Autonomous Platooning

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***Abstract*—The idea of platooning is to have a leader vehicle which is either autonomous or non-autonomous, leading a string of vehicles (which follows the leader) in such a way that an optimal distance is maintained between the vehicles so as to move safely and efficiently on the road. Linking cars into a trainlike group can save fuel, fit more cars on the road, and potentially improve safety. This system also allows for a closer headway between vehicles by eliminating reacting distance needed for human reaction. This will substantially shorten the commutes during peak periods. In a platoon vehicles will be able to leave and join the string any time. The other vehicles will adapt to this change even if the vehicle entering the string is not a part of the platoon. Another advantage of platooning is greater fuel economy by reducing air drag and unnecessary acceleration by the driver to maintain distance between vehicles. Platooning also increase the comfort of the passenger, as the ride is much smoother with fewer changes in acceleration. The constant monitoring and updates among the platoon result in a higher level of safety.**

I. Objectives

The aim of the lab is to use the MATLAB and simulink to simulate and validate autonomous platoon modelling and control. This work has been planned into three main sections: Section II is dedicated to the lateral control of a car-like vehicle. Here, we develop the kinematic model of a car-like vehicle and platoon on a pre-defined path, establish the lateral control law using chained system theory and exhibit the main properties through simulations. In Section III, we consider a leader-follower platoon, and establish the longitudinal control using exact linearisation techniques. Finally, in Section IV, we simulate a platoon of 5 vehicles involving lateral and longitudinal controls, and test several control strategies. The concluding remarks and suggestions for future work are summarized in Section V.

II. Lateral Control of a Car-Like Vehicle

A. Modelling Assumptions

Urban vehicles involved in platooning applications are supposed to move at quite low speed on asphalted roads. Therefore dynamic effects can be neglected and a kinematic model can satisfactorily describe their behaviour. In this work, the kinematic tricycle model is considered : the two actual front wheels are replaced by a unique virtual wheel located at the mid-distance between the actual wheels. The notations used are illustrated on figure 1 and detailed below:

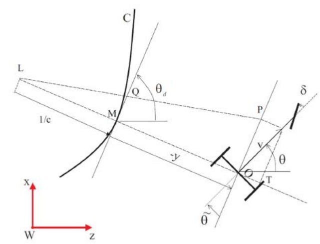

Figure 1. Unicycle Model Description

- C is the common reference path, defined in an absolute frame
- S is the curvilinear coordinate of point M along C, (it corresponds to the distance covered along C)
- O is the center of the vehicle rear axle
- M is the closest point on C to O
- c is the curvature of path C at M
- θ is the heading of the vehicle
- θ_d is the orientation of the tangent at M
- $\tilde{\theta} = \theta - \theta_d$ is the angular deviation of the vehicle wrt S
- y is the lateral deviation of the vehicle wrt S
- δ is the front steering wheel angle of the vehicle

B. Vehicle State-Space Model

In order to ensure that the distance travelled is monotonous and is perfectly consistent when following paths having high curvature, the model is expressed in curvilinear space instead of in cartesian space. The configuration of the ith vehicle can be described without ambiguity by the state space vector $(s_i, y_i, \tilde{\theta}_i)$. The control variables are longitudinal velocity and steering angle : $U = (v, \delta)^T$

In the unicycle model shown in Figure 1 before, we consider the triangle LOP. MQ and OP are parallel. From Thales theorem, we have:

$$\frac{LM}{LO} = \frac{MQ}{OP}$$
$$\frac{\frac{1}{c}}{\frac{1}{c} - y} = \frac{\dot{s}}{vcos\theta}$$

That gives,

$$\dot{s} = \frac{v\cos\tilde{\theta}}{1 - yc}$$

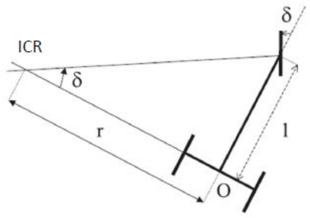

Figure 2. Wheelbase

Also,

$$-\dot{y} = -vsin\tilde{\theta}$$

$$\dot{y} = vsin\tilde{\theta}$$

From Fig 2 we find that δ is related as:

$$\tan\delta = \frac{l}{r}$$

The time-derivative of θ (from Figure 1):

$$\frac{d\theta}{dt} = \frac{v}{l}tan\delta$$

For curvature c

$$c = \frac{d\theta_d}{ds} = \frac{d\theta_d}{dt}\frac{dt}{ds}$$

$$\dot{\theta}_d = c\dot{s} = c\frac{vcos\theta}{1-yc}$$

The above relations can be summarised as below:

$$\begin{cases} \dot{s} = \frac{vcos\theta}{1-yc} \\ \dot{y} = vsin\tilde{\theta} \\ \dot{\tilde{\theta}} = v\left(\frac{tan\delta}{l} - \frac{cos\theta}{1-yc}\right) \end{cases} \tag{1}$$

This model can then be used to control the car with the variable v and δ. The model is tested with two simple trajectories, a line and a cirle, in the simulation part given later.

Platooning objectives can then be described as ensuring the convergence of y and $\tilde{\theta}$ to zero, by means of δ (lateral control), and maintaining the gap between two successive vehicles to a fixed value d, by means of controlling v (longitudinal control).

C. Chained System

We have for objective to make a lateral control law. To do that, we have to make first our system linear. We use the chained system theory. The system is an order 3 system. It can therefore be modelled under the following equations :

$$\begin{cases} \dot{a_1} = m_1 \\ \dot{a_2} = a_3 \cdot m_1 \\ \dot{a_3} = m_2 \end{cases} \tag{2}$$

The state mapping $A = \oslash(X)$ chosen for this problem is :

$$\begin{cases} a_1 = s_1 \\ a_2 = y \end{cases}$$

For the variable a_3 we then don't have any choice.

$$a_3 = \frac{\dot{a_2}}{\dot{a_1}}$$

With the model previously given on equation 1, we have :

$$a_3 = \frac{\tan(\theta)}{1 - y \cdot c(s)}$$

Finally the state mapping is expressed under the form :

$$\begin{cases} a_1 = s_1 \\ a_2 = y \\ a_3 = \frac{\tan(\theta)}{1-y\cdot c(s)} \end{cases} \tag{3}$$

From this we can then express the control mapping $M = \oslash(u)$. We replace the derivative over time $\frac{d}{dt}$ by the derivative over the curvilinear abscissa $\frac{d}{da_1}$. We have the relation $\frac{d}{dt} = \frac{d}{ds}\frac{ds}{dt}$. By dividing the 2 by $\dot{s}$ we decouple the longitudinal behaviour and the lateral behaviour. The equations 2 become :

$$\begin{cases} a_1' = 1 \\ a_2' = a_3 \\ a_3' = m_3 = \frac{m_2}{m_1} \end{cases} \tag{4}$$

From equation 2 we have :

$$m_1 = \frac{V\cos\tilde{\theta}}{1 - y \cdot c(s)} \tag{5}$$

and :

$$\begin{aligned} m_2 &= \dot{a_3} \\ &= -\dot{c}y\tan(\tilde{\theta}) - c\dot{y}\tan(\tilde{\theta}) + (1-cy)\frac{\dot{\tilde{\theta}}}{\cos^2\tilde{\theta}} \end{aligned}$$

Using the equation 1, m_2 can be expressed as :

$$\begin{aligned} m_2 = &-\frac{vgy\sin(\tilde{\theta})}{1-yc} - cv\sin\tilde{\theta}\tan\tilde{\theta} \\ &+ \left(\frac{tan\delta}{l} - \frac{cos\theta}{1-yc}\right)\frac{(1-cy)v}{\cos^2\tilde{\theta}} \end{aligned} \tag{6}$$

and m_3 is defined as being :

$$\begin{aligned} m_3 &= \frac{m_2}{m_1} \\ &= -gy\tan(\tilde{\theta}) - c\tan^2\tilde{\theta}(1-yc) \\ &+ \left(\frac{tan\delta}{l} - \frac{cos\theta}{1-yc}\right)\frac{(1-cy)^2}{\cos^3\tilde{\theta}} \end{aligned} \tag{7}$$

D. Control Law Design

We chose then to use a proportional derivative control law to control the variable a_2 leading to a control of the variable y and $\tilde{\theta}$.

The variable m_3 is controlled with the following relation :

$$m_3 = -K_d \cdot a_3 - K_p \cdot a_2 \quad , \quad K_p, K_d \in \mathbb{R}_+^* \tag{8}$$

Taking the derivation of the third equation of 4 over s and introducing its second equation, we get :

$$a_2'' + K_d \cdot a_2' + K_p \cdot a_2 = 0 \tag{9}$$

This equation will command our system. The first remark is that this equation doesn't depend on the time and therefore of the speed of the car. The only assumption for the speed is that it is not null. The solution of this equation give an exponential decrease of a_2 and a_2' toward 0 according to the distance travelled. If we look the equation given in the mapping 3, the variable y and θ will converge to zero if $y \neq 1/c$. We will check with simulation if we can observe this behaviour, meaning that the lateral behaviour is no dependent of the speed of the vehicle. We will moreover show that the coefficient of the proportional derivative controller is linked to the convergence distance. This can be proven by solving the equation 9.

Once the control law established, we have to implement it on the driving angle δ. By introducing equation 8 into 7, we express the command variable δ.

$$
\begin{aligned}
&-lK_d a_3 - lK_p a_2 + lgy\tan(\tilde{\theta}) + lc\tan^2\tilde{\theta}(1-yc) \\
&= \left(\tan\delta - \frac{l\cos\theta}{1-yc}\right)\frac{(1-cy)^2}{\cos^3\tilde{\theta}}
\end{aligned}
$$

And finally :

$$
\begin{aligned}
\delta = \tan^{-1}&\left(\frac{\cos^3\tilde{\theta}}{(1-cy)^2}\Big(-lK_d a_3 - lK_p a_2\right. \\
&\left.+ lgy\tan(\tilde{\theta}) + lc\tan^2\tilde{\theta}(1-yc)\Big)\frac{l\cos\tilde{\theta}}{1-yc}\right) \quad (10)
\end{aligned}
$$

E. Simulations

The kinematic modelling of the car-like vehicle is tested on two paths.

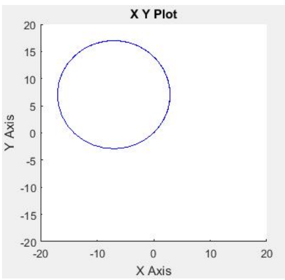

Figure 3. Path followed with constant non null steer angle

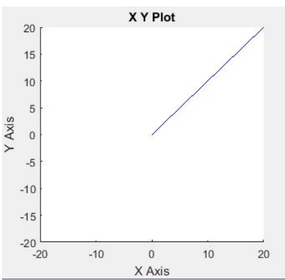

Figure 4. Path followed with null steer angle

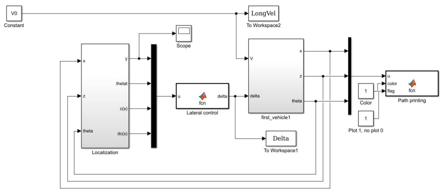

Figure 5. Block2

We use Simulink modelling from MATLAB to perform lateral control. The first block localisation takes a reference path, the current position of the car and computes the lateral distance, the lateral angle,the curvature and its derivative. The lateral control law is implemented in the second block. It outputs the angle δ applied on the car. The model of the car previously derived gives then the position according to the speed and steer angle.

A varying speed (shown in 14) is taken to demonstrate that velocity doesn't have any effect on the path followed. It affects only the trajectory, i.e the behaviour of the car according to the time is affected by the speed.

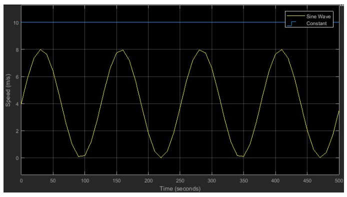

Figure 6. Variable speed used in lateral control demonstration

The different values taken for K_p and K_d are:

	Curve 1	Curve 2	Curve 3
K_p	1	3	0.42
K_d	2	4	0.1
Color	Red	Blue	Yellow

Figure 7. Legend for gains used

We try first our control on the first track which is a straight line.

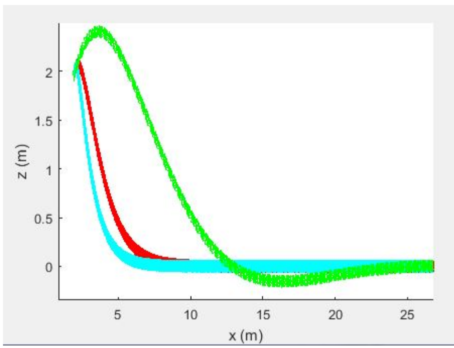

Figure 8. Effect of gains on lateral behaviour in line following

Then we test the same control on a curved path.

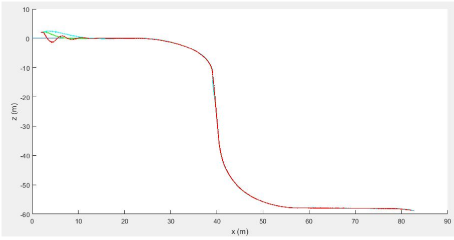

Figure 9. Effect of gains on lateral behaviour in curved path (TrajZadams)

F. Discussions

III. LEADER-FOLLOWER LONGITUDINAL CONTROL

The structure of chained form equations allows to address independently lateral and longitudinal control. Steering control laws δ can first be designed to achieve the lateral guidance of each vehicle within the platoon. In these control laws, v just appears as a free parameter. Since conversion of equations (1) into chained form is exact, all non-linearities are explicitly taken into account. Control variable v can then be designed to achieve longitudinal control.

A. Control Law Design

Consider a platoon of a leader-follower moving along a curve C as shown below: The longitudinal

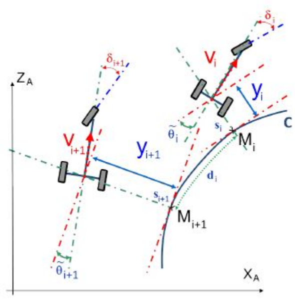

Figure 10. Lead Follower [Martinet]

error between two vehicles is $d_i = s_i - s_{i+1}$. Under the Constant Spacing (CS) platoon policy, the objective is to maintain a fixed distance d between any two cars, so the control becomes $c_{i+1} = e^i_{i+1} = d_i - d = s_i - s_{i+1} - d$. This gives the kinematic longitudinal as $\dot{e}^i_{i+1} = \dot{s}_i - \dot{s}_{i+1}$

Using equations (1), substituting the values for $\dot{s}$, we get:

$$\dot{e}^i_{i+1} = \frac{v_i cos\theta_i}{1 - y_i c(s_i)} - \frac{v_{i+1} cos\theta_{i+1}}{1 - y_{i+1} c(s_{i+1})}$$

Applying linearisation technique and using exponential convergence given by $\dot{e}^i_{i+1} = -ke^i_{i+1}$ (where $k > 0$), the longitudinal control law can be finally expressed as:

$$V_{i+1} = \left(\frac{1 - y_{i+1} c(s_{i+1})}{cos\tilde{\theta}_{i+1}}\right)\left(\frac{v_i cos\tilde{\theta}_i}{1 - y_i c(s_i)} + ke^i_{i+1}\right) \quad (11)$$

In this given case, having only a single leader-follower, we have $i = (0, 1)$.

B. Simulations and Discussions

The objective of this section is to show that this control work and what are the pros and cons of the control we will apply. To do so, we use two car. The first two car have the previous lateral control. The speed imposed to the first car is a simple constant. The second car is controlled with the control we just design. The parameter k have to be tested on the simulation to see its influence. Theoretically it play on the convergence time. On figure 11, we plot the distance between two car in function of time. We do it with 4 different gain.

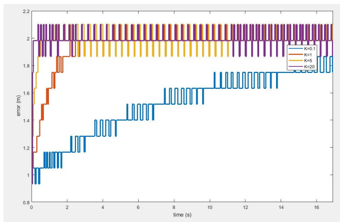

Figure 11. Variation of the distance between the car according to time and different gain

The first conclusion is that the distance of the car converge to the desired value. We had for example here a desired distance d of 2. We see then that the time of convergence is increasing when the gain is decreasing. The time of convergence at 5% is a function of the gain. It can be proven that it is the inverse function : $1/x$.

We see then a last phenomenon. The response of the system is very noisy. The reason of this behaviour is the discrete path. For each step of calculation, the curvilinear abscissa is calculated from the position of the vehicle in the bloc localisation. The problem is that the curvilinear abscissa is not continuous because the path definition is discrete. To calculate the curvilinear abscissa of the car, the bloc localisation search the minimum distance between the car and all the point of the track. The point with the minimal distance correspond then to the estimated curvilinear abscissa of the car. The problem is that the car will never be in reality at this curvilinear abscissa. The real curvilinear abscissa of the car is the projection of the position of the car on the real continuous curve of the path. That is why we have then a bad resolution on our figure 11. We see it particularly on the longitudinal control because it use directly the curvilinear abscissa which have a bad resolution (10 cm compared to the 2m we want to keep between the car). But it affect to the lateral control. Indeed all the information coming from the other and current car are then discrete information : The curvilinear abscissa, the curvature and its derivative, the distance to the path. To improve the result, it is possible to use interpolation. The problem to use interpolation is that the path can be very complicated and we can't make an interpolation on all the path. We have then to select a part of the path around the position of the car. To do that we select first the closest point to the path data. We select the two previous and following point on the path to follow. We have then five point to interpolate. We can use a cubic interpolation to have a good shape around the position of the car. We can then use a mathematical projection to find the curvilinear abscissa, the curvature and the distance to the curve. This way the information obtain will be more continuous.

IV. AUTONOMOUS PLATOONING

In this section, we consider a platoon of 5 vehicles and apply the developed knowledge in previous sections, to autonomously control it.

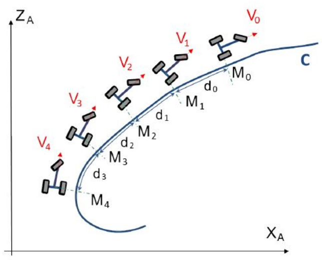

Figure 12. Platoon of 5 vehicles [Martinet]

In platoon mode the inter-distance between vehicles maybe regulated by different control strategies.

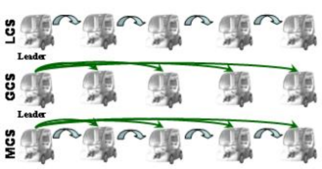

Figure 13. Longitudinal control strategies [Martinet]

A. Local strategy (LCS)

The standard approach to control a platoon is based on a local strategy, i.e. each vehicle is controlled from the unique data received from the single immediate front vehicle.

The longitudinal error in this method is computed as:

$$e_{i+1}^{1} = s_1 - s_{i+1} - d \tag{12}$$

The longitudinal velocity is obtained by taking derivatives of the error and using equation 1:

$$v_{i+1} = \left(\frac{1 - y_{i+1}c(s_{i+1})}{cos\tilde{\theta}_{i+1}}\right)\left(\frac{v_i cos\tilde{\theta}_1}{1 - y_i c(s_i)} + k\dot{e}_{i+1}^{1}\right) \tag{13}$$

Such a local approach presents drawbacks,like errors accumulation: the regulation errors, introduced by sensors noises, are growing from the first vehicle to the last one leading to unacceptable oscillations.

B. Global strategy (GCS)

To overcome the problems in LCS, inter-vehicles communication has to be considered and a global control strategy applied in order to more widely capture the platoon behavior. Information transmitted from the immediate front vehicle and the leader one are used to control each fleet element according to a global strategy.

The longitudinal error in this method is computed as:

$$e_{i+1}^{1} = s_1 - s_{i+1} - id \tag{14}$$

The longitudinal velocity is obtained by taking derivatives of the error and using equation 1:

$$v_{i+1} = \left(\frac{1 - y_{i+1}c(s_{i+1})}{cos\tilde{\theta}_{i+1}}\right)\left(\frac{v_1 cos\tilde{\theta}_1}{1 - y_1 c(s_1)} + k\dot{e}_{i+1}^{1}\right) \tag{15}$$

C. Mixed strategy (MCS)

Since both the local and global methods have drawbacks, a hybrid mixed strategy is used that takes advantages of both approaches depending on the context evolution [2].

The longitudinal error in this method is computed as:

$$e_{i+1} = (\sigma_{i+1})e_{i+1}^{1} + (1 - \sigma_{i+1})e_{i+1}^{i} \tag{16}$$

where,

$$\sigma_{i+1}(z_{i+1}) = \frac{1}{1 + e^{-az_{i+1}}}$$

σ_{i+1} is a sigmoid function and permits to provide predominance at each error, depending on the context. For the proper design of σ_{i+1}, a minimum security inter-distance d_s is introduced that always must be observed between 2 vehicles. When distance between vehicles i and $i+1$ is close to this limit, collision risk is important. Therefore, local approach must prevail over the absolute reference one. σ_{i+1} must then be close to 0 when e_{i+1}^{i} is close to $-d + d_s$. On the contrary, when inter-vehicles distance is close to d,absolute reference approach can be safely used, σ_{i+1} must then be close to 1 when e_{i+1}^{i} is close to 0. The sigmoid function σ is driven by variable z_{i+1} defined by: $z_{i+1} = e_{i+1}^{i} + \frac{d-d_s}{2}$

The longitudinal velocity maybe derived by taking derivatives of the error,

$$\dot{s}_{i+1} = (\sigma_{i+1})\dot{s}_1 + (1 - \sigma_{i+1})\dot{s}_i + k\dot{e}_i$$

Using equation 1, we get:

$$\frac{v_{i+1}cos\tilde{\theta}_{i+1}}{1 - y_{i+1}c(s_{i+1})} = (\sigma_{i+1})\left(\frac{v_1 cos\tilde{\theta}_1}{1 - y_1 c(s_1)}\right) + (1 - \sigma_{i+1})\left(\frac{v_i cos\tilde{\theta}_i}{1 - y_i c(s_i)}\right) + k\dot{e}_i$$

$$v_{i+1} = \left(\frac{1 - y_{i+1}c(s_{i+1})}{cos\tilde{\theta}_{i+1}}\right)\left[(\sigma_{i+1})\left(\frac{v_1 cos\tilde{\theta}_1}{1 - y_1 c_(s_1)}\right) + (1 - \sigma_{i+1})\left(\frac{v_i cos\tilde{\theta}_i}{1 - y_i c(s_{i+i})}\right) + k\dot{e}_i\right] \tag{17}$$

D. Simulations

On this part the objective is to compare different platoon. We tested the three previously defined different platoon on three different path. There is a straight line, a robotic curve and the "z" trajectory. The inter-vehicle distance is set at two meters. The correction is made with the gain $K_p = 1$, $K_d = 2$ and $k = 1$. A speed profile is imposed on the first car. It is shown on figure 14. We interact then with the first or second car depending the case to make it brake and see the impact on the platoon. The detail are given on the next part.

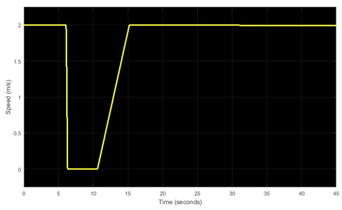

Figure 14. Speed imposed to the leader car

The results for platooning each of the control strategies on the different paths are given in the following page at the end of the report.

E. Analysis and Discussions

We can then analyse the figure we obtained. The first remark is that all car converge to the path whatever the initial position is. It is due to the lateral control. We can see it at the beginning of each trajectory from figure 4 to 17. On the figure 18 we can see that all the distance converge to 2 m between each car after we ordered a constant value on the speed of the first car between 0s and 5s. This is due to the different longitudinal control.

At 5s we imposed a speed of zero for the first car. All the car stopped immediately. We can see indeed that the distance between the car stopped at 2 m. At 11 second the speed increased again. From this first range of time between 0 and 11 second we can conclude that all platoon work correctly. The car follow each other on the path to follow following exactly the trajectory of the first car. The braking of the leader car is propagated throughout the entire platoon almost instantly that ensures that the imposed constant spacing remains unaffected.

At 20 second we tried to differentiate the platoon. We imposed a brake on the second car for local and global control and on the first car for hybrid control. At 30 second, the initial velocity is imposed back. On figure 4 to 17 we don't see any particular difference but the initial position at the beginning, all car follow the track. On the error 18 we can see differences. For hybrid and local control, the car brake a bit. We can see it because the distance to the previous car increased. The distance is not increasing more because we just changed the reference speed and not the parameter. The speed calculation still take into account information from the previous car. What we can see then is that the following car brake to and the take information from the previous car and the leader car is not any more the most important car (or still not important at all for the local platoon). With the global platoon we see that the third car, still referring to the leader will come into the second car. This platoon is then finally not a safe platoon.

V. CONCLUSION

This work presents the basic fundamentals of kinematic modelling and platooning of cars, and provides useful insights about autonomous control of platoons. The transformation from Cartesian to curvilinear space gives great flexibility. Use of chained system and linearisation techniques help decoupling lateral and longitudinal control, and allows for independently controlling the deviation and velocities. Among the various control strategies, we have seen that the local control is more robust to braking and acceleration changes within the platoon, while the global technique is very sensitive to disturbances. The best performance is observed with the mixed or hybrid control strategy, as expected.

As an extension of this current work, it may be useful to analyse the stability of the platoon control strategy. Theoretically, the local platoon is less stable than the hybrid control. That is why this control is a better control if we wanted to implement it on a real car. Variable spacing platoon strategies maybe considered for improved platoon performances. Additionally, the grid information containing environmental factors such as static and dynamic obstacles maybe taken into account, besides the inter-vehicular data communication, for a truly safe and reliable autonomous platooning.

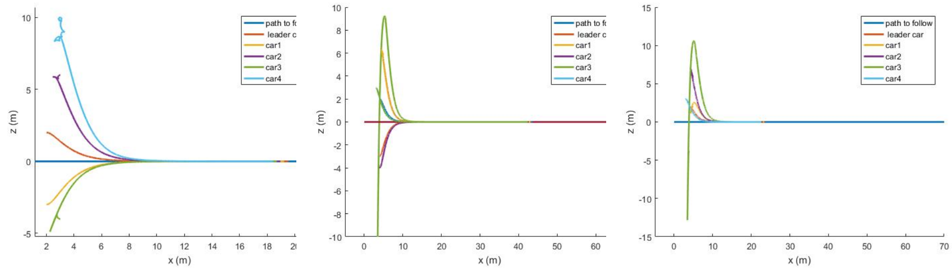

Figure 15. Line Path Following: Local, Global, Hybrid

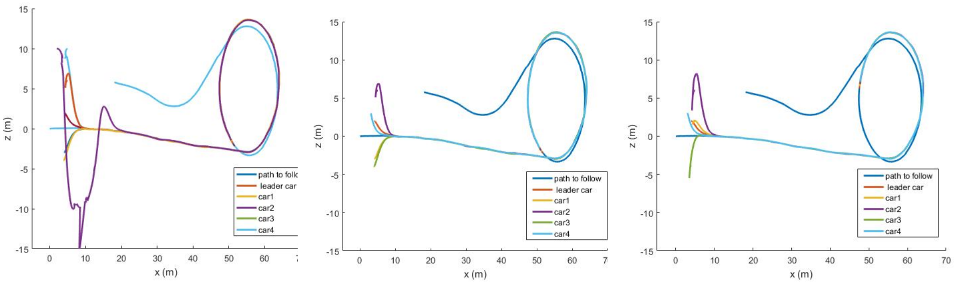

Figure 16. Robotics Path Following: Local, Global, Hybrid

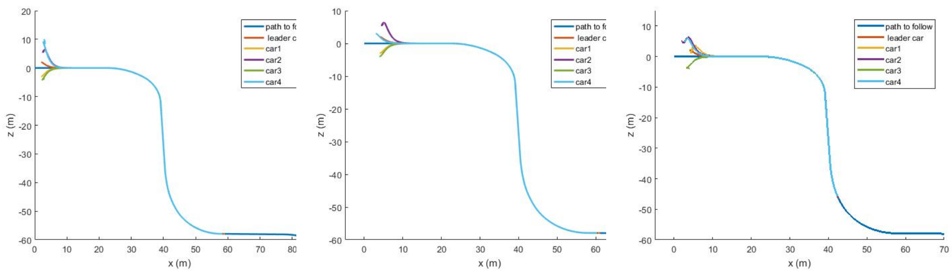

Figure 17. TrajZAdams Path Following: Local, Global, Hybrid

VI. REFERENCES

1. C. Samson. Control of chained systems: application to path following and time-varying point stabilization of mobile robots. IEEE Trans. on Automatic Control, 40(1):64–77, January 1995.
2. Philippe Martinet, Benoit Thuilot, Jonathan Bom. From autonomous navigation to platooning in urban context.

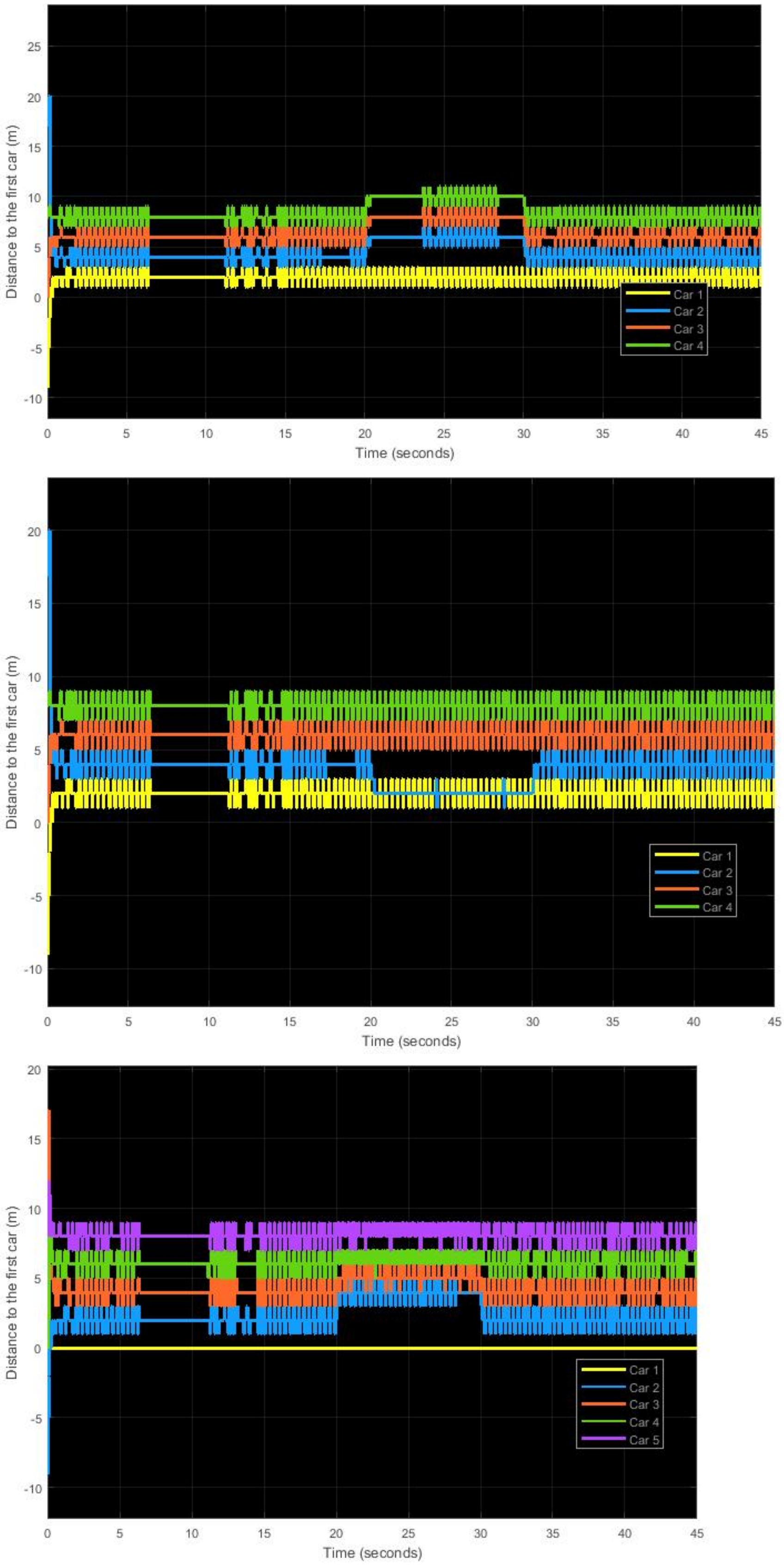

Figure 18. Error comparison with leader (on TrajZAdams Path):
Local, Global, Hybrid