Calculus

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0.1 Limits

0.1.1 general ?

Limit as a approaches x. The limit always gives the slope, so with the limit above we get the slope [m] as variable a approaches x.

$$\lim_{a \to x}$$

0.1.2 How to solve

A limit is solvable by pluggin in.

0.1.3 limits in form of derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This is how one can solve a limit, here in case of a derivative: Just plug in as said above, and if that's not possible just do polynomial division instead or add the "reverse" the denominator (the bottom thing of a fraction) I'll show it in a sec.

0.1.4 what one can do with limits:

$$\begin{aligned}
\lim_{x \to a} [f(x) \text{ś} g(x)] &= \lim_{x \to a} f(x) \text{ś} \lim_{x \to a} g(x) \\
\lim_{x \to a} [f(x) * g(x)] &= \lim_{x \to a} f(x) * \lim_{x \to a} g(x) \\
\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] &= \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \lim_{x \to a} g(x) \neq 0 \\
\lim_{x \to a} [f(x)]^n &= \left[\lim_{x \to a} f(x)\right]^n \to \lim_{x \to a} \sqrt[n]{f(x)} \to \sqrt[n]{\lim_{x \to a} f(x)}
\end{aligned}$$

0.2 Derivatives

0.2.1 in general

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

0.2.2 product rule

$$\frac{d}{dx}[f(x) * g(x)] = f'(x) * g(x) + f(x) * g'(x)$$

0.2.3 quotient rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) * f'(x) - f(x) * g'(x)}{[g(x)]^2}$$

0.2.4 derivatives with roots and how to solve them

Ex: $\frac{d}{dx}(\sqrt{3x})$
Rewrite $\sqrt{3x}$ as $(3x)^{\frac{1}{2}}$ using the fact that $\sqrt{y} = y^{\frac{1}{2}}$. Then rewrite $(3x)^{\frac{1}{2}}$ as $3^{\frac{1}{2}}x^{\frac{1}{2}}$ using the rule $(cx)^n = c^n x^n$. Compare $3^{\frac{1}{2}}x^{\frac{1}{2}}$ with the general form ax^b to see that the coefficient is $a = 3^{\frac{1}{2}}$ and the exponent is $b = \frac{1}{2}$. Plug $a = 3^{\frac{1}{2}}$ and $b = \frac{1}{2}$ into the formula $\frac{d}{dx}(ax^b) = bax^{b-1}$ (the bottom thing of a fraction).

$$\frac{d}{dx}(\sqrt{3x}) = \frac{d}{dx}(3^{\frac{1}{2}}x^{\frac{1}{2}}) = (\frac{1}{2})(3^{\frac{1}{2}})x^{\frac{1}{2}-1} = \frac{3^{\frac{1}{2}}}{2}x^{-\frac{1}{2}}$$

0.2.5 derivatives of trigonometric functions

$$\lim_{h\to 0}\frac{\sin(h)}{h} = 1, \lim_{h\to 0}\frac{1-\cos(h)}{h} = 0$$

Find $f'(x)$ for $f(x) = \sin(x)$,
$f(x+h) = \sin(x+h)$
$f(x) = \sin(x)$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \tfrac{f(x+h)-f(x)}{h} \\
\lim_{h\to 0} & \tfrac{\sin(x+h)-\sin(x)}{h} \\
\lim_{h\to 0} & \tfrac{\sin(x)\cos(h)+\cos(xj)\sin(h)}{h} \\
\lim_{h\to 0} & \tfrac{\sin(x)\cos(h)-\sin(x)}{h}\tfrac{\cos(x)\sin(h)}{h} \\
\lim_{h\to 0} & \tfrac{\sin(x)(\cos(h)-1)}{h} + \cos(x)\tfrac{\sin(h)}{h} \\
\lim_{h\to 0} & -\sin(x) * \tfrac{1-\cos(h)}{h} + \cos(x) * \tfrac{\sin(h)}{h} = 0 + \cos(x) * 1 = \cos(x) \\
= & \tfrac{d}{dx}\sin(x) = \cos(x)
\end{aligned}$$

trig derivatives to remember

$$\begin{aligned}
\frac{d}{dx}[\sin x] &= \cos(x) \\
\frac{d}{dx}[\cos x] &= -\sin(x) \\
\frac{d}{dx}[\tan x] &= \sec^2(x) \\
\frac{d}{dx}[\sec x] &= \sec(x)\tan(x) \\
\frac{d}{dx}[-\csc x] &= \csc(x)\cot(x) \\
\frac{d}{dx}[\cot x] &= \csc^2(x)
\end{aligned}$$

$$\frac{d}{dx}\left[\sin^{-1}\right] = \frac{1}{\sqrt{1-x^2}} \text{where} |x| < 1$$
$$\frac{d}{dx}\left[\cos^{-1}\right] = \frac{-1}{\sqrt{1-x^2}} \text{where} |x| < 1$$
$$\frac{d}{dx}\left[\tan^{-1}\right] = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\left[\cot^{-1}\right] = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}\left[\sec^{-1}\right] = \frac{1}{|x|\sqrt{x_2-1}} \text{where} |x| > 1$$
$$\frac{d}{dx}\left[\csc^{-1}\right] = \frac{-1}{|x|\sqrt{x^2-1}} \text{where} |x| > 1$$
$$\frac{d}{dx}[e^{ax}] = ae^{ax}$$
$$\frac{d}{dx}[\ln(ax)] = \frac{1}{x}$$
$$\frac{d}{dx}[\log_b x] = \frac{1}{x \ln b}$$
$$\frac{d}{dx}[b^x] = b^x \ln b$$
$$\frac{d}{dx}[\sin h] = \cos h$$
$$\frac{d}{dx}[\cos h] = \sin h$$
$$\frac{d}{dx}[\tan h] = \sec h^2 = \frac{1}{\cos h^2}$$

Note:-

Things may be missing