

# Introduction to GLMs

## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
	Configuring R . . . . .	1
1.1	Welcome . . . . .	2
1.2	What you should already know . . . . .	2
1.2.1	Epi 202: probability models . . . . .	2
1.2.2	Epi 203: inference for one or several homogenous populations . . . . .	3
1.2.3	Stat 108: linear regression models . . . . .	3
1.3	What we will cover in this course . . . . .	4
1.4	Motivations for regression models . . . . .	4
1.4.1	Example: Adelie penguins . . . . .	4
1.4.2	Linear regression . . . . .	4
1.4.3	Curved regression lines . . . . .	5
1.4.4	Multiple regression . . . . .	6
1.4.5	Modeling non-Gaussian outcomes . . . . .	8
1.4.6	Why don't we use linear regression? . . . . .	9
1.4.7	Zoom out . . . . .	10
1.4.8	log transformation of dose? . . . . .	11
1.4.9	Logistic regression . . . . .	12
1.5	Structure of regression models . . . . .	12

## 1 Introduction

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### Configuring R

Functions from these packages will be used throughout this document:

```
library(conflicted) # check for conflicting function definitions
# library(printr) # inserts help-file output into markdown output
library(rmarkdown) # Convert R Markdown documents into a variety of formats.
library(pander) # format tables for markdown
library(ggplot2) # graphics
library(ggfortify) # help with graphics
library(dplyr) # manipulate data
library(tibble) # `tibble`'s extend `data.frame`'s
library(magrittr) # `">%>%` and other additional piping tools
library(haven) # import Stata files
library(knitr) # format R output for markdown
library(tidyr) # Tools to help to create tidy data
library(plotly) # interactive graphics
library(dobson) # datasets from Dobson and Barnett 2018
library(parameters) # format model output tables for markdown
library(haven) # import Stata files
```

```

library(latex2exp) # use LaTeX in R code (for figures and tables)
library(fs) # filesystem path manipulations
library(survival) # survival analysis
library(survminer) # survival analysis graphics
library(KMsurv) # datasets from Klein and Moeschberger
library(parameters) # format model output tables for
library(webshot2) # convert interactive content to static for pdf
library(forcats) # functions for categorical variables ("factors")
library(stringr) # functions for dealing with strings
library(lubridate) # functions for dealing with dates and times

```

Here are some R settings I use in this document:

```

rm(list = ls()) # delete any data that's already loaded into R

conflicts_prefer(dplyr::filter)
ggplot2::theme_set(
  ggplot2::theme_bw() +
    # ggplot2::labs(col = "") +
  ggplot2::theme(
    legend.position = "bottom",
    text = ggplot2::element_text(size = 12, family = "serif")))

knitr::opts_chunk$set(message = FALSE)
options('digits' = 6)

panderOptions("big.mark", ",")
pander::panderOptions("table.emphasize.rownames", FALSE)
pander::panderOptions("table.split.table", Inf)
conflicts_prefer(dplyr::filter) # use the `filter()` function from dplyr() by default
legend_text_size = 9
run_graphs = TRUE

```

## 1.1 Welcome

Welcome to Epidemiology 204: Quantitative Epidemiology III (Statistical Models).

Epi 204 is a course on **regression modeling**.

## 1.2 What you should already know

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### Warning

Epi 202, Epi 203, and Sta 108 are prerequisites for this course. If you haven't passed one of these courses, talk to me ASAP.

### 1.2.1 Epi 202: probability models

- Probability distributions
  - binomial
  - Poisson
  - Gaussian
  - exponential
  
- Characteristics of probability distributions

- Mean, median, mode, quantiles
  - Variance, standard deviation, overdispersion
- 

- Characteristics of samples
  - independence, dependence, covariance, correlation
  - ranks, order statistics
  - identical vs nonidentical distribution (homogeneity vs heterogeneity)
  - Laws of Large Numbers
  - Central Limit Theorem for the mean of an iid sample

### 1.2.2 Epi 203: inference for one or several homogenous populations

- the maximum likelihood inference framework:
  - likelihood functions
  - log-likelihood functions
  - score functions
  - estimating equations
  - information matrices
  - point estimates
  - standard errors
  - confidence intervals
  - hypothesis tests
  - p-values
- Hypothesis tests for one, two, and  $>2$  groups:
  - t-tests/ANOVA for Gaussian models
  - chi-square tests for binomial and Poisson models
  - nonparametric tests:
    - \* Wilcoxon signed-rank test for matched pairs
    - \* Mann–Whitney/Kruskal–Wallis rank sum test for  $\geq 2$  independent samples
    - \* Fisher’s exact test for contingency tables
    - \* Cochran–Mantel–Haenszel–Cox log-rank test

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For all of the quantities above, and especially for confidence intervals and p-values, you should know how **both**:

- how to compute them
  - how to interpret them
- 

### 1.2.3 Stat 108: linear regression models

- building models for Gaussian outcomes
  - multiple predictors
  - interactions
- regression diagnostics
- fundamentals of R programming; e.g.:
  - Wickham, Çetinkaya-Rundel, and Grolemund (2023)
  - Dalgaard (2008)
- RMarkdown or Quarto for formatting homework<sup>1</sup>
  - LaTeX for writing math in RMarkdown/Quarto

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<sup>1</sup><https://r4ds.hadley.nz/quarto>

## 1.3 What we will cover in this course

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- Linear (Gaussian) regression models (review and more details)
- Regression models for non-Gaussian outcomes
  - binary
  - count
  - time to event
- Statistical analysis using R

We will start where Epi 203 left off: with linear regression models.

## 1.4 Motivations for regression models

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**Exercise 1.1.** Why do we need regression models?

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*Solution 1.1.*

- when there's not enough data to analyze every subgroup of interest individually
- especially when subgroups are defined using continuous predictors

### 1.4.1 Example: Adelie penguins

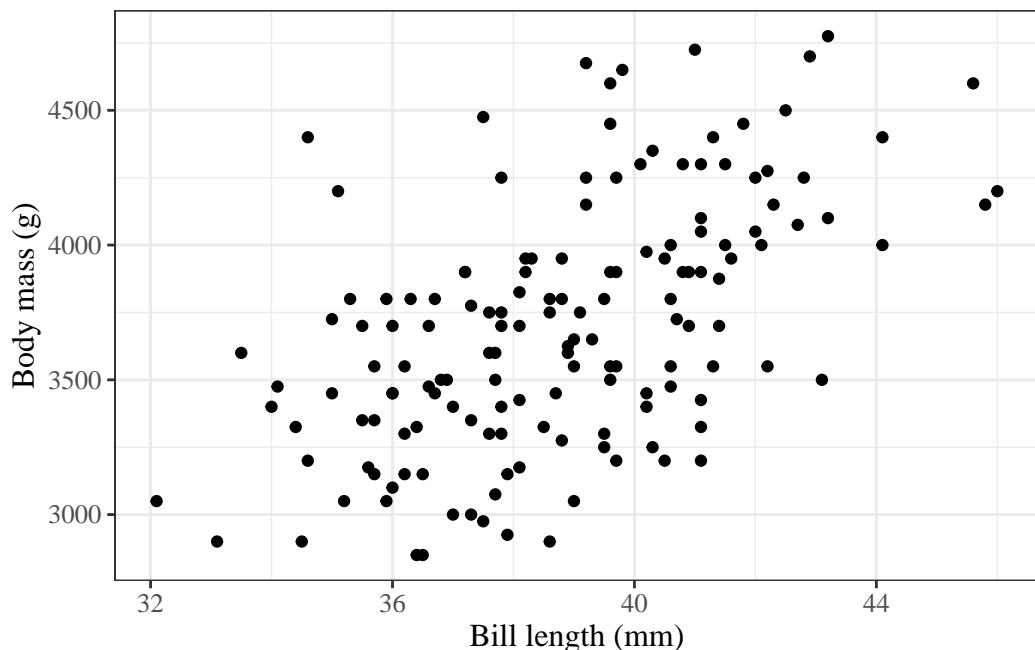


Figure 1: Palmer penguins

### 1.4.2 Linear regression

```
ggpenguins2 <-
  ggpenguins +
  stat_smooth(
    method = "lm",
```

```

    formula = y ~ x,
    geom = "smooth"
)

ggpenguins2 |> print()

```

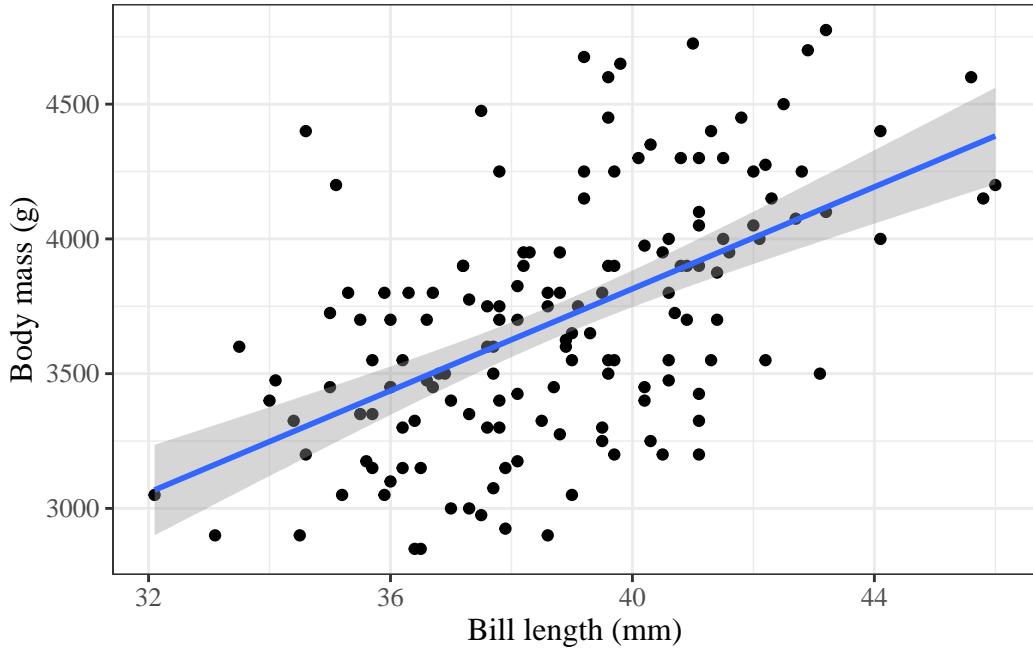


Figure 2: Palmer penguins with linear regression fit

#### 1.4.3 Curved regression lines

```

ggpenguins2 <- ggpenguins +
  stat_smooth(
    method = "lm",
    formula = y ~ log(x),
    geom = "smooth"
  ) +
  xlab("Bill length (mm)") +
  ylab("Body mass (g)")

ggpenguins2

```

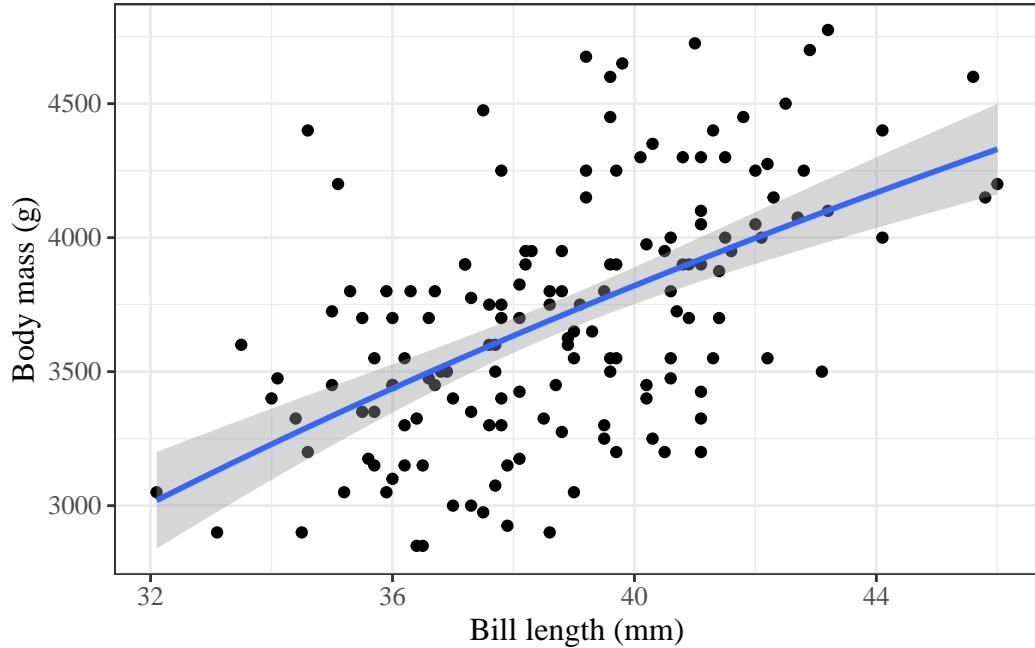


Figure 3: Palmer penguins - curved regression lines

#### 1.4.4 Multiple regression

```
ggpenguins <-
  palmerpenguins::penguins |>
  ggplot(
    aes(
      x = bill_length_mm,
      y = body_mass_g,
      color = species
    )
  ) +
  geom_point() +
  stat_smooth(
    method = "lm",
    formula = y ~ x,
    geom = "smooth"
  ) +
  xlab("Bill length (mm)") +
  ylab("Body mass (g)")
ggpenguins |> print()
```

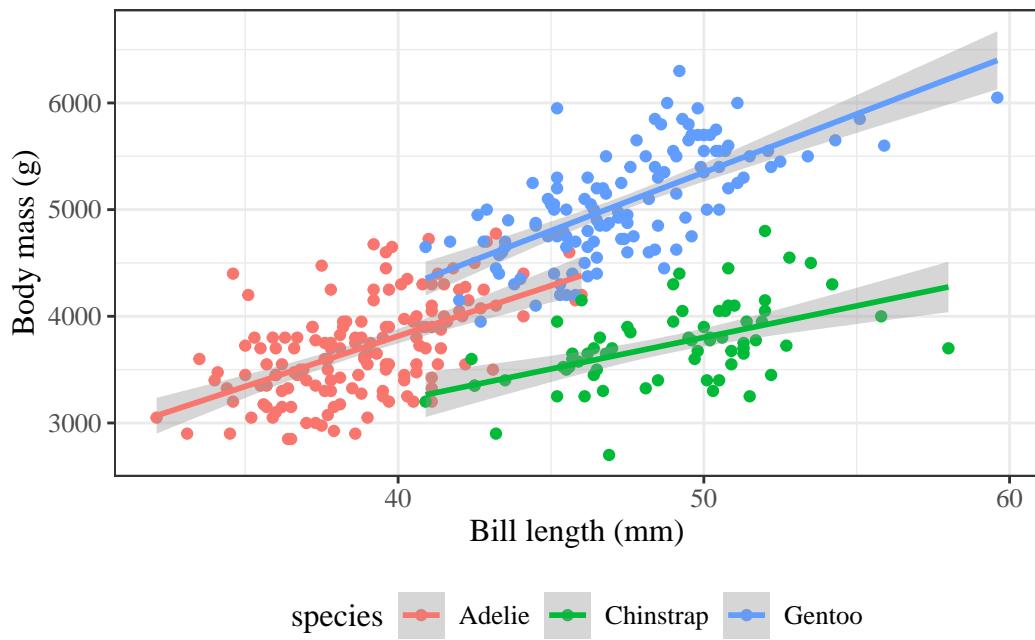


Figure 4: Palmer penguins - multiple groups

#### 1.4.5 Modeling non-Gaussian outcomes

```
library(glmx)
data(BeetleMortality)
beetles <- BeetleMortality |>
  mutate(
    pct = died / n,
    survived = n - died
  )

plot1 <-
  beetles |>
  ggplot(aes(x = dose, y = pct)) +
  geom_point(aes(size = n)) +
  xlab("Dose (log mg/L)") +
  ylab("Mortality rate (%)") +
  scale_y_continuous(labels = scales::percent) +
  # xlab(bquote(log[10]), bquote(CS[2])) +
  scale_size(range = c(1, 2))

print(plot1)
```

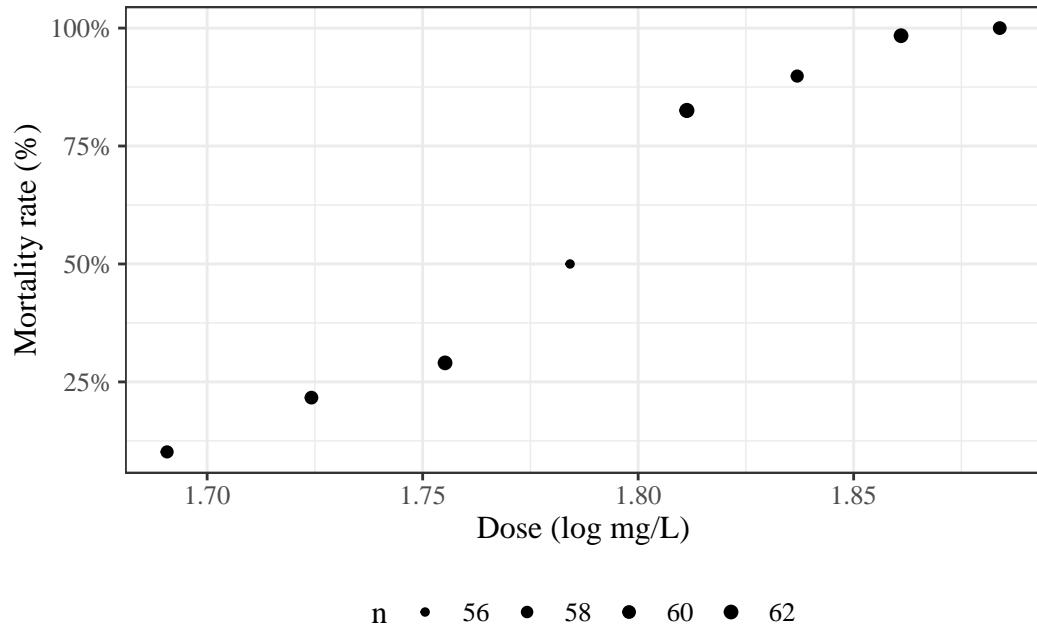


Figure 5: Mortality rates of adult flour beetles after five hours' exposure to gaseous carbon disulphide (Bliss 1935)

#### 1.4.6 Why don't we use linear regression?

```

beetles_long <-
  beetles |>
  reframe(
    .by = everything(),
    outcome = c(
      rep(1, times = died),
      rep(0, times = survived)
    )
  )

lm1 <-
  beetles_long |>
  lm(
    formula = outcome ~ dose,
    data = -
  )

range1 <- range(beetles$dose) + c(-.2, .2)

f_linear <- function(x) predict(lm1, newdata = data.frame(dose = x))

plot2 <-
  plot1 +
  geom_function(fun = f_linear, aes(col = "Straight line")) +
  labs(colour = "Model", size = "")
print(plot2)

```

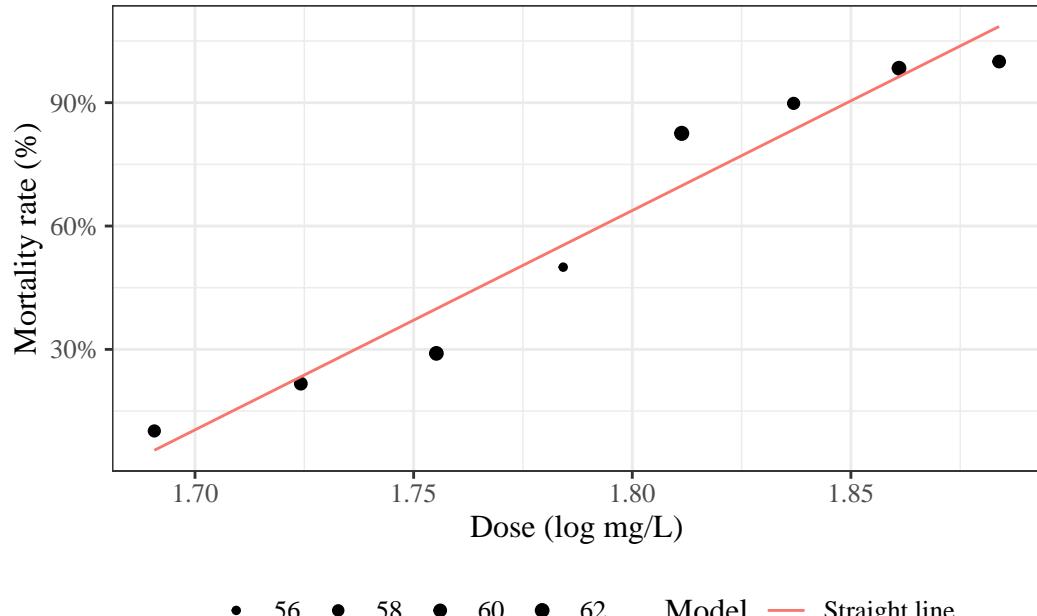


Figure 6: Mortality rates of adult flour beetles after five hours' exposure to gaseous carbon disulphide (Bliss 1935)

#### 1.4.7 Zoom out

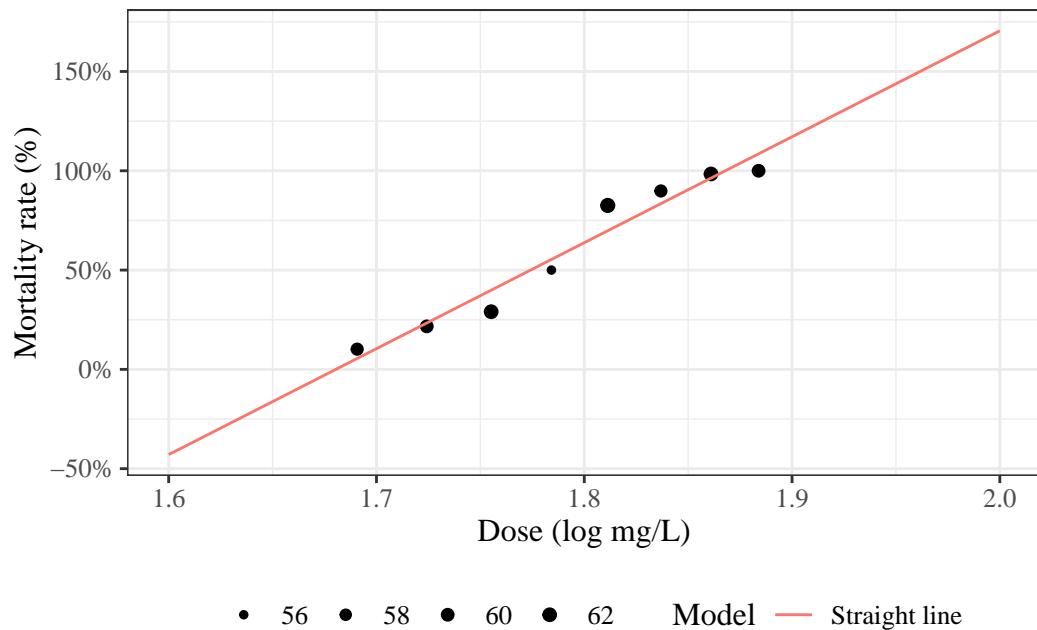


Figure 7: Mortality rates of adult flour beetles after five hours' exposure to gaseous carbon disulphide (Bliss 1935)

#### 1.4.8 log transformation of dose?

```
lm2 <-  
  beetles_long |>  
  lm(formula = outcome ~ log(dose), data = _)  
  
f_linearlog <- function(x) predict(lm2, newdata = data.frame(dose = x))  
  
plot3 <- plot2 +  
  expand_limits(x = c(1.6, 2)) +  
  geom_function(fun = f_linearlog, aes(col = "Log-transform dose"))  
  
print(plot3 + expand_limits(x = c(1.6, 2)))
```

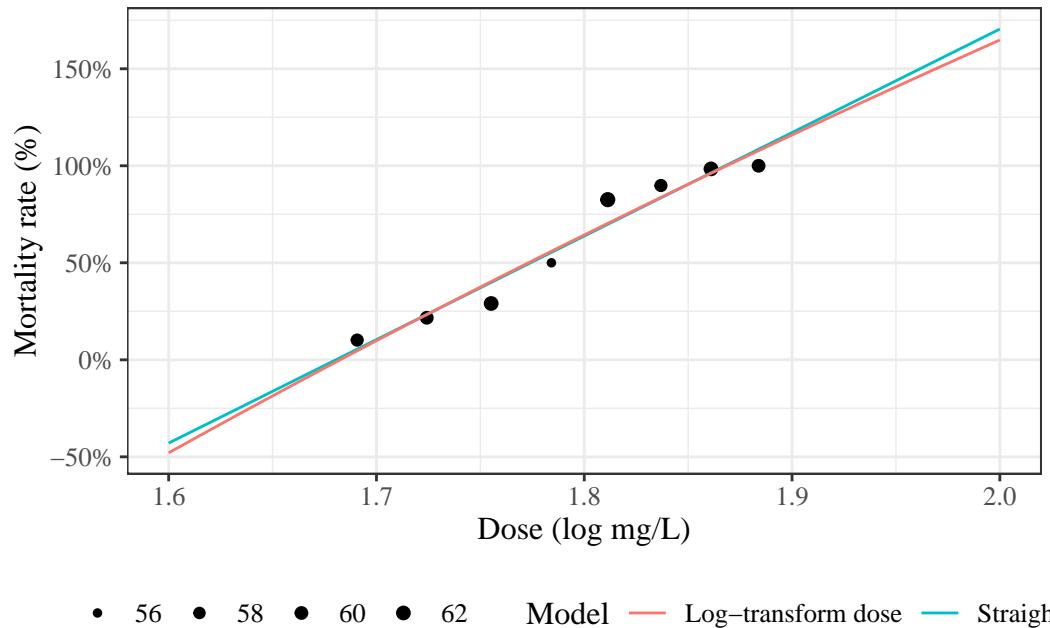


Figure 8: Mortality rates of adult flour beetles after five hours' exposure to gaseous carbon disulphide (Bliss 1935)

### 1.4.9 Logistic regression

```

glm1 <- beetles |>
  glm(formula = cbind(died, survived) ~ dose, family = "binomial")

f <- function(x) {
  glm1 |>
    predict(newdata = data.frame(dose = x), type = "response")
}

plot4 <- plot3 + geom_function(fun = f, aes(col = "Logistic regression"))
print(plot4)

```

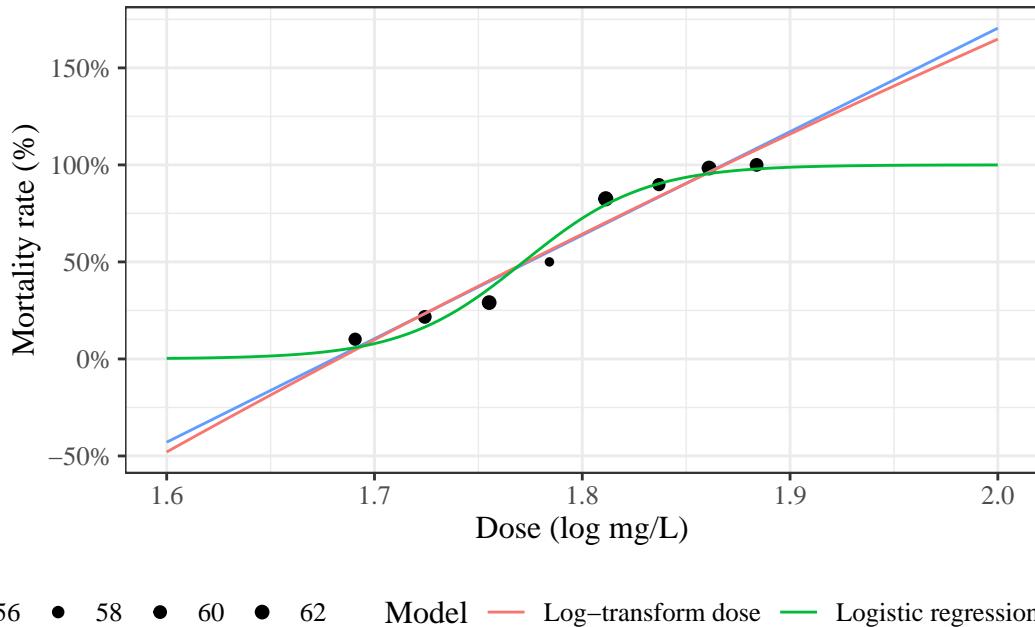


Figure 9: Mortality rates of adult flour beetles after five hours' exposure to gaseous carbon disulphide (Bliss 1935)

## 1.5 Structure of regression models

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**Exercise 1.2.** What is a regression model?

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**Definition 1.1** (Regression model). Regression models are conditional probability distribution models:

$$P(Y|\tilde{X})$$


---

**Exercise 1.3.** What are some of the names used for the variables in a regression model  $P(Y|\tilde{X})$ ?

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**Definition 1.2** (Outcome). The outcome variable in a regression model is the variable whose distribution is being described; in other words, the variable on the left-hand side of the “|” (“pipe”) symbol.

The outcome variable is also called the **response variable**, **regressand**, **predicted variable**, **explained variable**, **experimental variable**, **output variable**, **dependent variable**, **endogenous variables**, **target**, or **label**.

and is typically denoted  $Y$ .

**Definition 1.3** (Predictors). The predictor variables in a regression model are the conditioning variables defining subpopulations among which the outcome distribution might vary.

Predictors are also called **regressors**, **covariates**, **independent variables**, **explanatory variables**, **risk factors**, **exposure variables**, **input variables**, **exogenous variables**, **candidate variables** (Dunn and Smyth (2018)), **carriers** (Dunn and Smyth (2018)), **manipulated variables**, or **features** and are typically denoted  $\tilde{X}$ .<sup>2</sup>

Table 1: Common pairings of terms for variables  $\tilde{X}$  and  $Y$  in regression models  $P(Y|\tilde{X})$ <sup>4</sup>

$\tilde{X}$	$Y$	usual context
input	output	
independent	dependent	
predictor	predicted or response	
explanatory	explained	
exogenous	endogenous	econometrics
manipulated	measured	randomized controlled experiments
exposure	outcome	epidemiology
feature	label or target	machine learning

**Exercise 1.4.** What is the general structure of a generalized linear model?

*Solution 1.2.* Generalized linear models have three components:

1. The **outcome distribution** family:  $p(Y|\mu(\tilde{x}))$
2. The **link function**:  $g(\mu(\tilde{x})) = \eta(\tilde{x})$
3. The **linear component**:  $\eta(\tilde{x}) = \tilde{x} \cdot \beta$

- 
1. The **outcome distribution** family (a.k.a. the **random component** of the model)
    - Gaussian (normal)
    - Binomial
    - Poisson
    - Exponential
    - Gamma
    - Negative binomial
  2. The **linear component** (a.k.a. the *linear predictor* or *linear functional form*) describing how the covariates combine to define subpopulations:

$$\eta(\tilde{x}) \stackrel{\text{def}}{=} \tilde{x}^\top \beta = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots$$

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<sup>2</sup>The “~” (“tilde”) symbol in the notation  $\tilde{X}$  indicates that  $\tilde{X}$  is a vector. See the appendices<sup>3</sup> for a table of notation used in these notes.

<sup>4</sup>adapted from [https://en.wikipedia.org/wiki/Dependent\\_and\\_independent\\_variables#Synonyms](https://en.wikipedia.org/wiki/Dependent_and_independent_variables#Synonyms)

- 
3. The **link function** relating the outcome distribution to the linear component, typically through the mean:
- identity:  $\mu(y) = \eta(\tilde{x})$
  - logit:  $\log\left\{\frac{\mu(y)}{1-\mu(y)}\right\} = \eta(\tilde{x})$
  - log:  $\log\{\mu(y)\} = \eta(\tilde{x})$
  - inverse:  $(\mu(y))^{-1} = \eta(\tilde{x})$
  - clog-log:  $\log\{-\log\{1-\mu(y)\}\} = \eta(\tilde{x})$

Components 2 and 3 together are sometimes called the **systematic component** of the model (for example, in Dunn and Smyth (2018)).

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- Dalgaard, Peter. 2008. *Introductory Statistics with r*. New York, NY: Springer New York. <https://link.springer.com/book/10.1007/978-0-387-79054-1>.
- Dunn, Peter K, and Gordon K Smyth. 2018. *Generalized Linear Models with Examples in R*. Vol. 53. Springer. <https://link.springer.com/book/10.1007/978-1-4419-0118-7>.
- Wickham, Hadley, Mine Çetinkaya-Rundel, and Garrett Grolemund. 2023. *R for Data Science*. "O'Reilly Media, Inc.". <https://r4ds.hadley.nz/>.