WARSAW UNIVERSITY OF TECHNOLOGY

APPLICATION OF LINEAR PROGRAMMING IN PRODUCTION PLANNING

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DEBALEENA MISRA
EMARO 174/EM

OPTIMIZATION TECHNIQUES
Prof. Wlodzimierz Ogryczak

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INTRODUCTION

Optimisation problems arise in almost all branches of industry or society, e.g. in product and process design, production, logistics, traffic control and even strategic planning. In an optimisation problem (OP), one tries to minimise or maximise a global characteristic of a process such as elapsed time or cost, by an appropriate choice of parameters which can be controlled, and under a set of constraints, linked for example to physical limits.

This project is aimed at using mathematical linear programming to find an optimal solution to the given problem statement. The general linear programming problem calls for optimizing (maximizing/minimizing) a linear function of variables called the *objective function* subject to a set of linear equations and inequalities called the c*onstraints* or r*estrictions*. This study is focused on the design of a production planning model that optimally allocates limited resources and fulfils the demands while maximizing the profit. Since the units cannot be fractional, hence, integer programming is applied, which is a subset of LP having all characteristics of an LP except for one caveat: *the solution to the LP must be restricted to integers.*

The first step involves the analysis of the problem statement in order to develop a mathematical model to fit the given data. This includes determining appropriate objectives, constraints, interrelationships and alternative course of action. Optimal solutions are then derived from the formulated mathematical model using the optimization solvers in the AMPL environment.

PROBLEM ANALYSIS

Problem Statement

A manufacturer produces three models (I, II and III) of a certain product. He uses two types of raw material (A and B) of which 2000 and 3000 units are available, respectively. The raw material requirement can be read from the following table

Raw Material	Model I	Model II	Model III
A	2	3	5
B	4	2	7

The labour time for each unit of model I is twice that of model II and three times that of model III. The entire labour force can produce the equivalent of 700 units of model I. A market survey indicates that the maximum demand of three models is 300, 300 and 250 units, respectively. Assume that the profit per unit of models I, II, III are: \$30, \$20 for first 200 units and \$15 for further units and \$60 for first 150 units and \$65 for further units, respectively. The manufacturer has already signed a contract on delivery 100 units of model III. Failing the contract delivery will result in penalty of \$100 per each undelivered unit. Formulate the simplest optimization model (LP or MILP) to determine the number of units of each product which will maximize the profit.

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Qualitative analysis of the problem statement

Extracting the primary elements from the problem statement

- A combination of 3 models for the product needs to be manufactured in an optimal ratio
- Only two types of limited raw materials maybe used for the entire production
- There is a limit on the labour force available for production
- The profit for model I is fixed, however using the economy of sales, model II has decreasing profit while model III has increasing nature of profits, as more number of units are produced.
- A contract has been signed to guarantee the delivery of a fixed number of units for model III, failing which a penalty amount is imposed per undelivered item.

MODEL FORMULATION

Using the relevant information, a mathematical model maybe formulated for our production system

Decision variables

These are the variables, whose quantitative values are to be found from the solution of the model in order to maximize or minimize the objective function.

As models II and III have the economy of sales and varying profits or penalty conditions, the number of units produced for them has been decomposed into sub-parts, in order to identify what proportion of profit is derived from each of them.

The list of decision variables used for the model are listed below:

x_1 = Number of units produced in model I (constant profit of 30$)

x_{21} = Number of units (up to 200) produced in model II with a profit of 20$
x_{22} = Number of units produced in model II with a profit of 15$

x_{31} = Number of units produced (up to 100) in model III with a profit of 60$, and penalty of 100$ for every undelivered item
x_{32} = Number of units produced (up to 150) in model III with a profit of 60$, without penalty
x_{33} = Number of units produced in model III with a profit of 65$, without any penalty

Constraints

These are any restriction on values that can be assigned to decision variables in terms of inequalities or equations. They can be listed as kinds given below:

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- <u>Limits for the sub-parts for Models II and III</u>

  – Model II units are divided into 2 types- first 200 units with profit of 20$ and further units with profit 15$. So limits for sub-parts for Model II have limits as below:

    $0 \leq x_{21} \leq 200$ and $x_{22} \geq 0$

  – Model III units are divided into 3 types- first 100 units with profit of 20$ and penalty of 100$ for each undelivered item, next 50 units with same profit but no penalty, while further units have profit of 25$ without any penalty. So the sub-parts for Model III have limits as below:

    $100y_1 \leq x_{31} \leq 100$, $50y_2 \leq x_{31} \leq 50$ and $0 \leq x_{33} \leq 100y_2$ where $y_i \in \{0,1\}$

- <u>Functional Constraints</u>

  – *Limited availability of raw materials*

    Raw Material A: $2x_1 + 3(x_{21} + x_{22}) + 5(x_{31} + x_{32} + x_{33}) \leq 2000$
    Raw Material B: $4x_1 + 2(x_{21} + x_{22}) + 7(x_{31} + x_{32} + x_{33}) \leq 3000$

  – *Limited labour force*

$$x_{11} + \frac{1}{2}(x_{21} + x_{22}) + \frac{1}{3}(x_{31} + x_{32} + x_{33}) \leq 700$$

- <u>Non-negativity constraints</u> – All the decision variables must be positive in nature $(x_i \geq 0)$

<u>Parameters</u>

These are the input constants to the model. In our case, these are the restrictions imposed to the decision variables by the maximum demands in the market.

- Model I: $x_1 \leq 300$
- Model II: $x_{21} + x_{22} \leq 300$
- Model III: $x_{31} + x_{32} + x_{33} \leq 250$

<u>Objective Function</u>

This is the measure of performance (profit) expressed as mathematical function of decision variables. We have to maximize the total profit generated by all the models of production.

The objective function to be optimized maybe represented as:

$$\max Z = 30x_1 + (20x_{21} + 15x_{22}) + (60x_{31} + 60x_{32} + 65x_{33}) - 100(100 - x_{33})$$

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FINDING THE SOLUTION

Use of AMPL

The objective function has been solved using AMPL that stands for “A Mathematical Programming Language", and is a high level language that translates mathematical statements that describe a mathematical program into a format readable by most optimization software packages. AMPL works like a compiler: the model and input are put into an intermediate form which can be read by a solver. The solver finds an optimal solution to the problem by reading in the intermediate file produced by AMPL and applying an appropriate algorithm. In this study, CPLEX solver was used

AMPL Code

```
var y1 binary;
var y2 binary;

#Part 1 DECISION VARIABLES

var X1 integer >= 0;
var X2_1 integer >= 0;
var X2_2 integer >= 0;
var X3_1 integer;
var X3_2 integer >= 0;
var X3_3 integer >= 0;

#PART 2 OBJECTIVE FUNCTION
maximize Profit: 30 * X1 + 20 * X2_1 + 15 * X2_2 + 60 * X3_1 + 60 * X3_2 + 65 * X3_3 - 100 * (100 - X3_1);

#PART 3 CONSTRAINTS
subject to Raw_Mat_A: 2 * X1 + 3 * (X2_1 + X2_2) + 5 * (X3_1 + X3_2 + X3_3) <= 2000;
subject to Raw_Mat_B: 4 * X1 + 2 * (X2_1 + X2_2) + 7 * (X3_1 + X3_2 + X3_3) <= 3000;
subject to Time: X1 + (1/2) * (X2_1 + X2_2) + (1/3) * (X3_1 + X3_2 + X3_3) <= 700;

subject to X1_maximum: X1 <= 300;
subject to X2_maximum: (X2_1 + X2_2) <= 300;
subject to X3_maximum: (X3_1 + X3_2 + X3_3) <= 250;

subject to X2_1_max: X2_1 <= 200;

subject to X3_1_min: X3_1 >= 100*y1;
subject to X3_1_max: X3_1 <= 100;

subject to X3_2_min: X3_2 >= 50*y2;
subject to X3_2_max: X3_2 <= 50;

subject to X3_3_min: X3_3 >= 0;
subject to X3_3_max: X3_3 <= 100*y2;
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Solution

- The optimal value of profit was found at 25,180$
- The optimum units produced for Models I, II and III are listed below:
  - Model I = 282
  - Model II = 61
  - Model III = 250

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CONCLUSION

In the global economy, proper organisation and planning of production are vital to retain the competitive edge of companies. Production planning is a key issue in the industry, and several interesting optimization problems can be studied. Computer-based optimisation techniques are the best means of obtaining viable solutions, but until now the linear (LP) and mixed integer programs (MILP) developed have been able to deal only with simple problems. Most of researches existing in the literature have been focused on applying optimization techniques or developing efficient heuristic approaches to overcome issues available in material requirement planning (MRP) context in order to generate a feasible production plan. The more important larger problems have generally been solved using ad-hoc heuristics which often produce incomplete and less satisfactory solutions. Deterministic modelling in forms of an integrated approach of the hybrid simulation/analytical modelling is also sometimes used to derive the advantages of both simulation and analytical modelling through a unique system. The obtained production plan can be simultaneously both mathematically optimal and practically feasible. In more complex multi-product multi-period production planning tasks, probabilistic modelling and fuzzy control is also applied. Thus, various model extensions are possible which adds to the complexity of algorithm development and helps attain more accurate optimization results, however for simplicity reasons often several assumptions need to be made as was applied in our case study. Today, the development of new algorithms, software and hardware is leading to the provision of mathematical applications and tools which allow the solution of these larger problems in acceptable times and helps improve efficiency of the manufacturing industry greatly.

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