

So, i still have to learn \LaTeX and also need to practice writing limits here, so i can also just explain to you how limits work.

A limit basically measures the slope at a curve at a point (of a function), so the tangent. a limit is written.

$$\lim_{n \rightarrow 0}$$

n approaching 0 basically, that means the variable n approaches 0, meaning two points of a secant are getting so close, it could nearly be a tangent. I hope you know tangent/secant, if not just look them up it's pretty easy. Tangent is the curve of a function shown with a line that crosses the function once, a secant crosses a function twice. (that probably makes no sense) this can be used e.g. here:

$$\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

you might know this thing for normally getting the slope of a curve, then written:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

i have no idea how to explain this here: basically the top of the fractions are the same because $f(x)$ is the same as a y , the $+h$ is the difference between y_1 and y_2 in the fractions right above, and the h as a denominator (i hope it's spelled this way) is the difference between the y s which is the same as $x_2 - x_1$. the $\lim_{h \rightarrow 0}$ saying $x \rightarrow h$, means h is getting closer and closer to 0, to the point that it is this close, it could even be zero. Goal is always to plug 0 in, which cant be dont while h is in the denominator. so most times the first step is to plug the values and functions you have in and cross out the denominator.

EX

Find the equation of the tangent line to $y = \frac{3}{x}$ at $(3,1)$.

Here we already have $x_0 = 3$, $f(x) = \frac{3}{x}$ and $f(x+h) = \frac{3}{3+h}$

$$\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

↓ here you just enter the given variables

$$\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

here you solve the functions

↓

$$\lim_{h \rightarrow 0} \frac{\frac{3}{3+h} - 1}{h}$$

↓

$$\lim_{h \rightarrow 0} \frac{\frac{3}{3+h} - \frac{3+h}{3+h}}{h}$$

the $\frac{3+h}{3+h}$ fraction is exactly the same as 1 because nominator and dominator are the same

↓

$$\lim_{h \rightarrow 0} \frac{\frac{3-3-h}{3+h}}{h}$$

that above is just some algebra, i will not explain it because i would not know how.

↓

$$\lim_{h \rightarrow 0} \frac{\frac{-h}{3+h}}{\frac{h}{1}}$$

again some basic algebra, h and $\frac{h}{1}$ are the same.

↓

$$\lim_{h \rightarrow 0} \frac{\cancel{h}}{3+h} * \frac{1}{\cancel{h}}$$

here just some crossing out

↓

$$\lim_{h \rightarrow 0} \frac{-1}{3+h}$$

now you have the h not alone in the denominator and you can insert the h (=0)

↓

$$\frac{-1}{3+0} = -\frac{1}{3}$$

soo, here you know have the slope of a curve at a point, so exactly what the goal was, go get the tangent equation you now just set in your slope into the line formula.

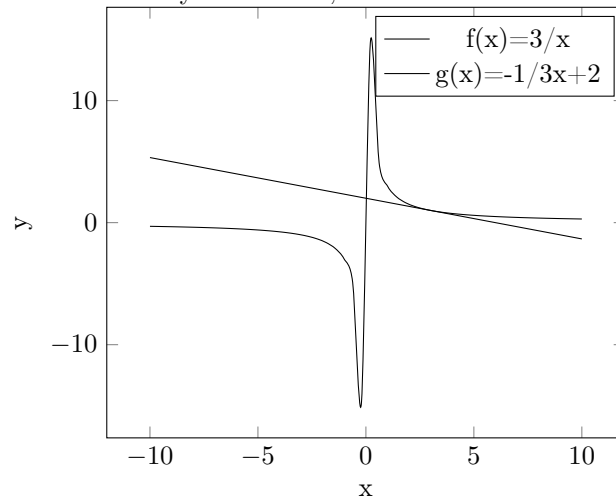
$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{3}(x - 3)$$

$$y - 1 = -\frac{1}{3}x + 1$$

$$y = -\frac{1}{3}x + 2$$

So that's your formula, let me illustrate it:



So that is the exact slope of the curve at the point (3,1). That is all limits are there for, at least i think and hope so. On the pic you see: red graph (the main function), blue graph (the tangent, which was solved for) and lastly green point is the point where the slope is searched.