences*, 283-298.
- Stephens, L. & Clement, J. (2010). Documenting the use of expert scientific reasoning processes by high school physics students. *Physical Review Special Topics Physics Education Research*, 6(2): *URL:* http://link.aps.org/doi/10.1103/PhysRevSTPER.6.020122
- Strauss, A., & Corbin, J. (1998). *Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory*. Sage Publications. Thousand Oaks, CA., pp. 12 & 101.
- Trickett, S. and Trafton, J. G. (2002) The instantiation and use of conceptual simulations in evaluating hypotheses: Movies-in-the-mind in scientific reasoning. In Wayne Gray and Christian Schunn, Eds., *Proceedings of the Twenty-Fourth Annual Conference of the Cognitive Science Society* 22, 878-883. Mahwah, NJ: Erlbaum.
- van Zee, E. & Minstrell, J. (1997). Reflective discourse: Developing shared understandings in a physics classroom. *International Journal of Science Education*, 19, 209-228.
- Williams, E.G. (2011), Fostering high school physics students' construction of explanatory mental models for electricity: Identifying and describing whole-class discussion-based teaching strategies, Doctoral Dissertation. University of Massachusetts, Amherst.
- Williams, E.G., and Clement, J. (2013). From research to practice: Fostering pre-service science teachers' skills in facilitating effective whole class discussions. Paper presented at NARST, 2013.