

1 Conditional Probability Review

- (a) $1/5$
- (b) $26/50 = 13/25$
- (c) $6/50 = 3/25$
- (d) $P(F*B)/P(B) = 3/25 * 25/13 = 3/13$
- (e) $3/25 * 1/5 = 15/25 = 3/5$
- (f) $3/25 = 3/5 * 3/13 = 3/5 * 1/5$
- (g) $11/46$

2 Linear Regression

"""

PART (a)

I decided to use scikit-learn for this portion
of the homework.

"""

```
import numpy as np
from sklearn.linear_model import LinearRegression
from math import sqrt
def main():
    """
    PART (b)
    The incercept is 4.360802923546112
    The learned theta vector is [-0.13429383  1.84768377 -0.89658481]
    The RMSE value is 0.18970922173315746
    Below is the code i wroe for this
    """
```

```
x = np.array(
    [[3, 9, 2],
     [6, 9, 1],
     [7, 7, 7],
     [8, 6, 4],
     [1, 0, 8]])
y = np.array([19,19,10,11,-3])

theta = np.array([1,0,8]).reshape(1,-1)

model = LinearRegression().fit(x, y)

ans = model.predict(theta)

print(ans)

inter = model.intercept_
c = model.coef_
```

```
print(inter)
print(c)
```

```
fx = []
```

```
for row in x:
    theta = np.array(row).reshape(1,-1)
    fx.append(model.predict(theta))
```

```
i = 0
sigma = 0
while i < 5:
    sigma+= (((fx[i] - y[i])*(fx[i] - y[i]))/5)
    i+=1
rms = sqrt(sigma)
print(rms)
```

```
"""
```

PART (c)

The label for the instance [3,3,5] is 5.01804869

Code below

```
"""
```

```
unlabeled = np.array([3,3,5]).reshape(1,-1)
```

```
ansC = model.predict(unlabeled)
```

```
print(ansC)
```

```
"""
```

PART (D)

The learned theta vector does not change if the rows of x and y are permuted because the data is not interpreted as being order specific so it is the same exact data

Code below

```
"""
```

```
xP = np.array(
    [[6, 9, 1],
     [1, 0, 8],
     [7, 7, 7],
     [3, 9, 2],
     [8, 6, 4]])
```

```

yP = np.array([19,-3,10,19,11])

modelP = LinearRegression().fit(xP, yP)

ansP = modelP.coef_

print(ansP)

```

```

if __name__ == '__main__':
    main()

```

3 More Probability Review

(a) $P(k \text{ tails on first } k \text{ tosses} \mid 1\text{st head on } (k+1)\text{th toss})$
 $= (1-\lambda)^k \cdot \lambda$

(b) $P(\text{HT}) = \# \text{ of tosses needed to get the first head}$
 $E[P(\text{HT})] = \lambda + (1-\lambda)(E[P]+1)$ " $E[p] == E[P(\text{HT})]$ "
 $E[P] = \lambda + E[P] + 1 - \lambda E[P] - \lambda$
 $E[P] - 1 = E[P] - \lambda E[P]$
 $E[p] - 1 - \lambda E[P] = E[p]$
 $E[P] = 1/\lambda$

(c) $P(x \text{ heads} \mid n \text{ tosses}) = \binom{n}{x} \cdot \lambda^x (1-\lambda)^{n-x}$
 $= \sum_{x=0}^n P(x \text{ heads} \mid n \text{ tosses})$
 $E[P(x \mid n)] = n\lambda$ by the linearity of expectation

4 A Continuous Variable plus Baye's Rule

9:50 PM 10/28/2020

$$(a) E[P(x \text{ at midday})] = \int_0^1 x(n+1)x^n dx$$

$$= \int_0^1 x(nx^n + x^n) dx$$

$$= \int_0^1 nx^{n+1} + x^{n+1} dx$$

$$= \int_0^1 nx^{n+1} dx + \int_0^1 x^{n+1} dx$$

$$= n \left(\frac{x^{n+2}}{n+2} \right) + \frac{x^{n+2}}{n+2} \Big|_0^1$$

$$= n \left(\frac{1}{n+2} \right) + \frac{1}{n+2}$$

$$= \frac{n}{n+2} + \frac{1}{n+2} = \boxed{\frac{n+1}{n+2}}$$

(b)

Since we hear no thunder before sunset $x=1$ since the probability of hearing thunder before sunset is $(1-x)$.

• $P(A)$ = expected at midday

$P(B) = 1$ since no thunder before sunset

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{(n+1)x^n \cdot 1}{(n+1)x^n} = 1 \quad ?$$

$$\int_0^1 dx = 1 - 0 = 1$$