4. Display the numbers as a matrix.

$$\begin{bmatrix} 0 & 1 & 2 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

on Resources

Interactivity: The adjacency matrix (int-6466)

studyon

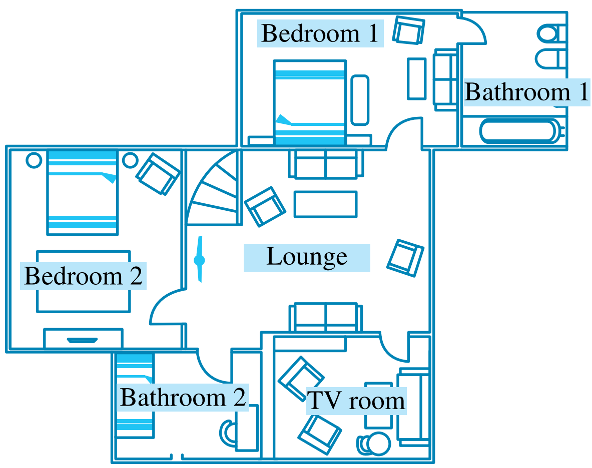
Units 1 & 2 > AOS 3 > Topic 2 > Concepts 1 & 2

Networks, vertices and edges Concept summary and practice questions

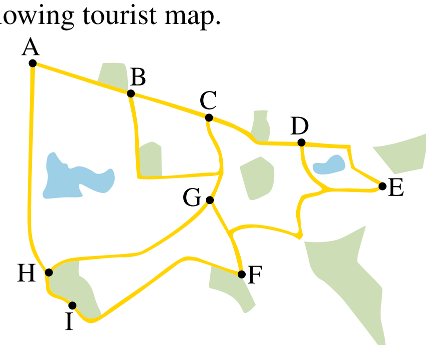
Isomorphic graphs and matrices Concept summary and practice questions

Exercise 5.2 Definitions and terms

1. **WE1** The diagram shows the plan of a floor of a house. Draw a graph to represent the possible ways of travelling between each room of the floor.

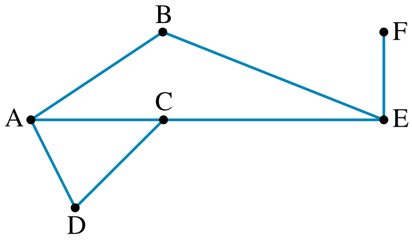


2. Draw a graph to represent the following tourist map.

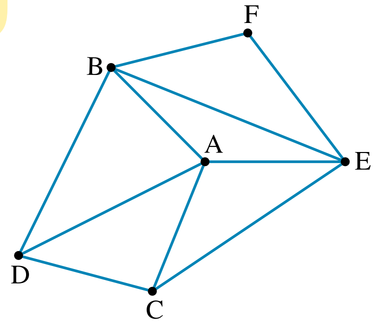


3. WE2 For each of the following graphs, verify that the number of edges is equal to half the sum of the degree of the vertices.

a.

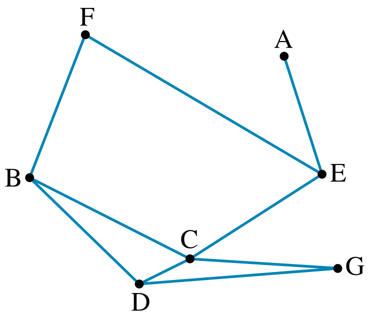


b.

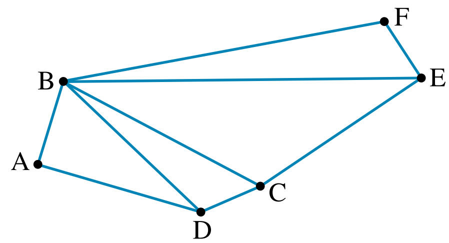


4. For each of the following graphs, verify that the number of edges is equal to half the sum of the degree of the vertices.

a.

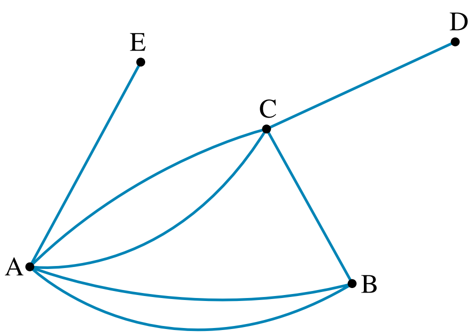


b.

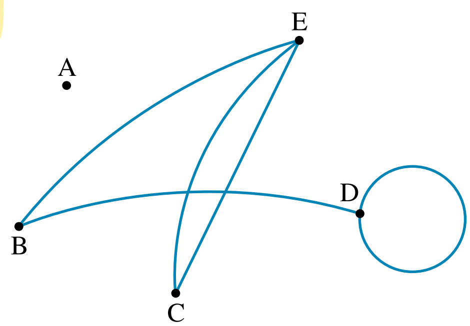


5. Identify the degree of each vertex in the following graphs.

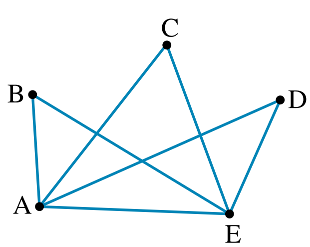
a.



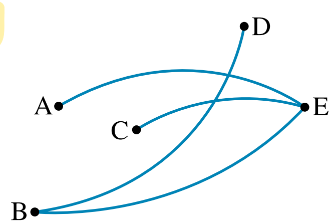
b.



c.

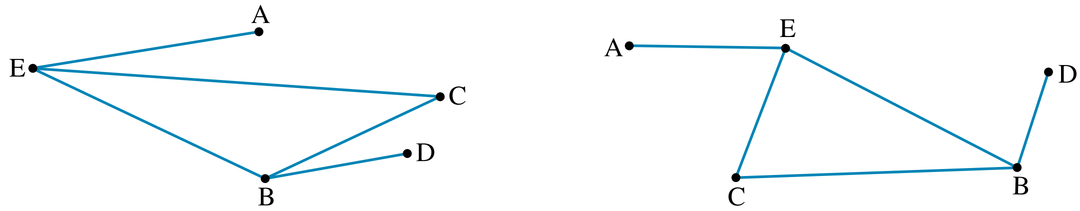


d.



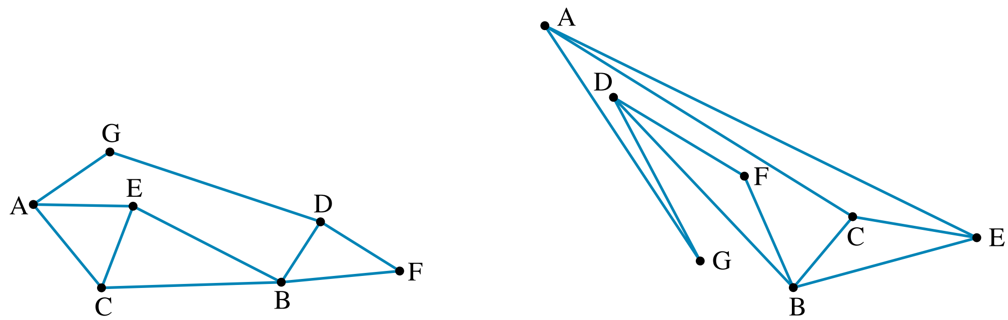
6. WE3 Confirm whether the following pairs of graphs are isomorphic.

a.



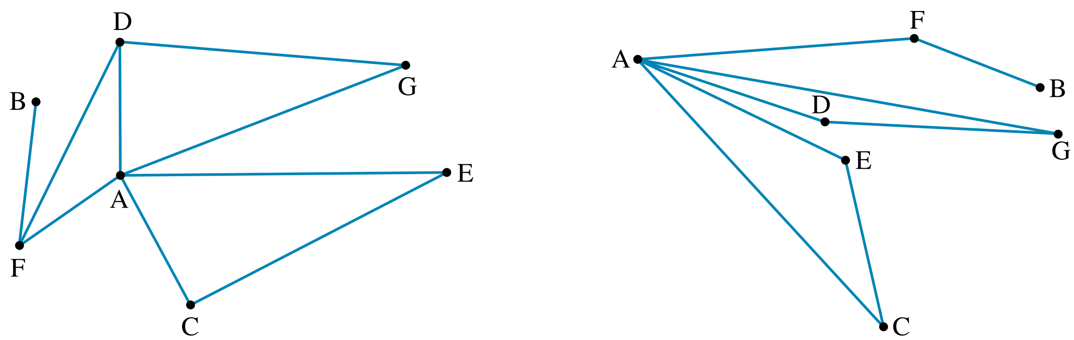
Graph 1 Graph 2

b.



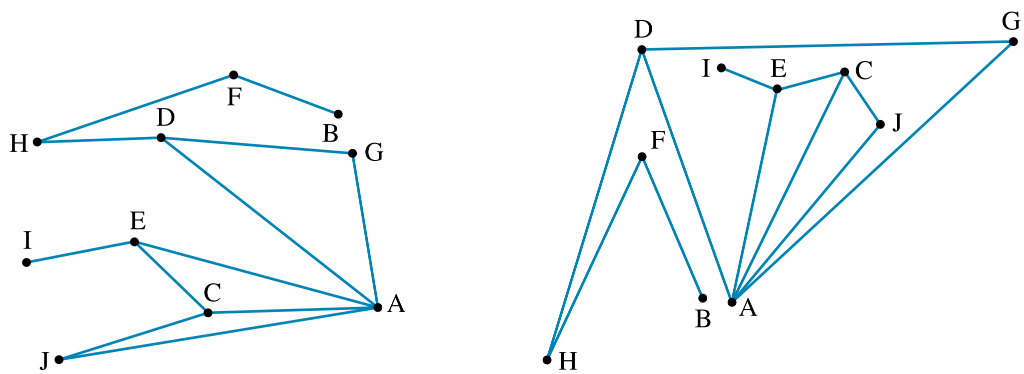
Graph 1 Graph 2

c.



Graph 1 Graph 2

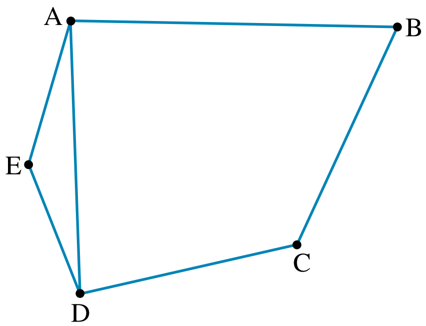
d.



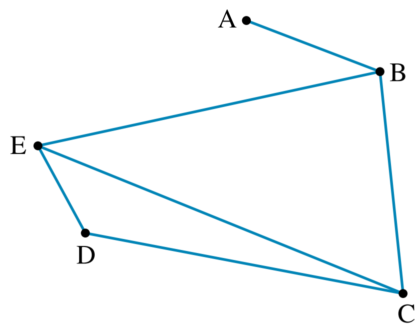
Graph 1 Graph 2

7. Explain why the following pairs of graphs are not isomorphic:

a.

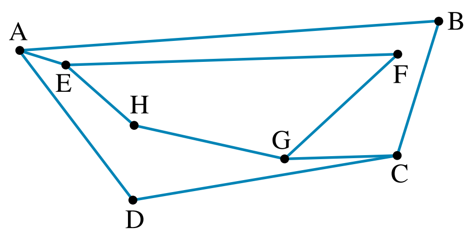


Graph 1

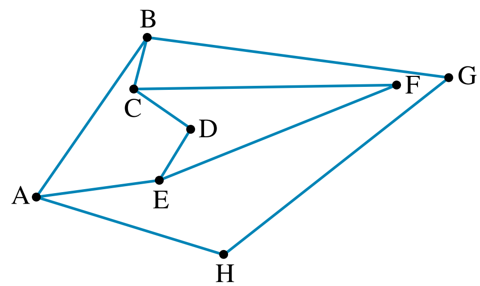


Graph 2

b.

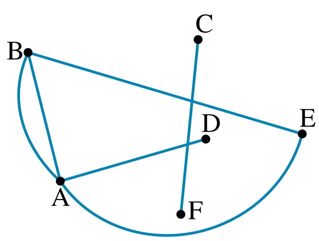


Graph 1

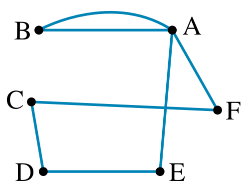


Graph 2

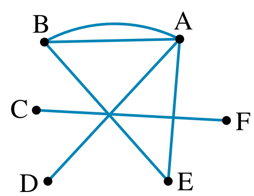
8. Identify pairs of isomorphic graphs from the following.



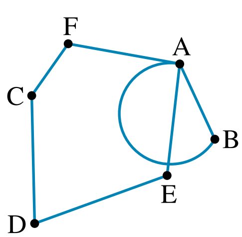
Graph 1



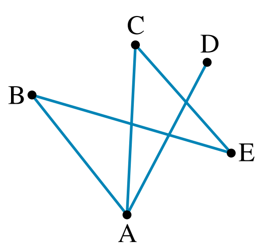
Graph 2



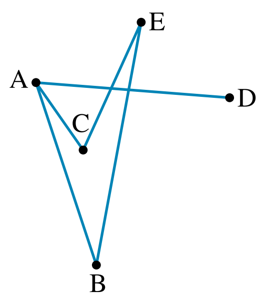
Graph 3



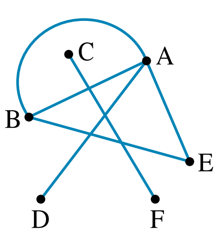
Graph 4



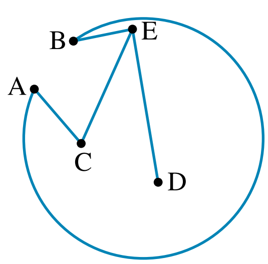
Graph 5



Graph 6

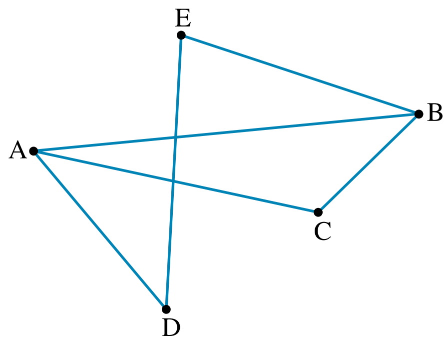


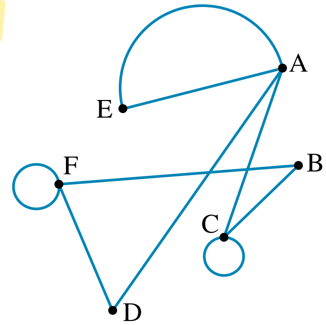
Graph 7

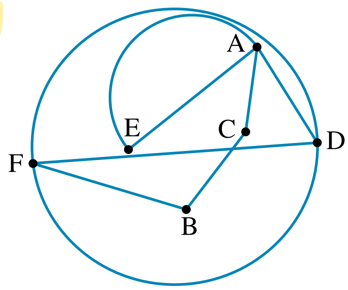


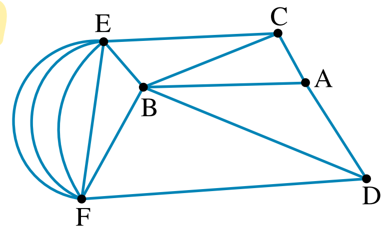
Graph 8

9. WE4 Construct adjacency matrices for the following graphs.

a. 

b. 

c. 

d. 

10. Draw graphs to represent the following adjacency matrices.

a. $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix}$

11. Construct the adjacency matrices for each of the graphs shown in question **13**.

12. Complete the following adjacency matrices.

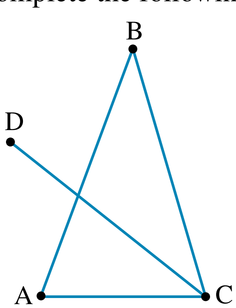
a. $\begin{bmatrix} 0 & 0 & \\ 0 & 2 & 2 \\ 1 & & 0 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 & & 0 \\ & 0 & & \\ 0 & 1 & 0 & 1 \\ & 2 & & 0 \end{bmatrix}$

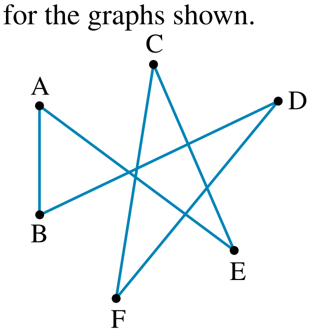
c. $\begin{bmatrix} 0 & & 1 & & 0 \\ 0 & 0 & & & 0 \\ & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & & & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 \\ & & & 0 & 0 \\ 0 & & & 0 & 1 \end{bmatrix}$

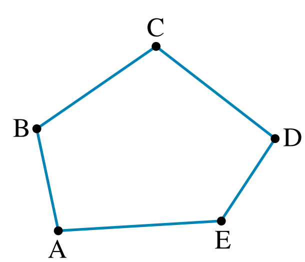
13. Complete the following table for the graphs shown.



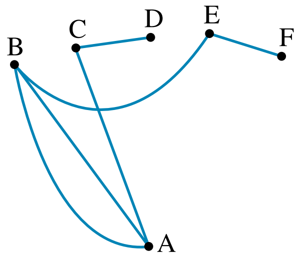
Graph 1



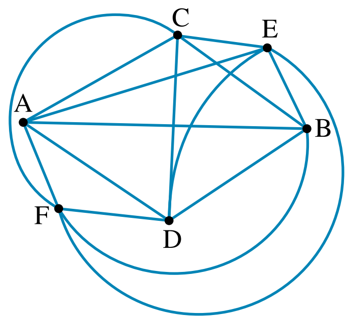
Graph 2



Graph 3



Graph 4



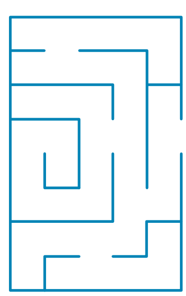
Graph 5

	Simple	Complete	Connected
Graph 1	No	No	Yes
Graph 2			
Graph 3			
Graph 4			
Graph 5			

14. Enter details for complete graphs in the following table.

Vertices	Edges
2	
3	
4	
5	
6	
n	

15. Draw a graph of:
    a. a simple, connected graph with 6 vertices and 7 edges
    b. a simple, connected graph with 7 vertices and 7 edges, where one vertex has degree 3 and five vertices have degree 2
    c. a simple, connected graph with 9 vertices and 8 edges, where one vertex has degree 8.
16. By indicating the passages with edges and the intersections and passage endings with vertices, draw a graph to represent the maze shown in the diagram.


Maze

17. Five teams play a round robin competition.

a. Draw a graph to represent the games played.

b. What type of graph is this?

c. What does the total number of edges in the graph indicate?

18. The diagram shows the map of some of the main suburbs of Beijing.

a. Draw a graph to represent the shared boundaries between the suburbs.

b. Which suburb has the highest degree?

c. What type of graph is this?

19. The map shows some of the main highways connecting some of the states on the west coast of the USA.

Resources

Interactivity: Planar graphs (int-6467)
Interactivity: Euler's formula (int-6468)

studyon

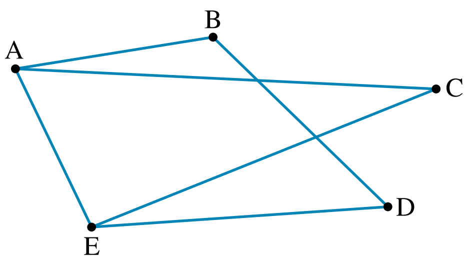
Units 1 & 2 > AOS 3 > Topic 2 > Concept 3

Faces, vertices, edges and Euler's formula Concept summary and practice questions

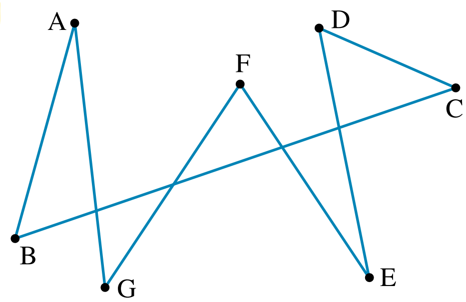
Exercise 5.3 Planar graphs

1. WE5 Redraw the following graphs so that they have no intersecting edges.

a.

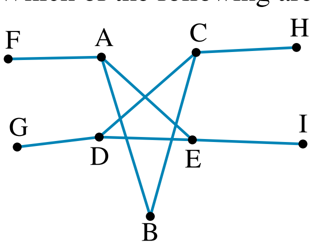


b.

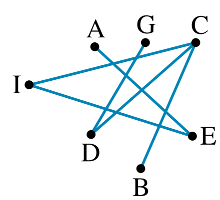


2. WC Which of the following are planar graphs?

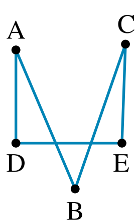
a. A.



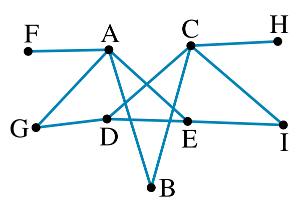
B.



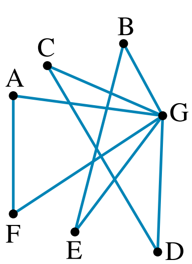
C.



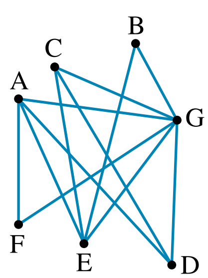
D.



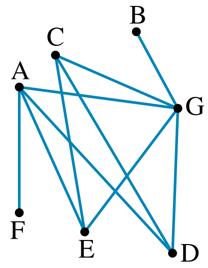
b. A.



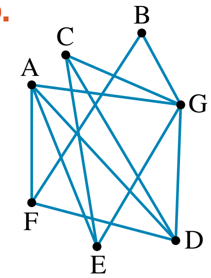
B.



C.

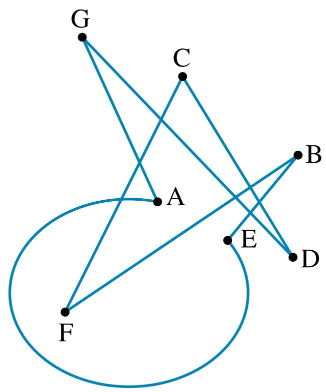


D.

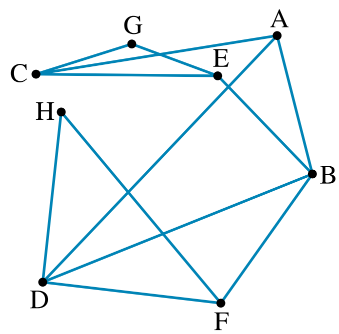


3. Redraw the following graphs to show that they are planar.

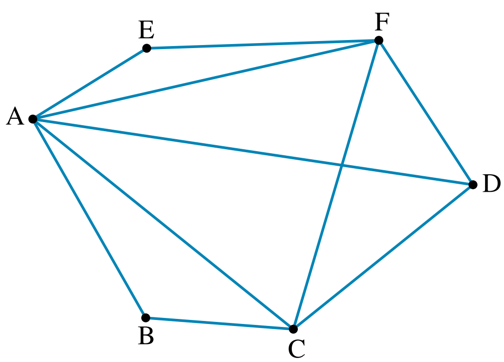
a.



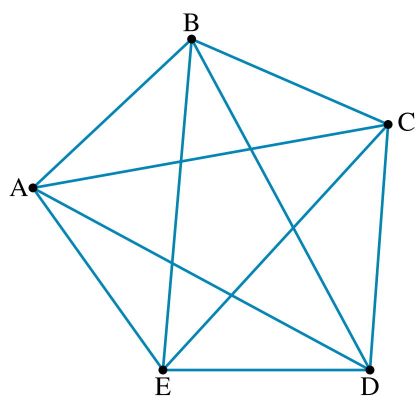
b.



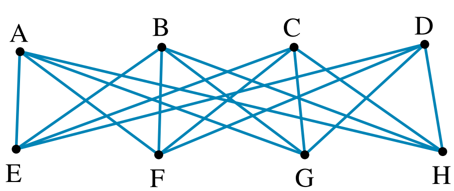
4. Which of the following graphs are not planar?



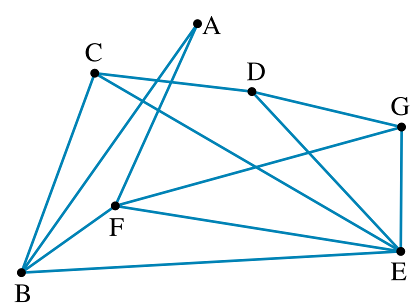
Graph 1



Graph 2



Graph 3



Graph 4

5. **WE6** How many faces will there be for a connected planar graph of:

a. 8 vertices and 10 edges

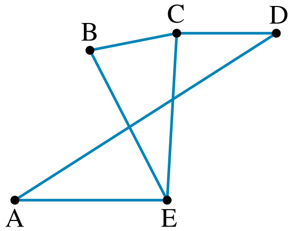
b. 11 vertices and 14 edges?

6. a. For a connected planar graph of 5 vertices and 3 faces, how many edges will there be?

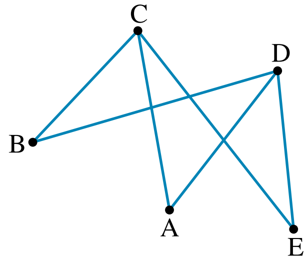
b. For a connected planar graph of 8 edges and 5 faces, how many vertices will there be?

7. For each of the following planar graphs, identify the number of faces:

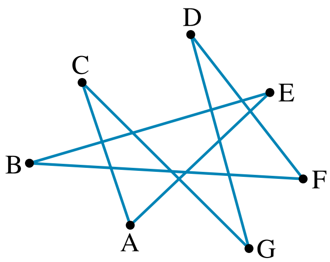
a.



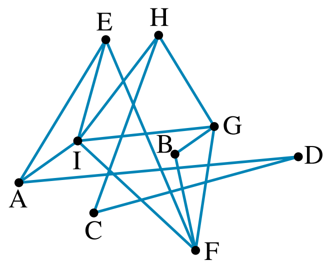
b.



c.



d.



8. Construct a connected planar graph with:

a. 6 vertices and 5 faces

b. 11 edges and 9 faces.

9. Use the following adjacency matrices to draw graphs that have no intersecting edges.

a. $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

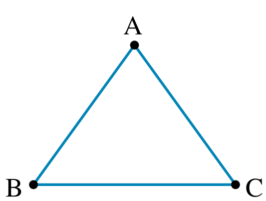
b. $\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

10. For the graphs in question **9**:

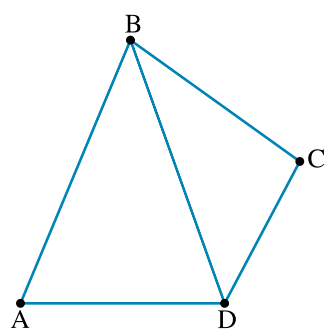
i. identify the number of enclosed faces

ii. identify the maximum number of additional edges that can be added to maintain a simple planar graph.

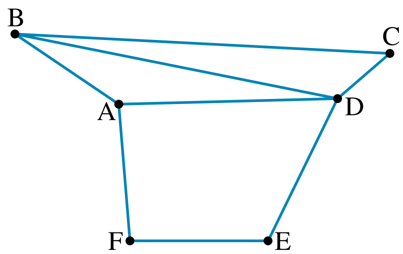
11. a. Use the planar graphs shown to complete the table.



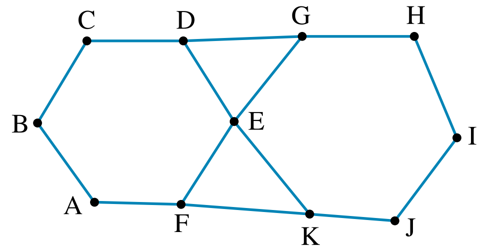
Graph 1



Graph 2



Graph 3

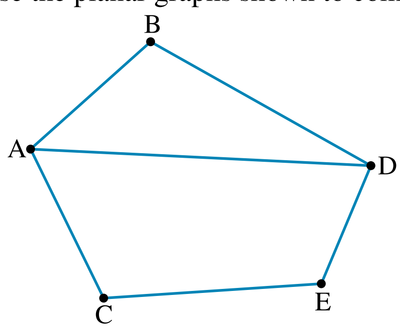


Graph 4

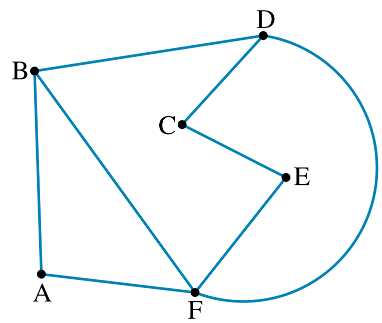
Graph	Total edges	Total degrees
Graph 1		
Graph 2		
Graph 3		
Graph 4		

b. What pattern is evident from the table?

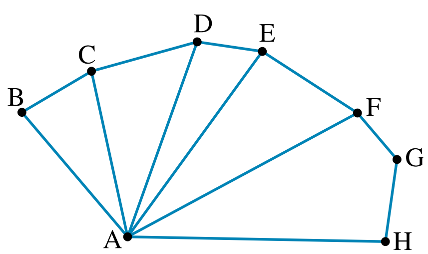
12. a. Use the planar graphs shown to complete the table.



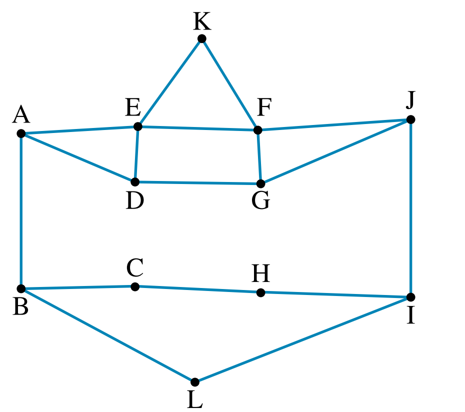
Graph 1



Graph 2



Graph 3

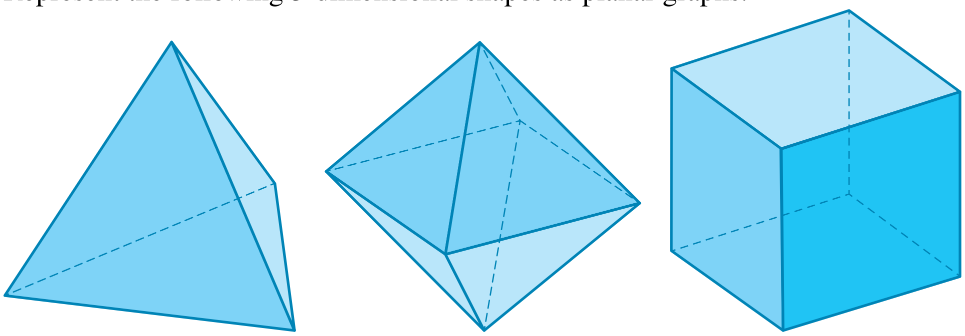


Graph 4

Graph	Total vertices of even degree	Total vertices of odd degree
Graph 1		
Graph 2		
Graph 3		
Graph 4		

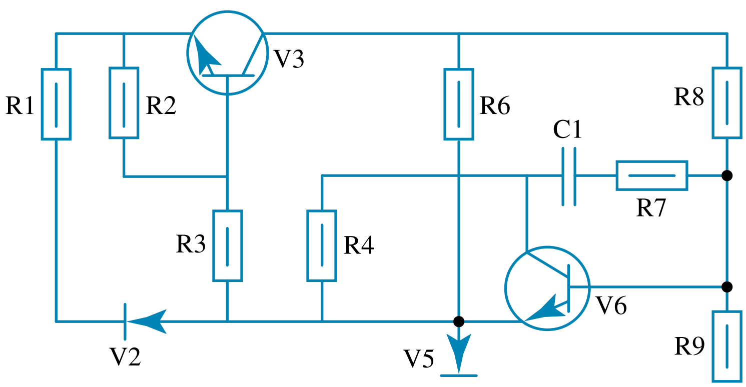
b. Is there any pattern evident from this table?

13. Represent the following 3-dimensional shapes as planar graphs.



Tetrahedron Octahedron Cube

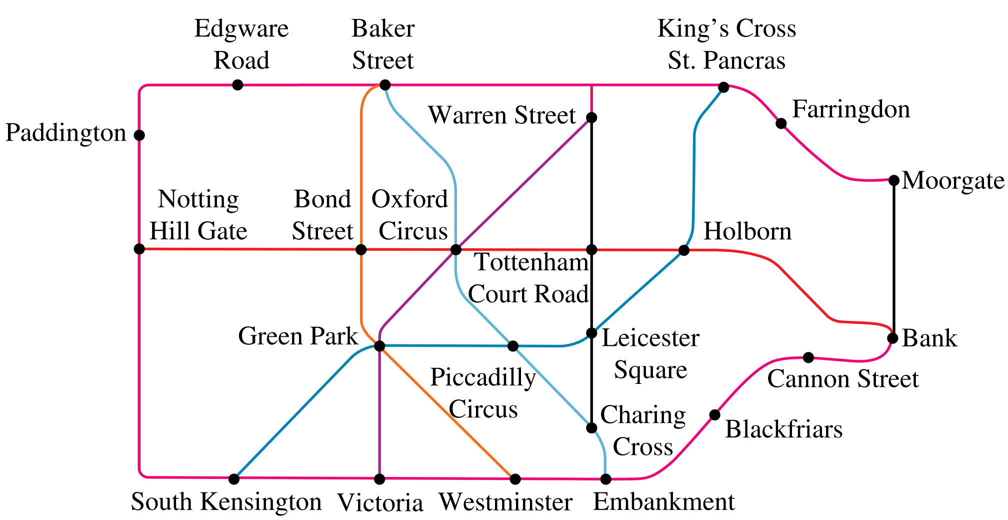
14. A section of an electric circuit board is shown in the diagram.



a. Draw a graph to represent the circuit board, using vertices to represent the labelled parts of the diagram.

b. Is it possible to represent the circuit board as a planar graph?

15. The diagram shows a section of the London railway system.



on Resources

Interactivity: Traversing connected graphs (int-6469)
Interactivity: Euler trails and Hamiltonian paths (int-6470)

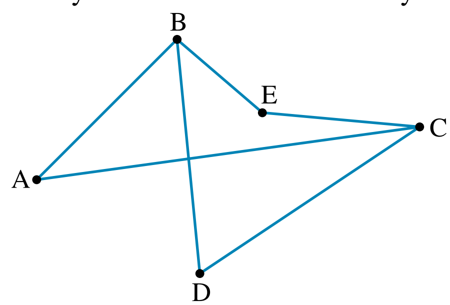
studyon

Units 1 & 2 › AOS 3 › Topic 2 › Concept 4

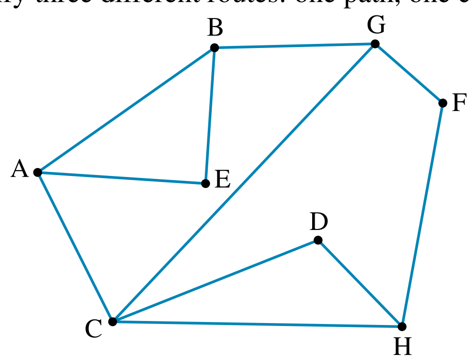
Euler trails and circuits Concept summary and practice questions

Exercise 5.4 Connected graphs

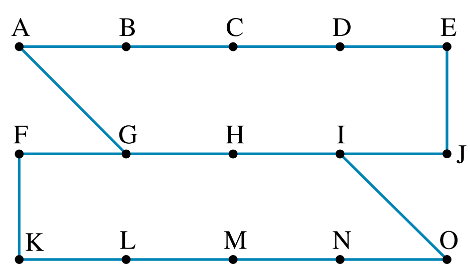
1. WE7 In the following network, identify two different routes: one cycle and one circuit.



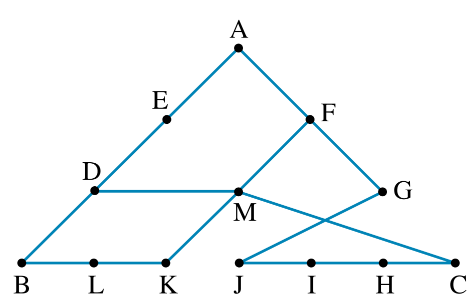
2. In the following network, identify three different routes: one path, one cycle and one circuit.



3. Which of the terms walk, trail, path, cycle and circuit could be used to describe the following routes on the graph shown?
   a. AGHIONMLKFGA
   b. IHGFKLMNO
   c. HIJEDCBAGH
   d. FGHIJEDCBAG

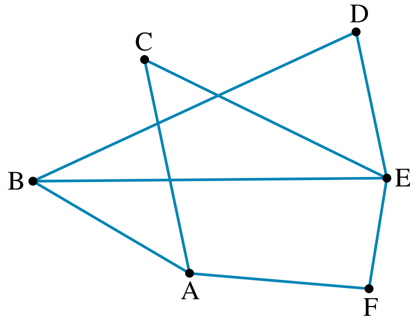


4. Use the following graph to identify the indicated routes.
   a. A path commencing at M, including at least 10 vertices and finishing at D
   b. A trail from A to C that includes exactly 7 edges
   c. A cycle commencing at M that includes 10 edges
   d. A circuit commencing at F that includes 7 vertices

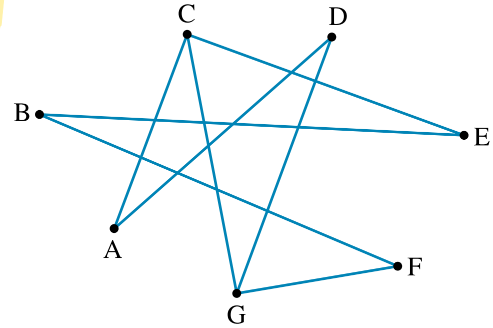


5. **WE8** Identify an Euler trail and a Hamiltonian path in each of the following graphs.

a.

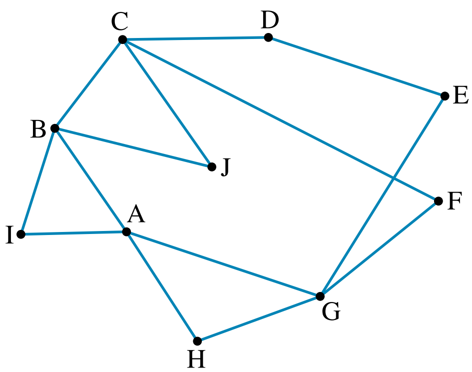


b.

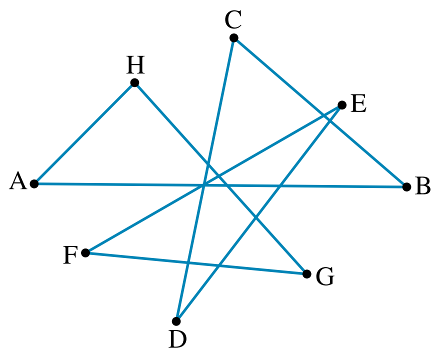


6. Identify an Euler circuit and a Hamiltonian cycle in each of the following graphs, if they exist.

a.

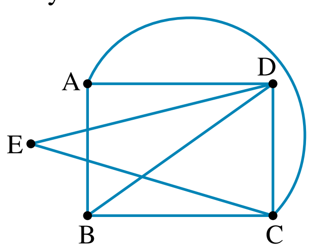


b.

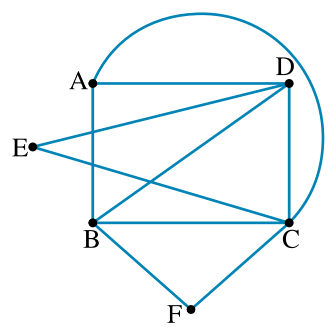


7. a. Identify which of the following graphs have an Euler trail.

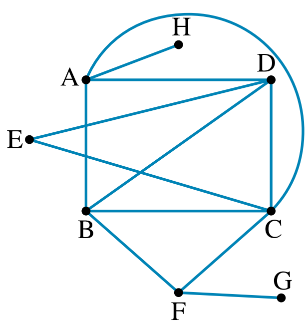
i.



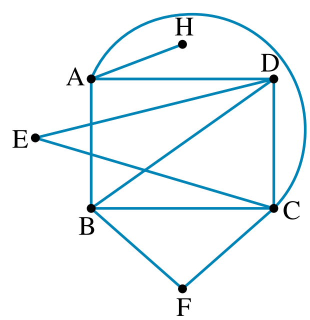
ii.



iii.



iv.



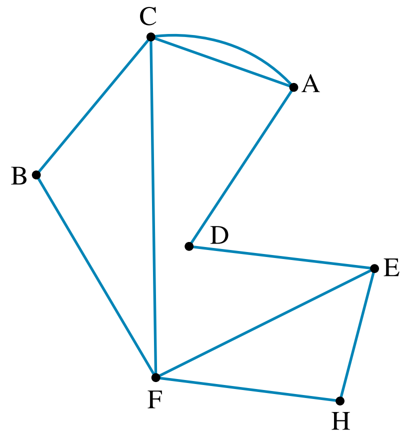
b. Identify the Euler trails found.

8. a. Identify which of the graphs from question **7** have a Hamiltonian cycle.

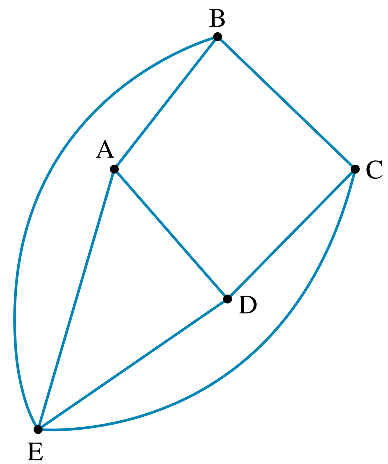
b. Identify the Hamiltonian cycles found.

9. a. Construct adjacency matrices for each of the graphs in question **7**.
   b. How might these assist with making decisions about the existence of Euler trails and circuits, and Hamiltonian paths and cycles?

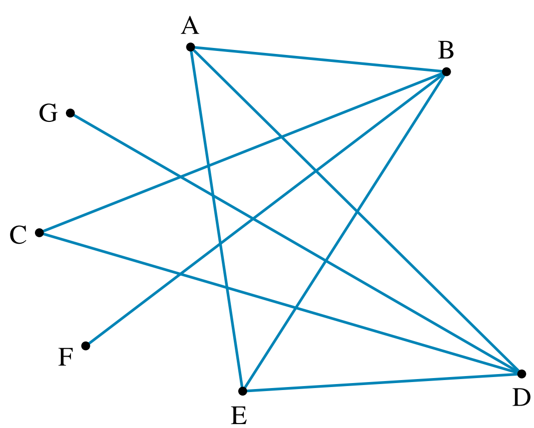
10. In the following graph, if an Euler trail commences at vertex A, at which vertices could it finish?



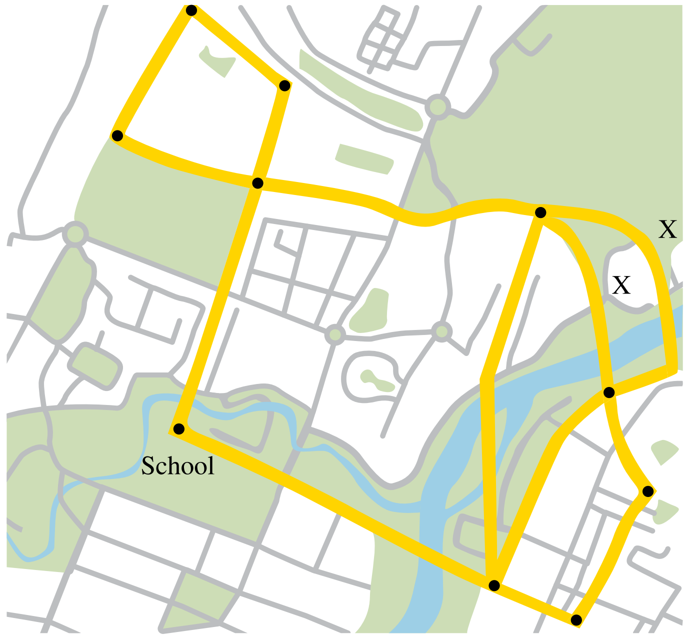
11. In the following graph, at which vertices could a Hamiltonian path finish if it commences by travelling from:
   a. B to E
   b. E to A?



12. In the following graph, other than from G to F, between which 2 vertices must you add an edge in order to create a Hamiltonian path that commences from vertex:
   a. G
   b. F?

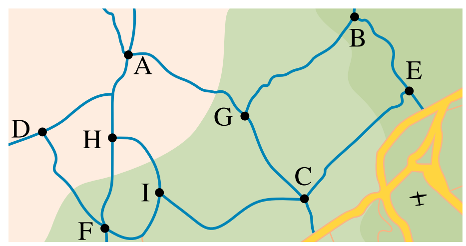


13. On the map shown, a school bus route is indicated in yellow. The bus route starts and ends at the school indicated.



a. Draw a graph to represent the bus route.
b. Students can catch the bus at stops that are located at the intersections of the roads marked in yellow. Is it possible for the bus to collect students by driving down each section of the route only once? Explain your answer.
c. If road works prevent the bus from travelling along the sections indicated by the Xs, will it be possible for the bus to still collect students on the remainder of the route by travelling each section only once? Explain your answer.

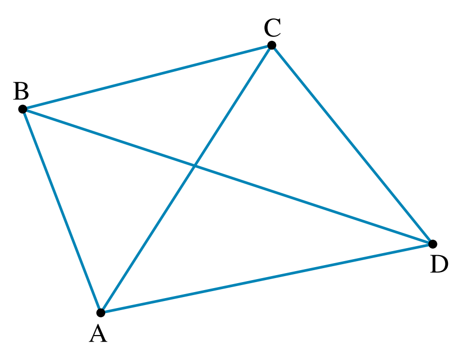
14. The map of an orienteering course is shown. Participants must travel to each of the nine checkpoints along any of the marked paths.



a. Draw a graph to represent the possible ways of travelling to each checkpoint.
b. What is the degree of checkpoint H?
c. If participants must start and finish at A and visit every other checkpoint only once, identify two possible routes they could take.
d. i. If participants can decide to start and finish at any checkpoint, and the paths connecting D and F, H and I, and A and G are no longer accessible, it is possible to travel the course by moving along each remaining path only once. Explain why.
   ii. Identify the two possible starting points.

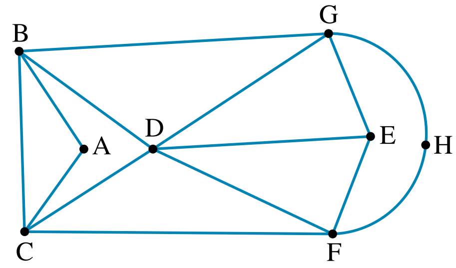
15. a. Use the following complete graph to complete the table to identify all of the Hamiltonian cycles commencing at vertex A.

	Hamiltonian circuit
1.	ABCDA
2.	
3.	
4.	
5.	
6.	



b. Are any other Hamiltonian cycles possible?

16. The graph shown outlines the possible ways a tourist bus can travel between eight locations.



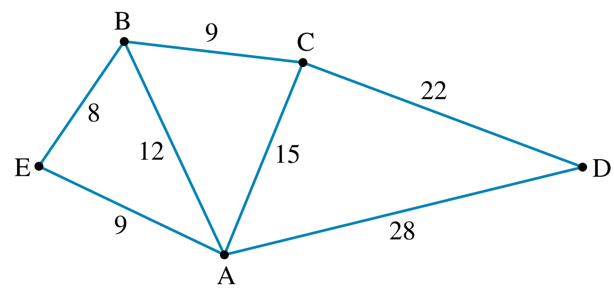
a. If vertex A represents the second location visited, list the possible starting points.
b. If the bus also visited each location only once, which of the starting points listed in part **a** could not be correct?
c. If the bus also needed to finish at vertex D, list the possible paths that could be taken.
d. If instead the bus company decides to operate a route that travelled to each connection only once, what are the possible starting and finishing points?
e. If instead the company wanted to travel to each connection only once and finish at the starting point, which edge of the graph would need to be removed?

5.5 Weighted graphs and trees

5.5.1 Weighted graphs

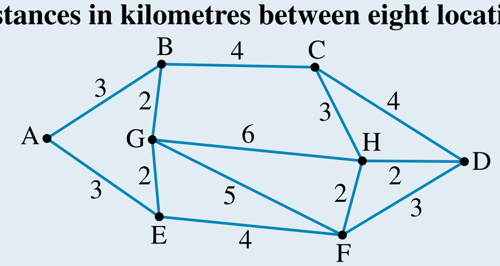
In many applications using graphs, it is useful to attach a value to the edges. These values could represent the length of the edge in terms of time or distance, or the costs involved with moving along that section of the path. Such graphs are known as **weighted graphs**.

Weighted graphs can be particularly useful as analysis tools. For example, they can help determine how to travel through a network in the shortest possible time.



WORKED EXAMPLE 9

The graph represents the distances in kilometres between eight locations.

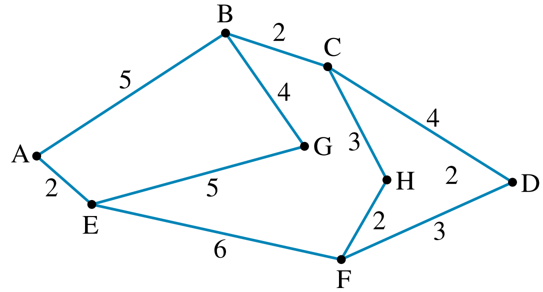


Identify the shortest distance to travel from A to D that goes to all vertices.

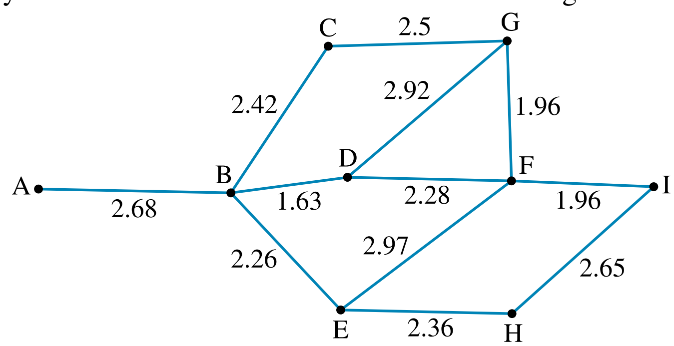
THINK	WRITE
1. Identify the Hamiltonian paths that connect the two vertices.	Possible paths: a. ABGEFHCD b. ABCHGEFD c. AEGBCHFD d. AEFGBCHD e. AEFHGBCD

Exercise 5.5 Weighted graphs and trees

1. WE9 Use the graph to identify the shortest distance to travel from A to D that goes to all vertices.

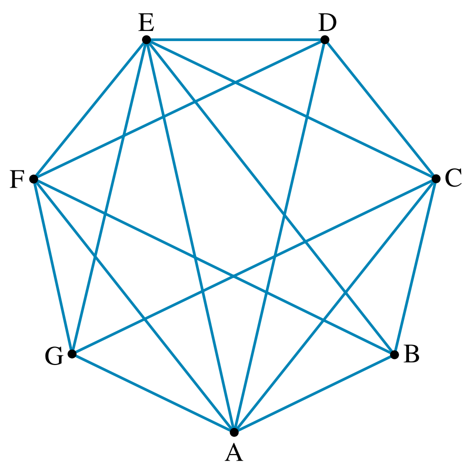


2. Use the graph to identify the shortest distance to travel from A to I that goes to all vertices.

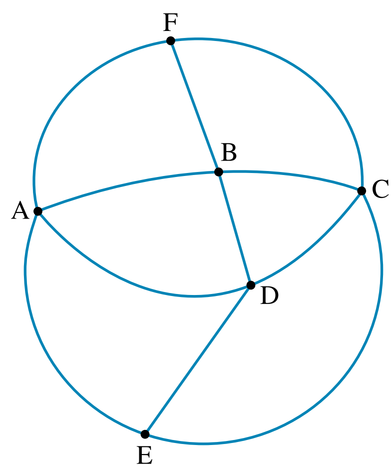


3. Draw three spanning trees for each of the following graphs.

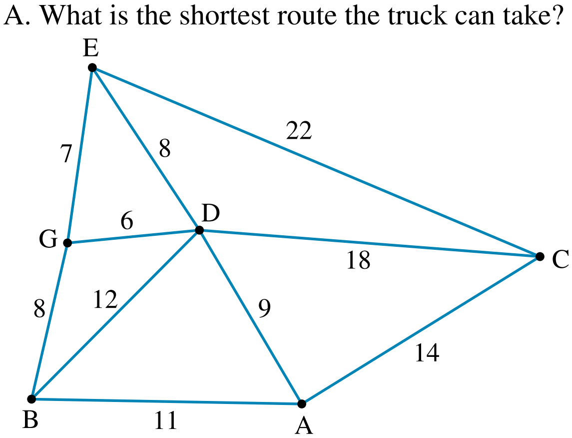
a.



b.

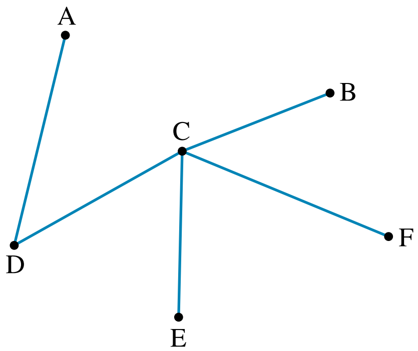


4. A truck starts from the main distribution point at vertex A and makes deliveries at each of the other vertices before returning to A. What is the shortest route the truck can take?

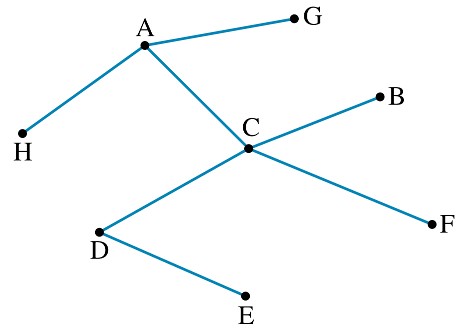


5. For the following trees:
   i. add the minimum number of edges to create an Euler trail
   ii. identify the Euler trail created.

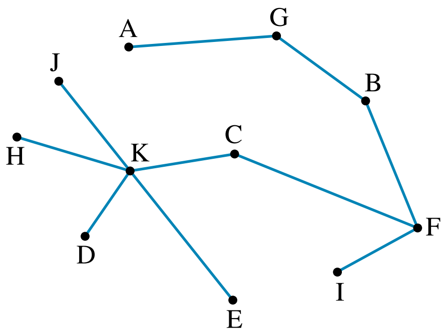
a.



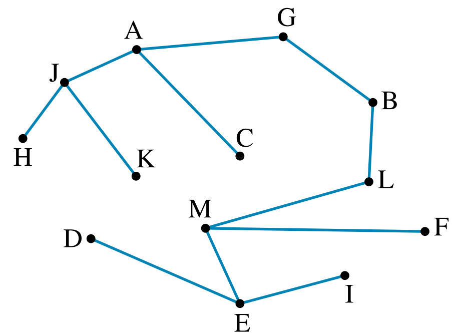
b.



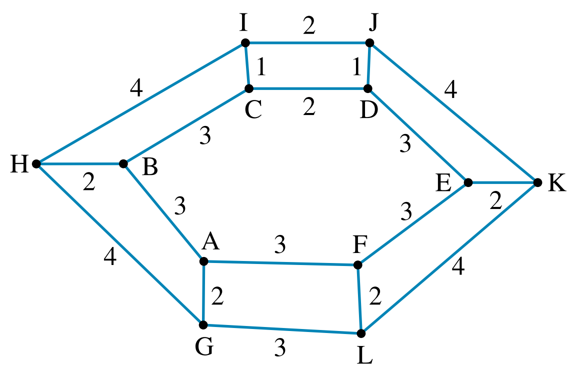
c.



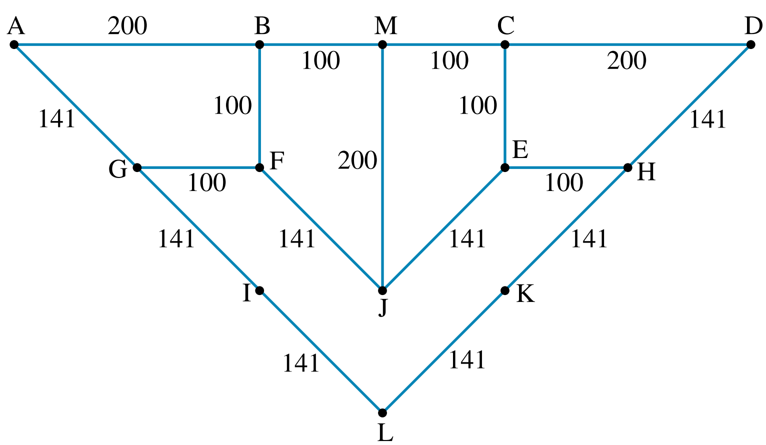
d.



6. WE10 Use Prim's algorithm to identify the minimum spanning tree of the graph shown.

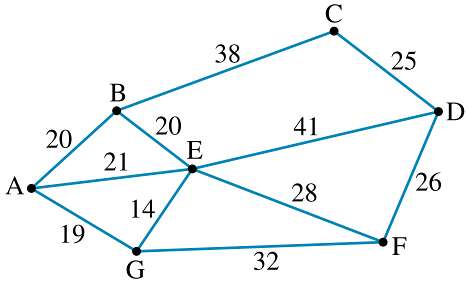


7. Use Prim's algorithm to identify the minimum spanning tree of the graph shown.

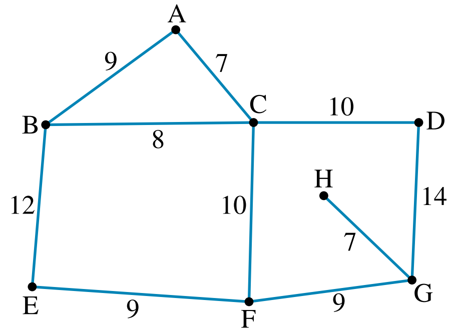


8. Identify the minimum spanning tree for each of the following graphs.

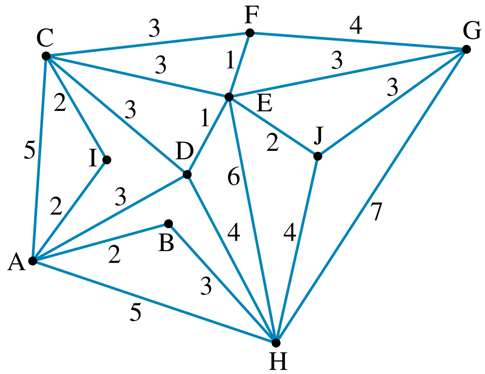
a.



b.



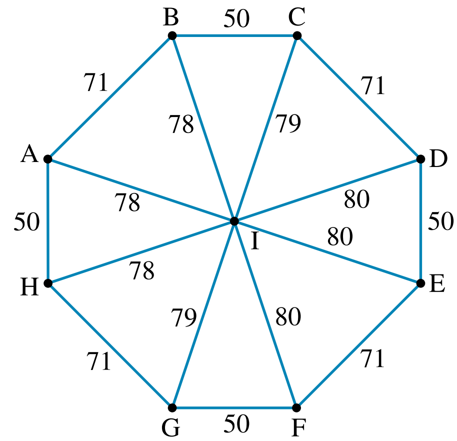
9. Draw diagrams to show the steps you would follow when using Prim's algorithm to identify the minimum spanning tree for the following graph.



10. Part of the timetable and description for a bus route is shown in the table. Draw a weighted graph to represent the bus route.

Bus stop	Description	Time
Bus depot	The northernmost point on the route	7:00 am
Northsea Shopping Town	Reached by travelling south-east along a highway from the bus depot	7:15 am
Highview Railway Station	Travel directly south along the road from Northsea Shopping Town.	7:35 am
Highview Primary School	Directly east along a road from the railway station	7:40 am
Eastend Medical Centre	Continue east along the road from the railway station.	7:55 am
Eastend Village	South-west along a road from the medical centre	8:05 am
Southpoint Hotel	Directly south along a road from Eastend Village	8:20 am
South Beach	Travel south-west along a road from the hotel.	8:30 am

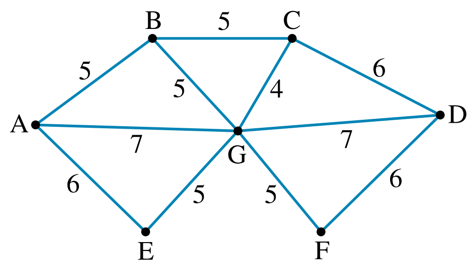
11. Consider the graph shown.



a. Identify the longest and shortest Hamiltonian paths.

b. What is the minimum spanning tree for this graph?

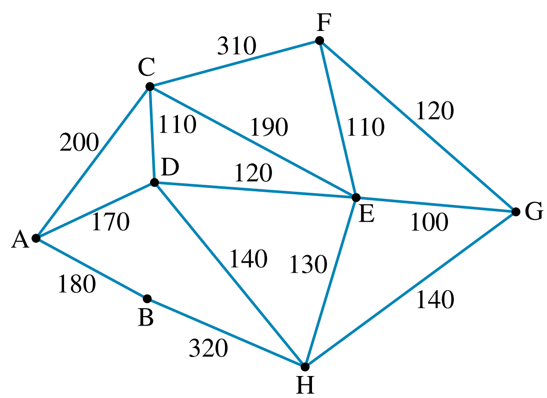
12. Consider the graph shown.



a. If an edge with the highest weighting is removed, identify the shortest Hamiltonian path.

b. If the edge with the lowest weighting is removed, identify the shortest Hamiltonian path.

13. The weighted graph represents the costs incurred by a salesman when moving between the locations of various businesses.



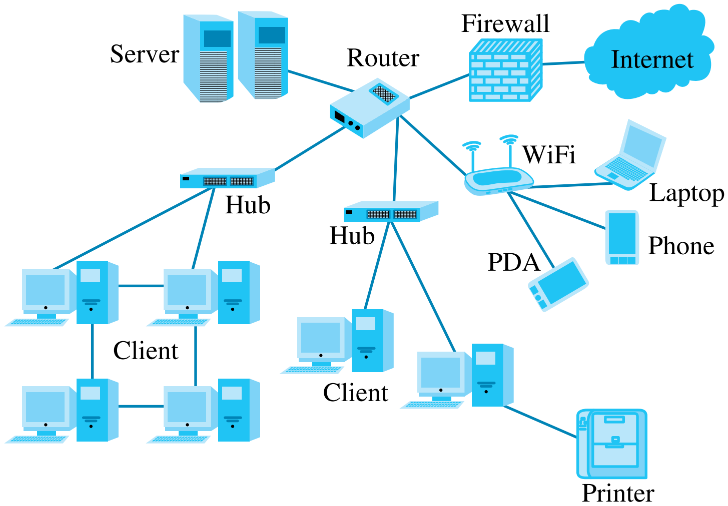
a. What is the cheapest way of travelling from A to G?

b. What is the cheapest way of travelling from B to G?

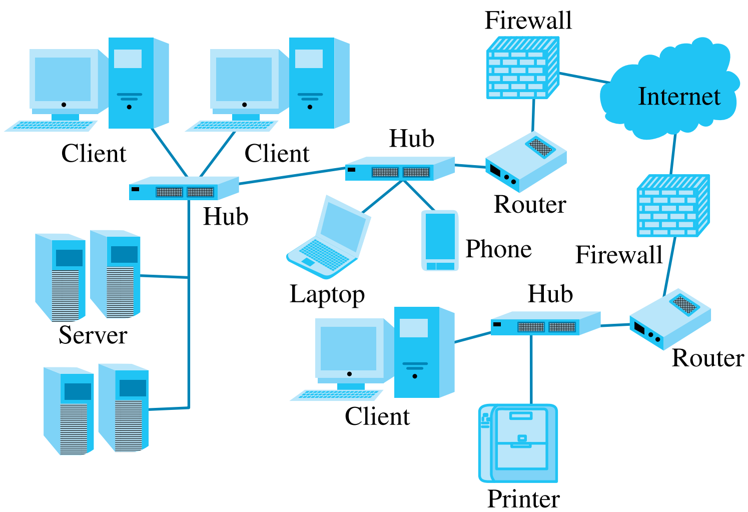
c. If the salesman starts and finishes at E, what is the cheapest way to travel to all vertices?

14. The diagrams show two options for the design of a computer network for a small business.

Option 1



Option 2



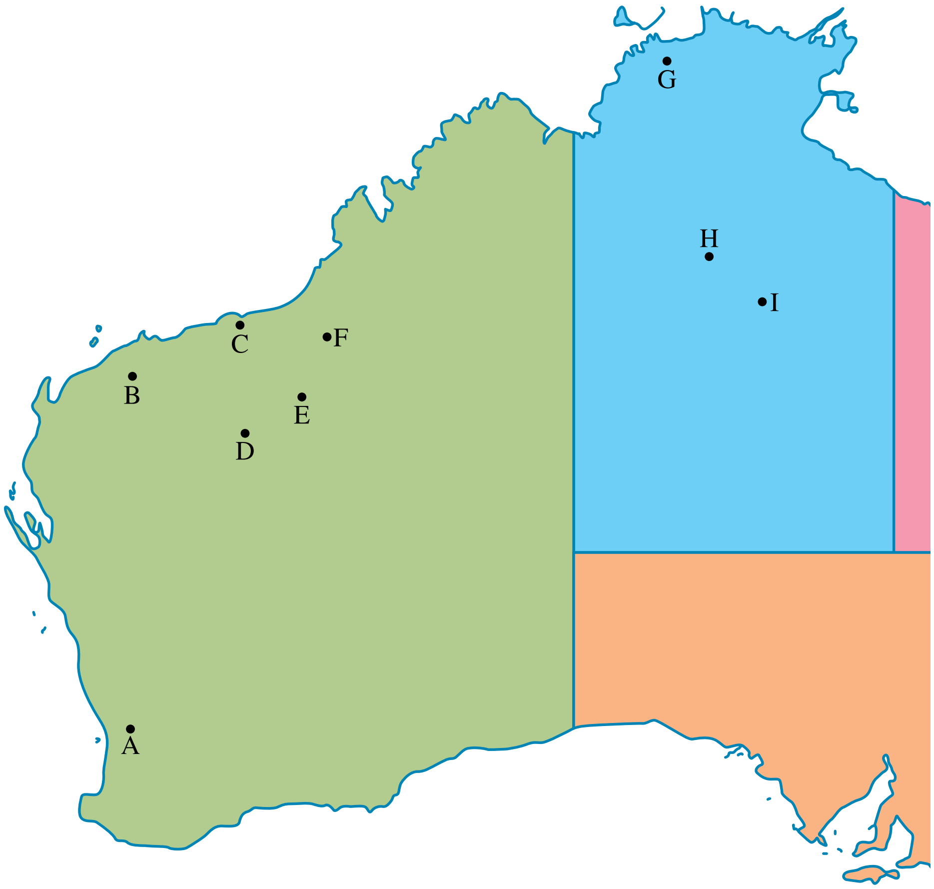
Information relating to the total costs of setting up the network is shown in the following table.

Connected to:	Server	Client	Hub	Router	Firewall	Wifi	Printer
Server			$995	$1050			
Client		$845	$355				$325
Hub			$365	$395			$395
Router	$1050		$395		$395	$395	
Laptop			$295			$325	
Phone			$295			$325	
PDA						$325	
Internet					$855		

a. Use this information to draw a weighted graph for each option.

b. Which is the cheapest option?

15. A mining company operates in several locations in Western Australia and the Northern Territory, as shown on the map.

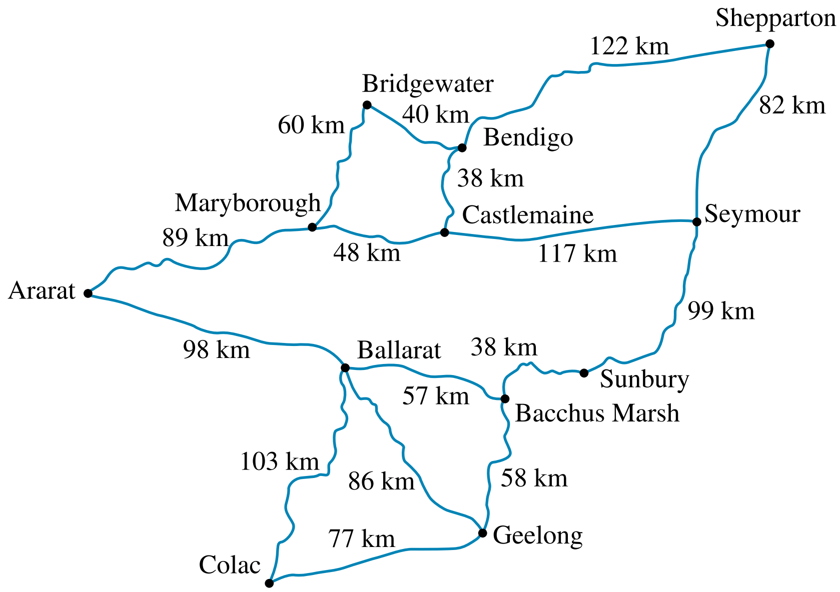


Flights operate between selected locations, and the flight distances (in km) are shown in the following table.

	A	B	C	D	E	F	G	H	I
A		1090		960			2600		2200
B	1090		360	375	435				
C		360							
D	960	375							
E		435							
F							1590	1400	
G	2600					1590		730	
H						1400	730		220
I	2200							220	

a. Show this information as a weighted graph.
b. Does a Hamiltonian path exist? Explain your answer.
c. Identify the shortest distance possible for travelling to all sites the minimum number of times if you start and finish at:
   i. A
   ii. G.
d. Draw the minimum spanning tree for the graph.

16. The organisers of the 'Tour de Vic' bicycle race are using the following map to plan the event.



a. Draw a weighted graph to represent the map.

b. If they wish to start and finish in Geelong, what is the shortest route that can be taken that includes a total of nine other locations exactly once, two of which must be Ballarat and Bendigo?

c. Draw the minimal spanning tree for the graph.

d. If the organisers decide to use the minimum spanning tree as the course, what would the shortest possible distance be if each location had to be reached at least once?

5.6 Review: exam practice

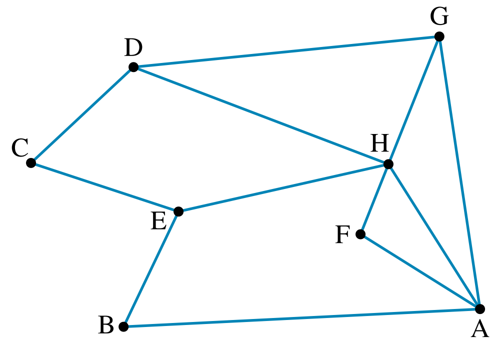
A summary of this topic is available in the Resources section of your eBookPLUS at www.jacplus.com.au.

Multiple choice

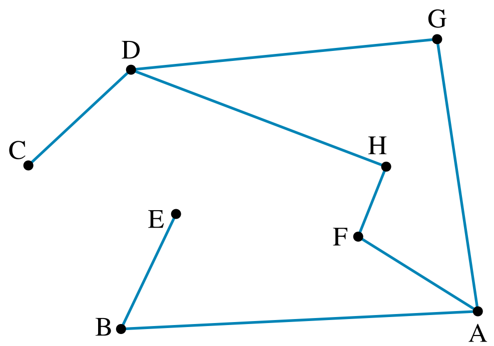
1. **MC** The minimum number of edges in a connected graph with eight vertices is:

A. 5 **B.** 6 **C.** 7 **D.** 8 **E.** 9

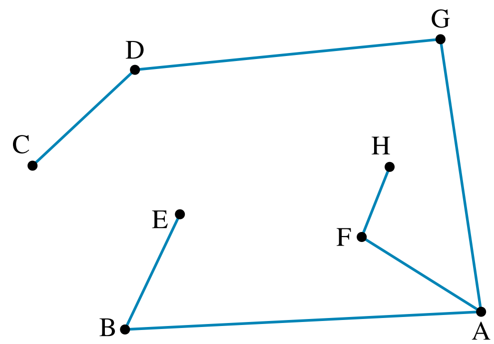
2. **MC** Which graph is a spanning tree for the following graph?



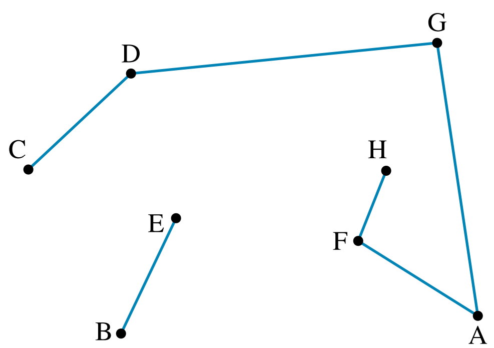
A.



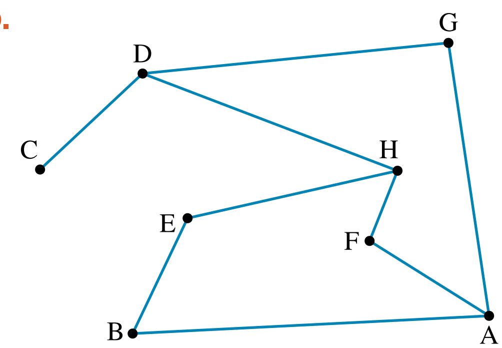
B.



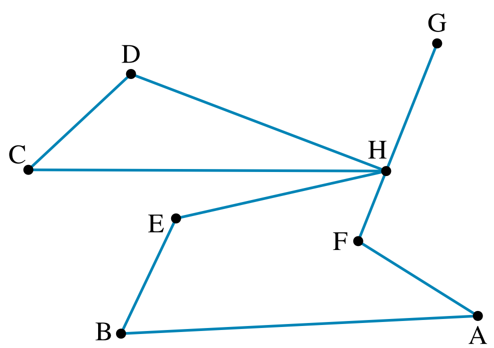
C.



D.



E.

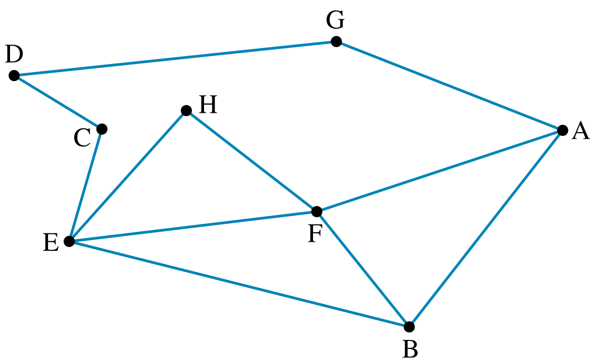


3. MC A connected graph with 9 vertices has 10 faces. The number of edges in the graph is:

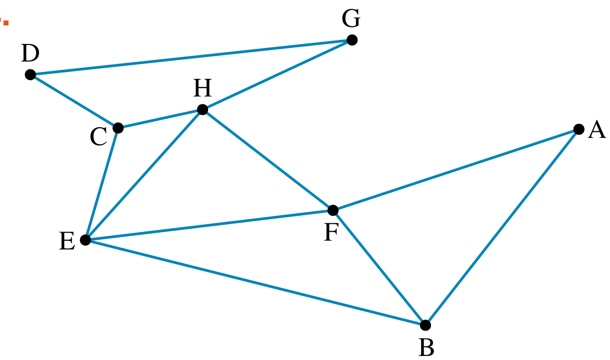
A. 15 B. 16 C. 17 D. 18 E. 19

4. MC Which of the following graphs will not have an Euler trail?

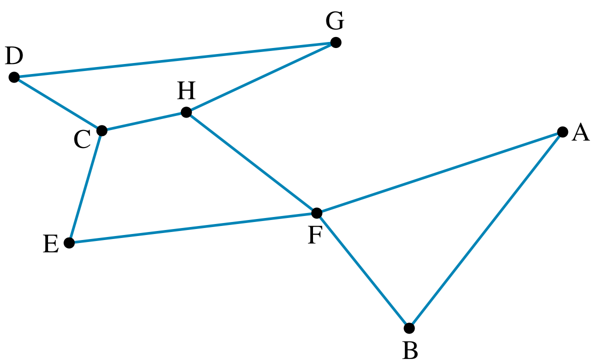
A.



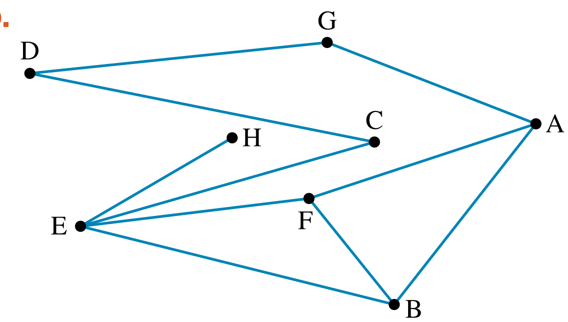
B.



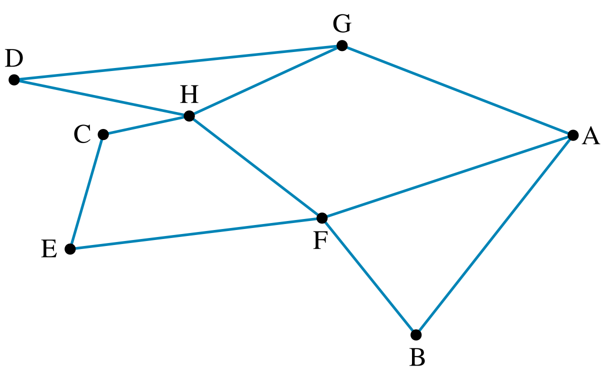
C.



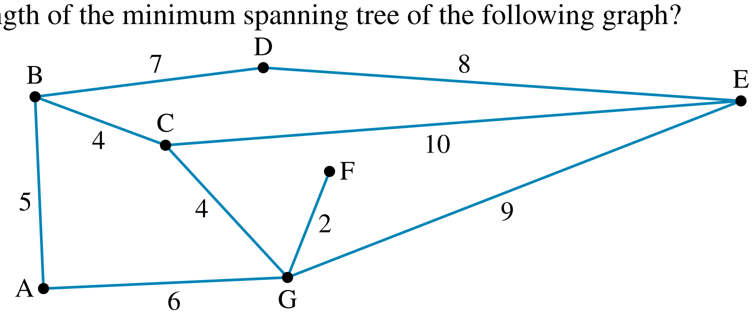
D.



E.

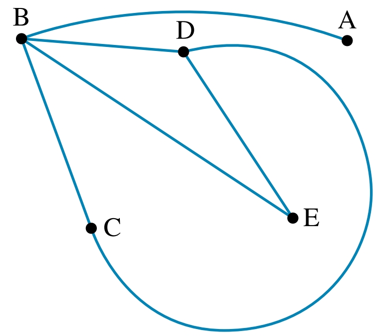


5. MC What is the length of the minimum spanning tree of the following graph?



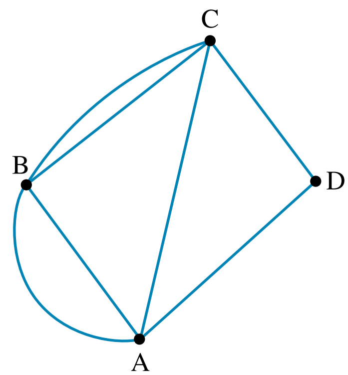
A. 33 B. 26 C. 34 D. 31 E. 30

6. MC An Euler circuit can be created in the following graph by adding an edge between the vertices:



A. A and D B. A and B C. A and C D. B and C E. A and E

7. MC The adjacency matrix that represents the following graph is:



A. $\begin{bmatrix} 0 & 2 & 2 & 2 \\ 2 & 0 & 2 & 0 \\ 2 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix}$

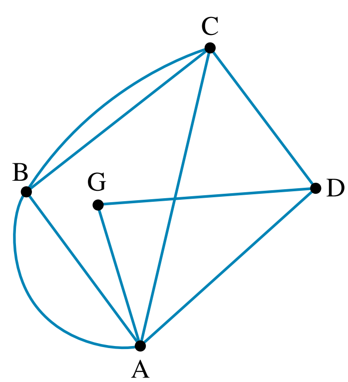
B. $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 1 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

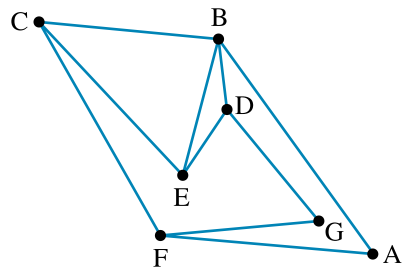
E. $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

8. MC The number of faces in the following planar graph is:



A. 6 B. 7 C. 8 D. 9 E. 10

9. MC A Hamiltonian cycle for the following graph is:



A. ABCEDGFA
B. ABDGFCEA
C. ABDGFCEDEBCFA
D. ABDGFCECFA
E. ABDGFA

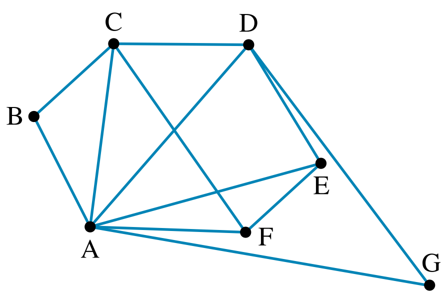
10. MC A complete graph with 7 vertices will have a total number of edges of:

A. 7
B. 8
C. 14
D. 42
E. 21

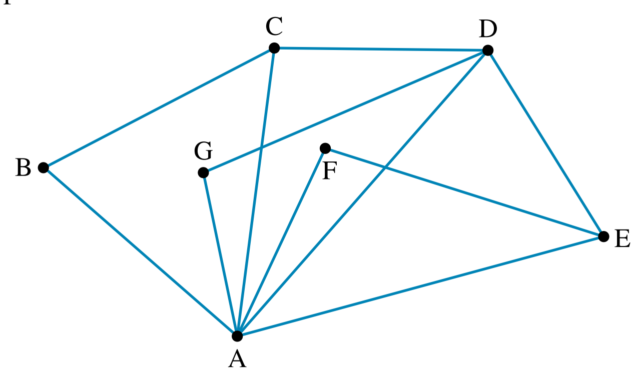
Short answer

1. a. Identify whether the following graphs are planar or not planar.

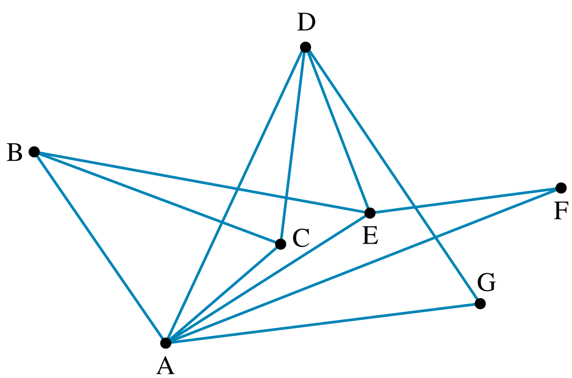
i.



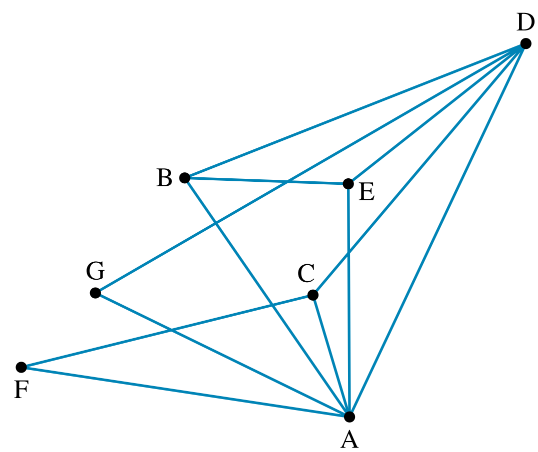
ii.



iii.



iv.



b. Redraw the graphs that are planar without any intersecting edges.

2. Complete the following adjacency matrices.

a. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ & 0 & & 0 \\ & 3 & 1 & \\ & & 1 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 & 2 & 1 & \\ & 0 & & 0 & 1 \\ & 2 & 0 & 2 & \\ & & & 2 & 2 \\ 1 & & 3 & & 0 \end{bmatrix}$

c. $\begin{bmatrix} 0 & & 1 & 3 & 1 & \\ 2 & 0 & & & 1 & \\ & 3 & 0 & 2 & & \\ & 1 & & 2 & 2 & 1 \\ & & 3 & & 3 & 1 \\ 2 & 0 & 1 & & & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & & & & 0 & & \\ 2 & 0 & & 1 & & & \\ 1 & 2 & 0 & 1 & 1 & 0 & \\ 3 & & & 0 & & & 1 \\ & 2 & & 0 & 0 & & 3 \\ 1 & 1 & & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & & & & 0 \end{bmatrix}$

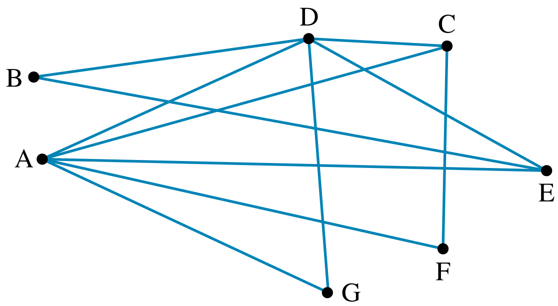
3. Identify which of the following graphs are:

i. simple

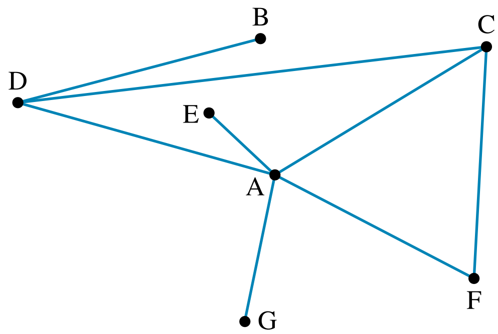
ii. complete

iii. planar.

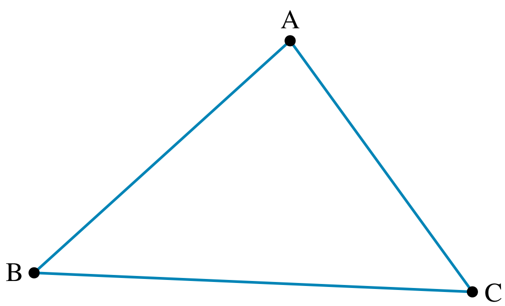
a.



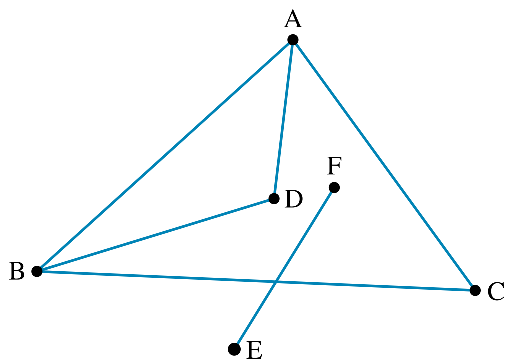
b.



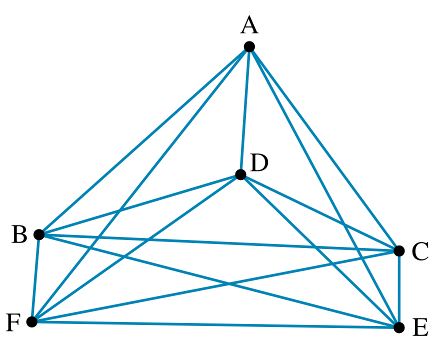
c.



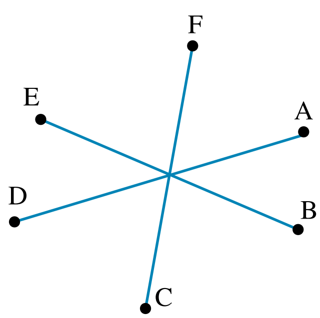
d.



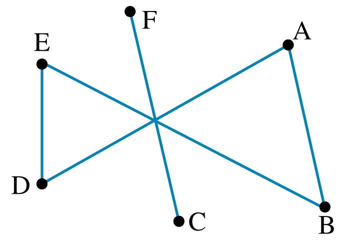
e.



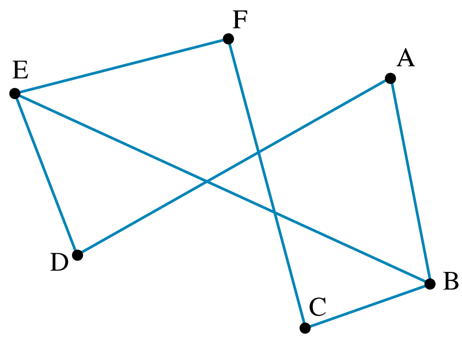
f.



g.

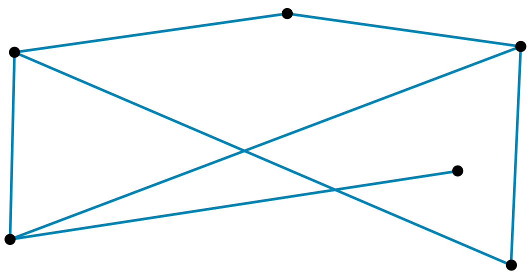


h.

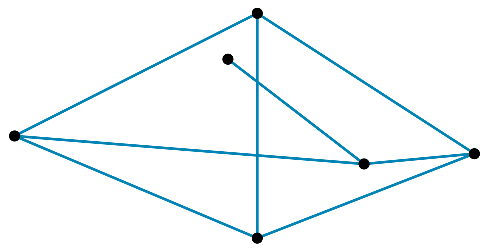


4. Which of the following graphs are isomorphic?

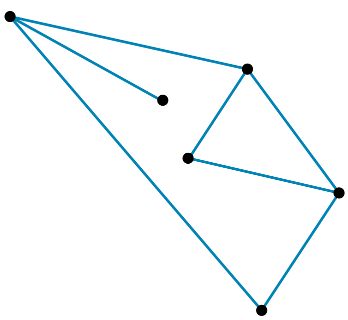
a.



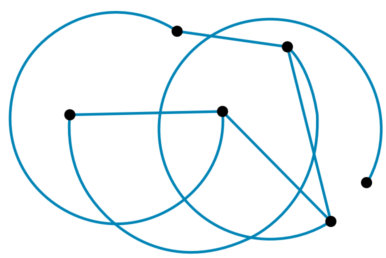
b.



c.



d.

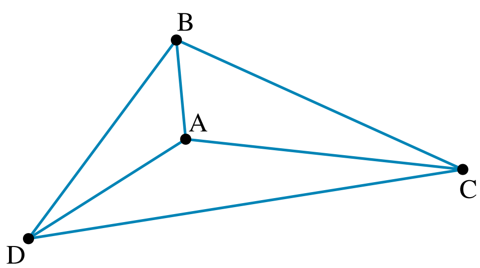


5. For each of the following graphs:

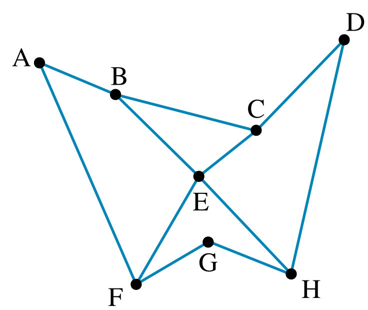
i. add the minimum number of edges to the following graphs in order to create an Euler trail

ii. state the Euler trail created.

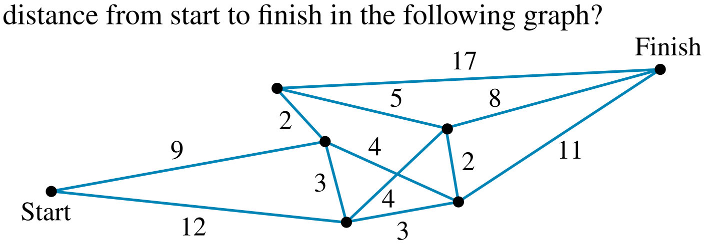
a.



b.



6. a. What is the shortest distance from start to finish in the following graph?



b. What is the total length of the shortest Hamiltonian path from start to finish?

c. Draw the minimum spanning tree for this graph.

Extended response

1. The flying distances between the capital cities of Australian mainland states and territories are listed in the following table.

	Adelaide	Brisbane	Canberra	Darwin	Melbourne	Perth	Sydney
Adelaide		2055	1198	3051	732	2716	1415
Brisbane	2055		1246	3429	1671	4289	982
Canberra	1198	1246		4003	658	3741	309
Darwin	3051	3429	4003		3789	4049	4301
Melbourne	732	1671	658	3789		3456	873
Perth	2716	4363	3741	4049	3456		3972
Sydney	1415	982	309	4301	873	3972	

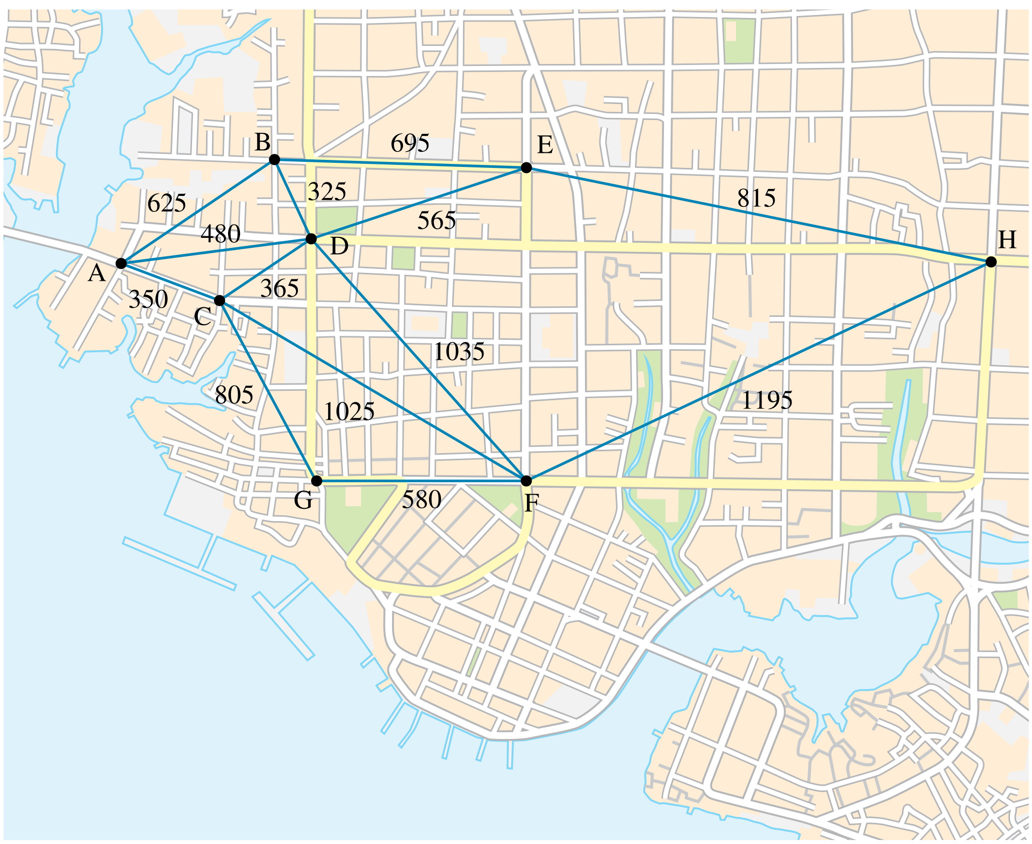
a. Draw a weighted graph to show this information.

b. If technical problems are preventing direct flights from Melbourne to Darwin and from Melbourne to Adelaide, what is the shortest way of flying from Melbourne to Darwin?

c. If no direct flights are available from Brisbane to Perth or from Brisbane to Adelaide, what is the shortest way of getting from Brisbane to Perth?

d. Draw the minimum spanning tree for the graph and state its total distance.

2. The diagram shows the streets in a suburb of a city with a section of underground tunnels shown in black. Weightings indicate distances in metres. The tunnels are used for utilities such as electricity, gas, water and drainage.

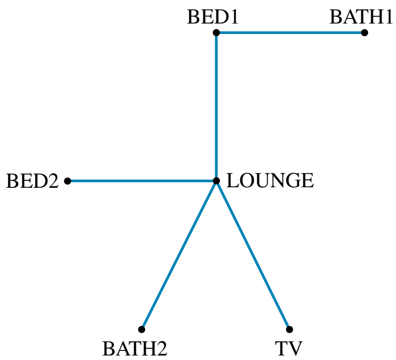


Answers

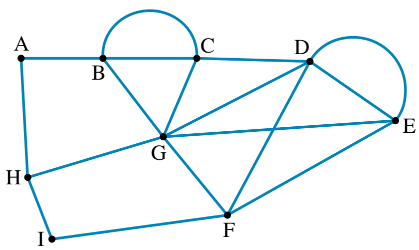
Topic 5 Graphs and networks

Exercise 5.2 Definitions and terms

1.



2.



3. a. Edges = 7; Degree sum = 14
b. Edges = 10; Degree sum = 20

4. a. Edges = 9; Degree sum = 18
b. Edges = 9; Degree sum = 18

5. a. deg(A) = 5; deg(B) = 3; deg(C) = 4; deg(D) = 1; deg(E) = 1
b. deg(A) = 0; deg(B) = 2; deg(C) = 2; deg(D) = 3; deg(E) = 3
c. deg(A) = 4; deg(B) = 2; deg(C) = 2; deg(D) = 2; deg(E) = 4
d. deg(A) = 1; deg(B) = 2; deg(C) = 1; deg(D) = 1; deg(E) = 3

6. a. The graphs are isomorphic.
b. The graphs are isomorphic.
c. The graphs are not isomorphic.
d. The graphs are isomorphic.

7. a. Different degrees and connections
b. Different connections

8. The isomorphic pairs are graphs **2** and **4**, and graphs **5** and **6**.

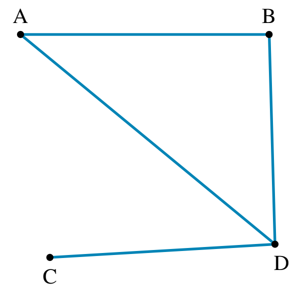
9. a. $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$

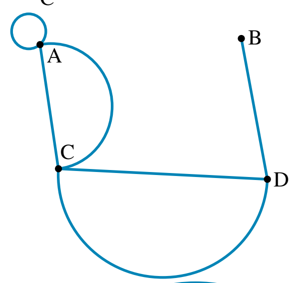
c. $\begin{bmatrix} 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 & 4 & 0 \end{bmatrix}$

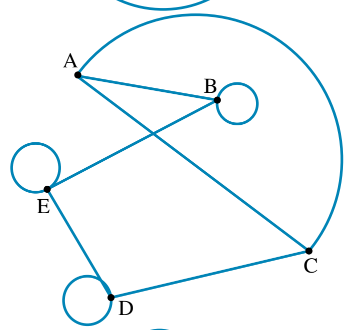
10. a.



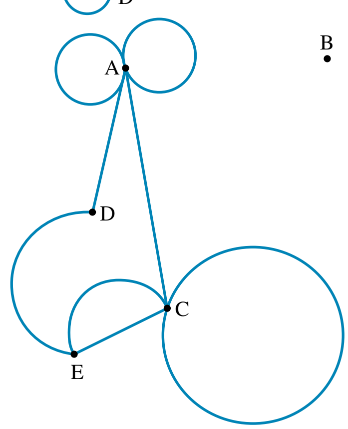
b.



c.



d.



11.

Graph 1:

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Graph 2:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Graph 3:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Graph 4:

$$\begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Graph 5:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

12. a. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 2 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

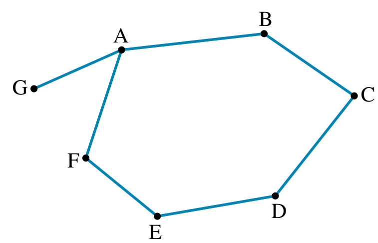
c. $\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

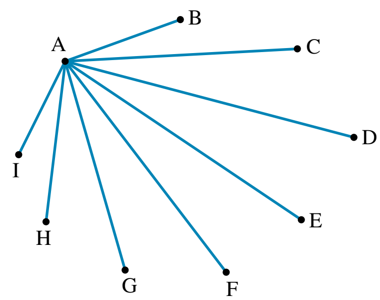
13.

Graph	Simple	Complete	Connected
Graph 1	Yes	No	Yes
Graph 2	Yes	No	Yes
Graph 3	Yes	No	Yes
Graph 4	No	No	Yes
Graph 5	No	Yes	Yes

b.



c.

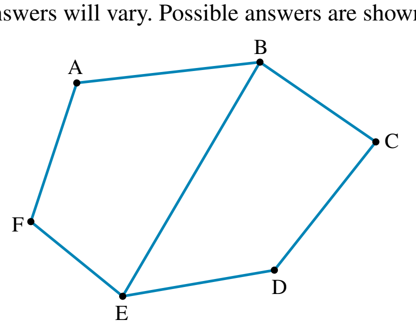


14.

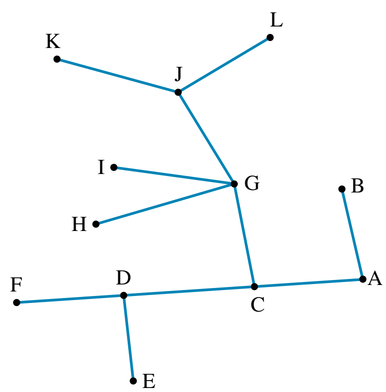
Vertices	Edges
2	1
3	3
4	6
5	10
6	15
n	$\frac{n(n-1)}{2}$

15. Answers will vary. Possible answers are shown.

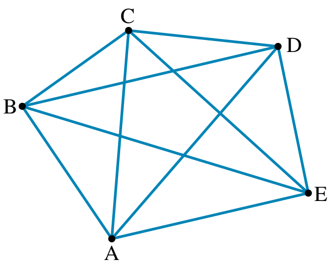
a.



16.



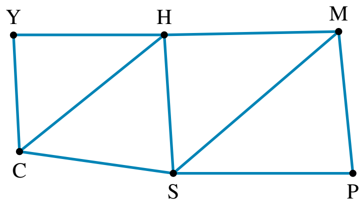
17. a.



b. Complete graph

c. Total number of games played

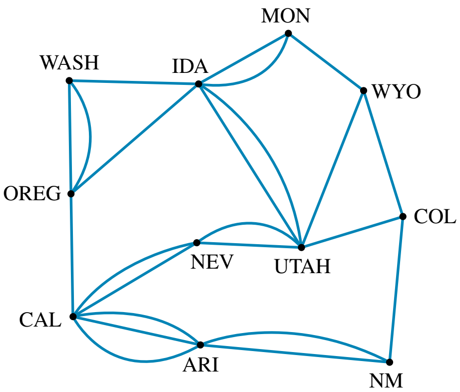
18. a.



b. Huairou and Shunyi

c. Simple connected graph

19. a.



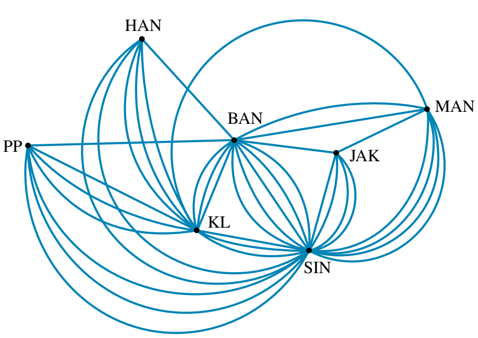
b.

$$\begin{array}{c} \\ \text{Wa} \\ \text{O} \\ \text{Ca} \\ \text{I} \\ \text{N} \\ \text{A} \\ \text{M} \\ \text{U} \\ \text{Wy} \\ \text{Co} \\ \text{NM} \end{array} \begin{array}{c} \begin{array}{ccccccccccc} \text{Wa} & \text{O} & \text{Ca} & \text{I} & \text{N} & \text{A} & \text{M} & \text{U} & \text{Wy} & \text{Co} & \text{NM} \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 3 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

c. California, Idaho and Utah

d. Montana

20. a.

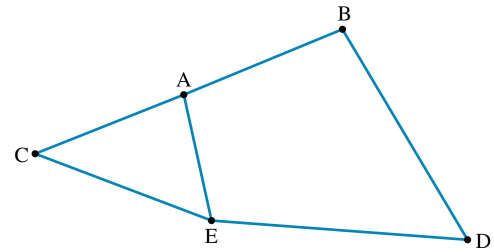


b. Directed, as it would be important to know the direction of the flight

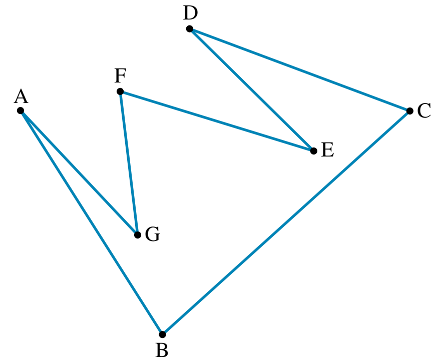
i. 10 **ii.** 7

Exercise 5.3 Planar graphs

1. a.

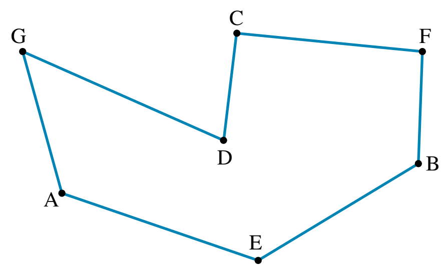


b.

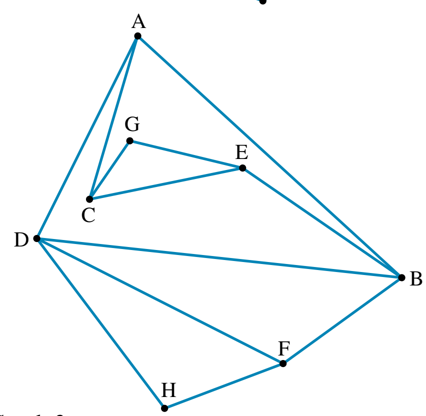


2. a. All of them **b.** All of them

3. a.



b.



4. Graph 3

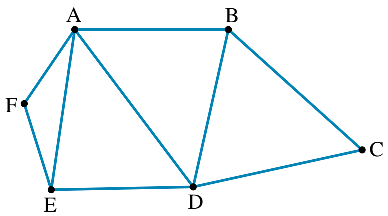
5. a. 4 **b.** 5

6. a. 6 **b.** 5

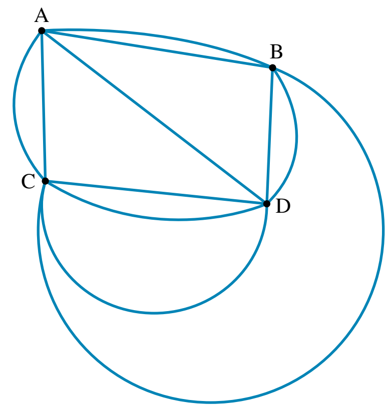
7. a. 3 **b.** 3

c. 2 **d.** 7

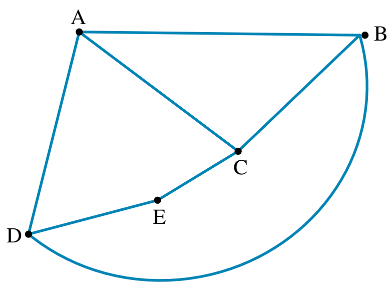
8. a.



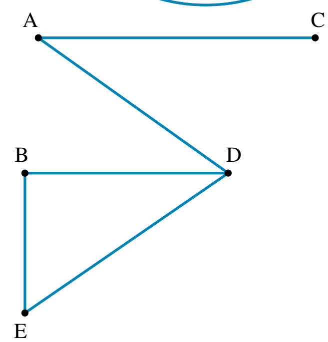
b.



9. a.



b.



10. a. **i.** 3 **ii.** 2 **b.** **i.** 1 **ii.** 4

11. a.

Graph	Total edges	Total degrees
Graph 1	3	6
Graph 2	5	10
Graph 3	8	16
Graph 4	14	28

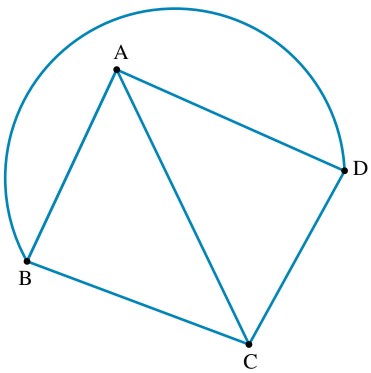
b. $\text{Total degrees} = 2 \times \text{total edges}$

12. a.

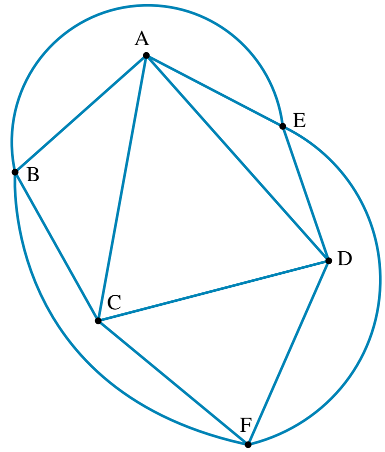
Graph	Total vertices of even degree	Total vertices of odd degree
Graph 1	3	2
Graph 2	4	2
Graph 3	4	4
Graph 4	6	6

b. No clear pattern evident.

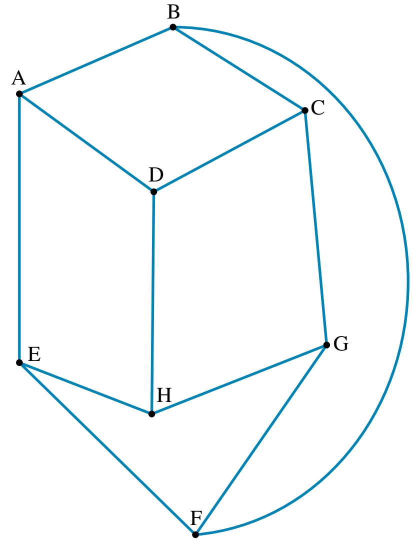
13.



Tetrahedron

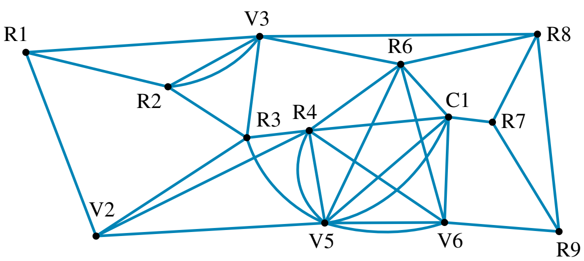


Octahedron



Cube

14. a.



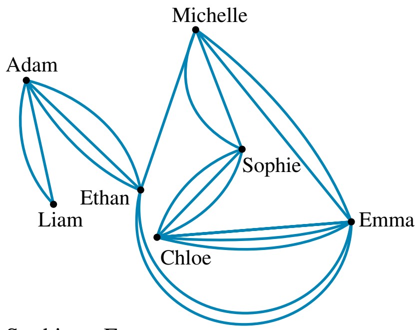
b. No

15. a.

	Pa	Ed	Bak	Wa	Ki	Fa	Mo	No	Bo	Ox	To	Ho	Ba	So	Vi	Gr	Pi	We	Em	Bl	Ca	Le	Ch
Pa	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ed	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bak	0	1	0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Wa	0	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
Ki	0	0	1	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
Fa	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mo	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
No	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
Bo	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
Ox	0	0	1	1	0	0	0	0	1	0	1	0	0	0	0	1	1	0	0	0	0	0	0
To	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0
Ho	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0
Ba	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
So	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
Vi	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0
Gr	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	0	1	1	0	0	0	0	0
Pi	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	1
We	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0
Em	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
Bl	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
Ca	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
Le	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	1
Ch	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0

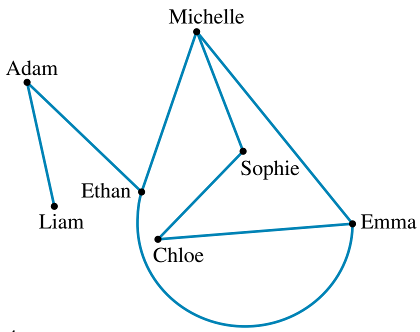
b. The sum of the rows represents the sum of the degree of the vertices, or twice the number of edges (connections).

16. a.



b. Sophie or Emma

c.



d. 4

Exercise 5.4 Connected graphs

1. Cycle: ABECA (others exist)
Circuit: BECD (others exist)

2. Path: ABGFHDC (others exist)
Cycle: DCGFHD (others exist)
Circuit: AEBGFHDCA (others exist)

3. a. Walk
b. Walk, trail and path
c. Walk, trail, path, cycle and circuit
d. Walk and trail

4. a. MCHIJGFAED
b. AEDBLKMC
c. MDEAFGJIHCM
d. FMCHIJGF

5. a. Euler trail: AFEDBECAB; Hamiltonian path: BDECAF
b. Euler trail: GFBECGDAC; Hamiltonian path: BECADGF

6. a. Euler circuit: AIBAHGFCJBCDEGA; Hamiltonian cycle: none exist
b. Euler circuit: ABCDEFGHA (others exist); Hamiltonian cycle: HABCDEFGH (others exist)

7. a. Graphs **i**, **ii** and **iv**
b. Graph **i**: ACDABDECB (others exist)
Graph **ii**: CFBCEDBADCA (others exist)
Graph **iv**: CFBCEDCADBAH (others exist)

8. a. Graphs **i** and **ii**
b. Graph **i**: CEDABC
Graph **ii**: CEDABGC

9. a. i.
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

ii.
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

iii.
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

iv.
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

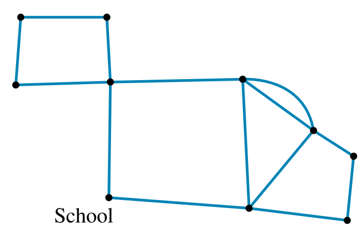
b. The presence of Euler trails and circuits can be identified by using the adjacency matrix to check the degree of the vertices. The presence of Hamiltonian paths and cycles can be identified by using the adjacency matrix to check the connections between vertices.

10. E

11. a. A or C
b. B or D

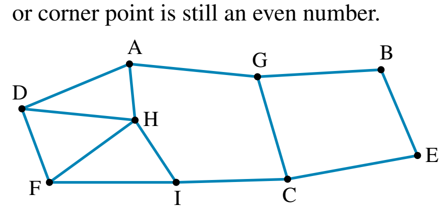
12. a. G to C
b. F to E

13. a.



b. Yes, because the degree of each intersection or corner point is an even number.
c. Yes, because the degree of each remaining intersection or corner point is still an even number.

14. a.



b. 4
c. i. ADHFICEBGA
ii. AHDFICEBGA
d. i. Yes, because two of the checkpoints have odd degree.
ii. H and C

15. a.

	Hamiltonian cycle
1.	ABCDA
2.	ABDCA
3.	ACBDA
4.	ACDBA
5.	ADBCA
6.	ADCBA

b. Yes, commencing on vertices other than A

16. a. B, C, D, F or G **b.** B or C **c.** None possible

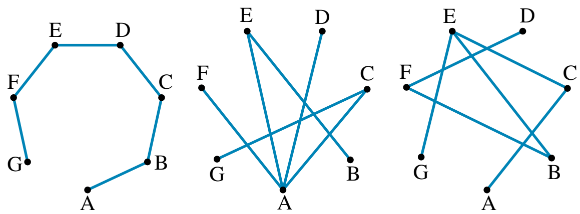
d. D or E **e.** D to E

Exercise 5.5 Weighted graphs and trees

1. 21

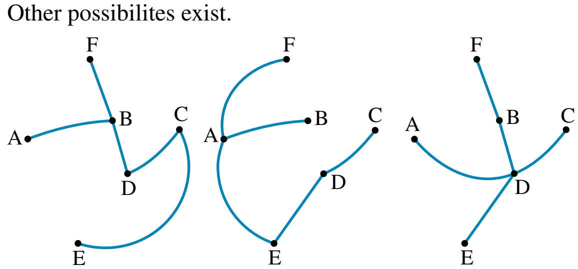
2. 20.78

3. a.



Other possibilites exist.

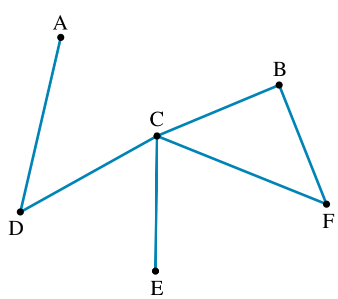
b.



Other possibilites exist.

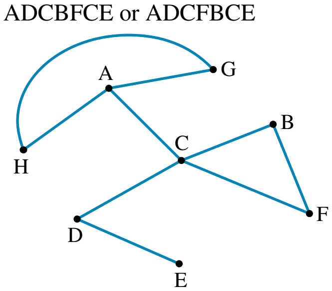
4. ABGEDCA or ACDEGBA (length 66)

5. a. i.



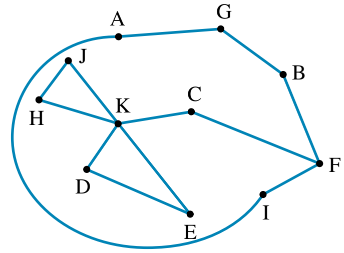
ii. ADCBFCE or ADCFBCE

b. i.



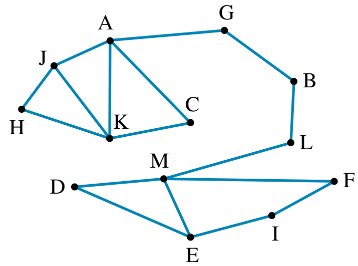
ii. AHGACBFCDE or similar

c. i.



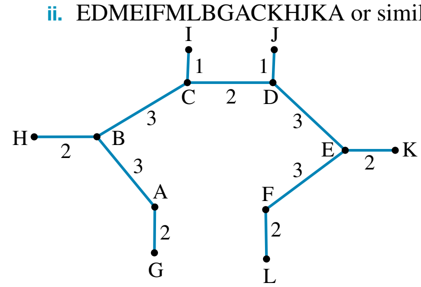
ii. KDEKHJKCFIAGBF or similar

d. i.

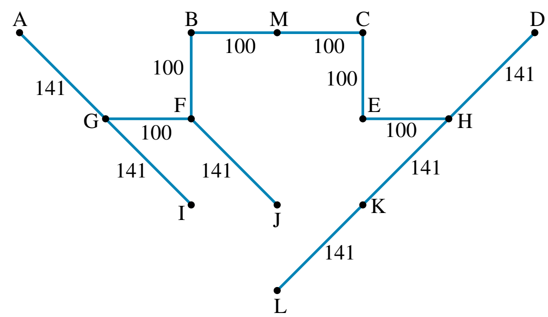


ii. EDMEIFMLBGACKHJKA or similar

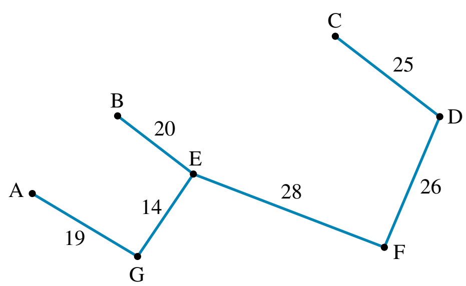
6.



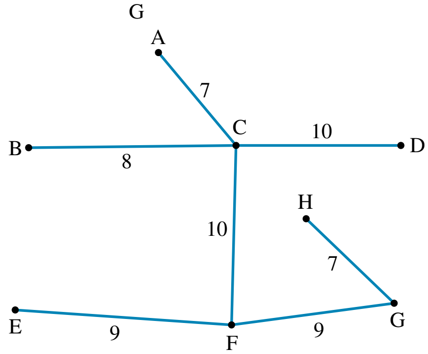
7.



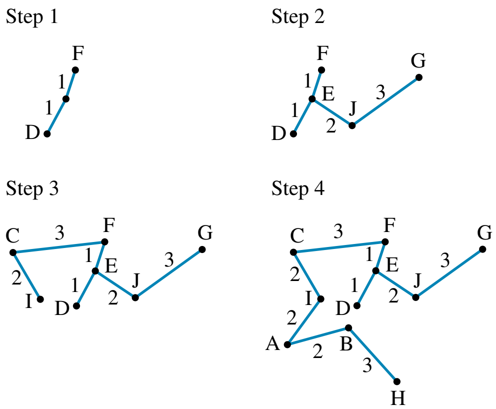
8. a.



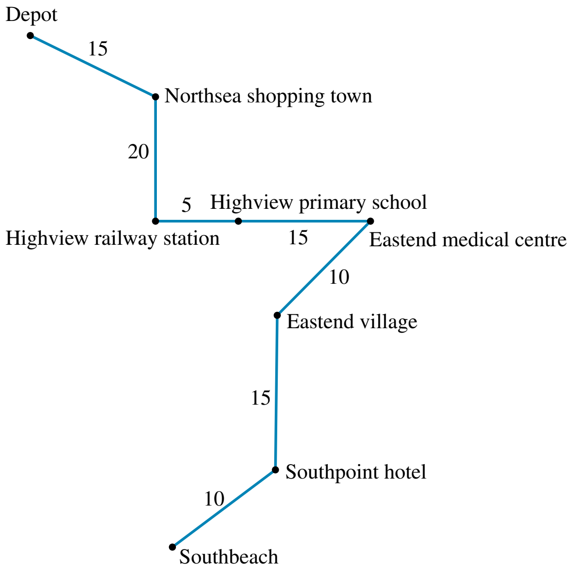
b.



9.

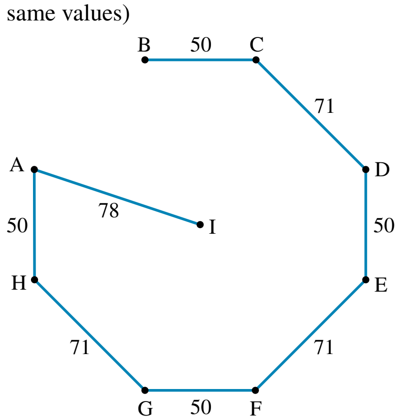


10.



11. a. Longest: IFEDCBAHG (or similar variation of the same values)
Shortest: IAHGFEDCB (or similar variation of the same values)

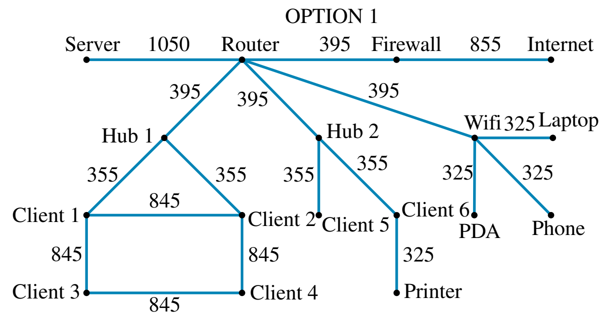
b.

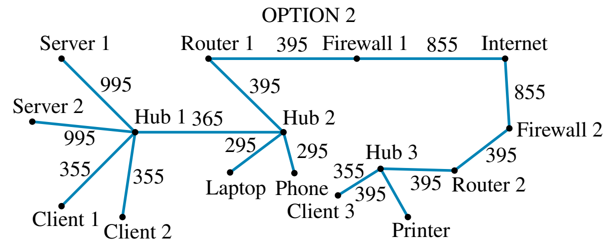


12. a. FDCGBAE (other solutions exist)
b. FDCBAEG (other solutions exist)

13. a. ADEG
b. BHG
c. EGFCDABHE

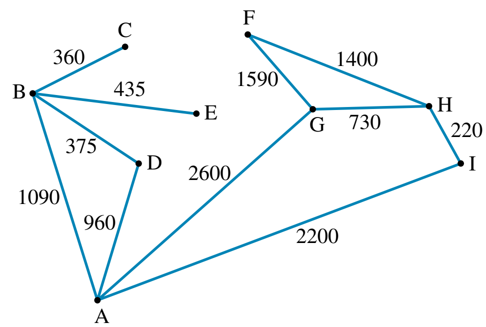
14. a.





b. Option 2

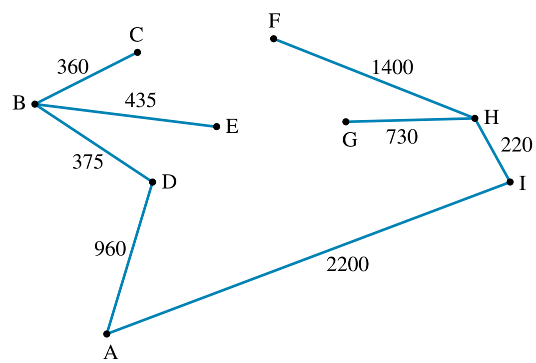
15. a.



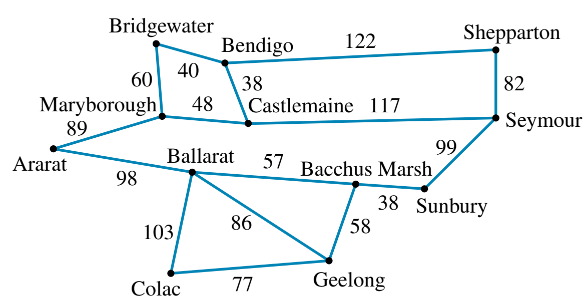
b. No; C and E are both only reachable from B.

c. i. 12 025 **ii.** 12 025

d.

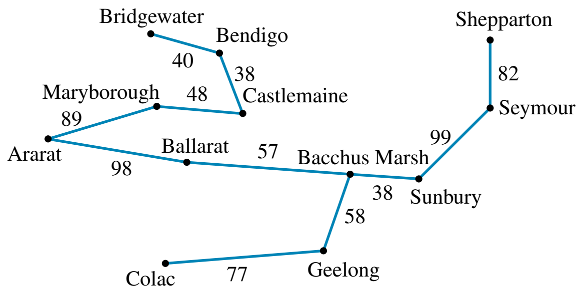


16. a.



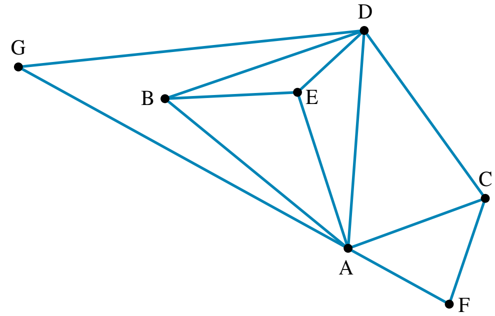
b. 723 km

c.



d. 859 km

iv.



5.6 Review: exam practice

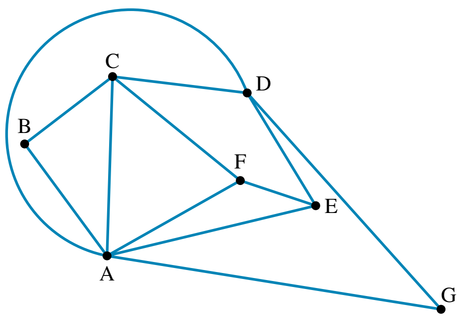
Multiple choice

1. C 2. B 3. C 4. D 5. E
6. A 7. E 8. A 9. A 10. E

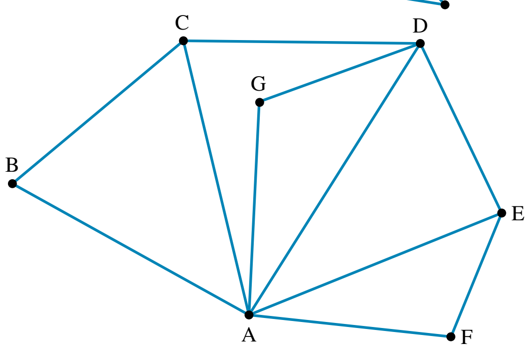
Short Answer

1. a. i. Planar **ii.** Planar
iii. Planar **iv.** Planar

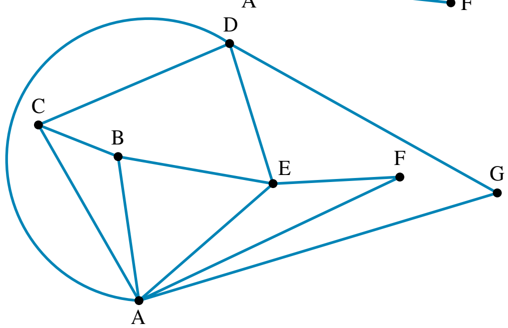
b. i.



ii.



iii.



2. a. $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 & 2 & 1 & 1 \\ 1 & 0 & 2 & 0 & 1 \\ 2 & 2 & 0 & 2 & 3 \\ 1 & 0 & 2 & 2 & 2 \\ 1 & 1 & 3 & 2 & 0 \end{bmatrix}$

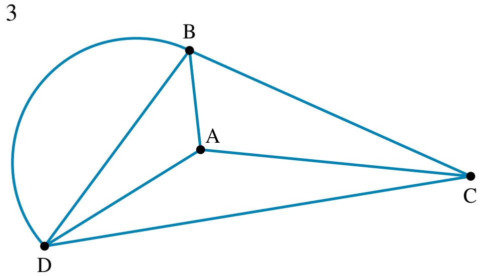
c. $\begin{bmatrix} 0 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 3 & 1 & 1 & 0 \\ 1 & 3 & 0 & 2 & 3 & 1 \\ 3 & 1 & 2 & 2 & 2 & 1 \\ 1 & 1 & 3 & 2 & 3 & 1 \\ 2 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 2 & 1 & 3 & 0 & 1 & 0 \\ 2 & 0 & 2 & 1 & 2 & 1 & 0 \\ 1 & 2 & 0 & 1 & 1 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 & 2 & 1 \\ 0 & 2 & 1 & 0 & 0 & 0 & 3 \\ 1 & 1 & 0 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 3 & 2 & 0 \end{bmatrix}$

3. a. Simple, planar **b.** Simple, planar
c. Simple, complete, planar **d.** Simple, planar
e. Simple, complete **f.** Simple, planar
g. Simple, planar **h.** Simple, planar

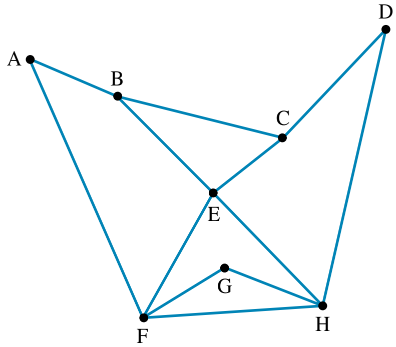
4. Graphs **a** and **d** are isomorphic.

5. a. i. 3



ii. ABDBCADC

b. i.

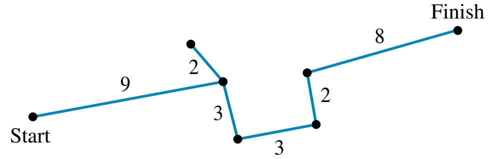


ii. BAFEHGFHDCEBC

6. a. 23

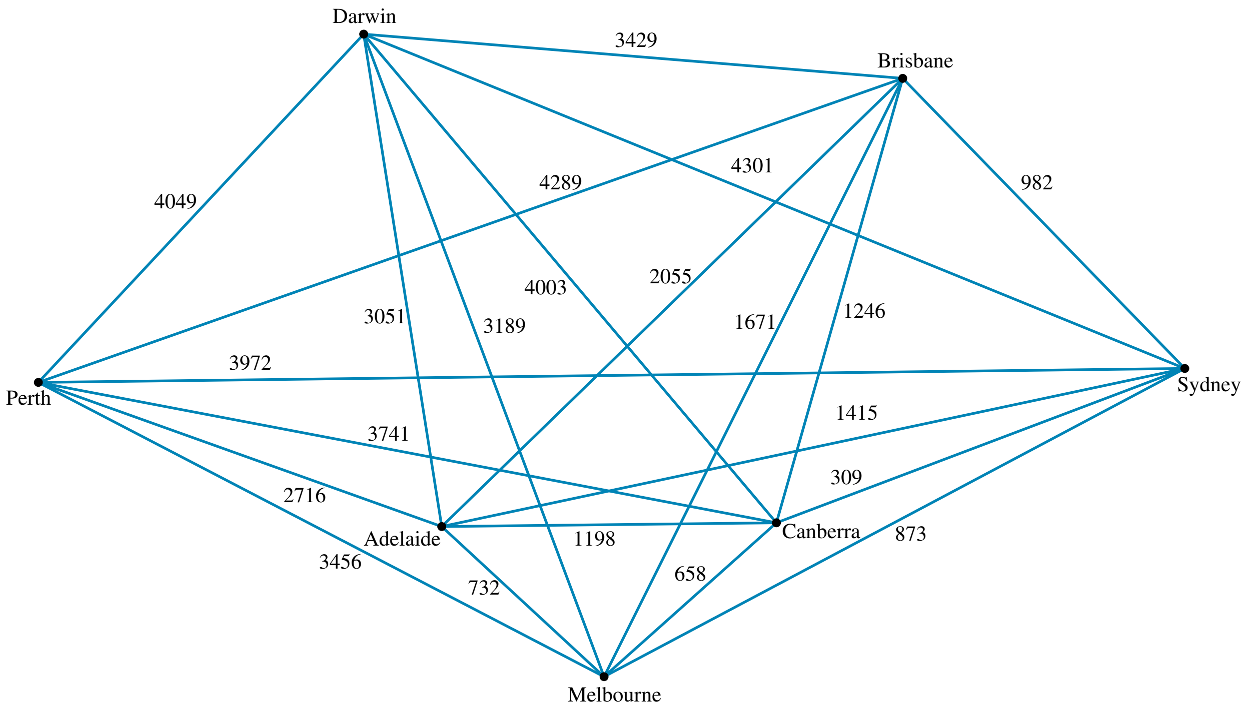
b. 34

c.



Extended response

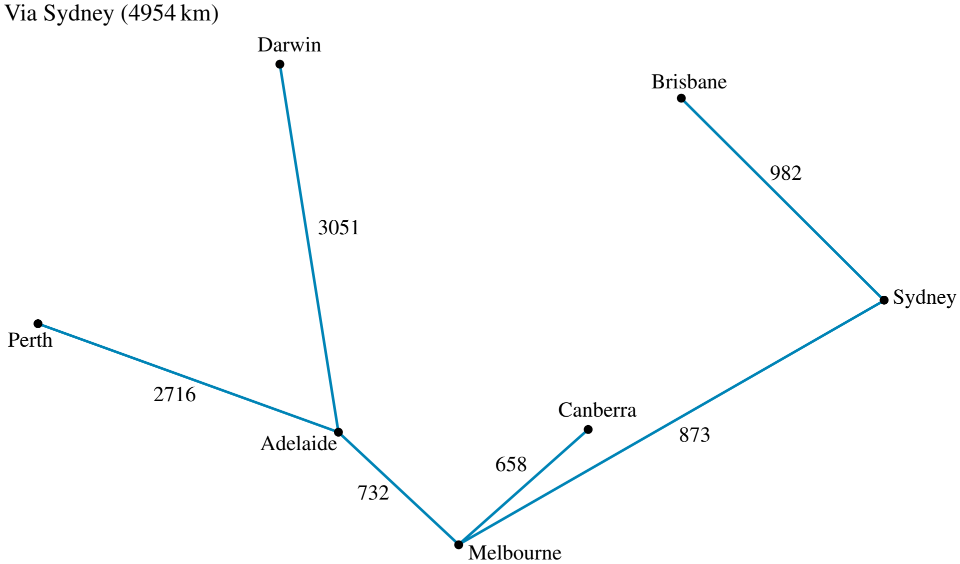
1. a.



b. Via Canberra (4661 km)

c. Via Sydney (4954 km)

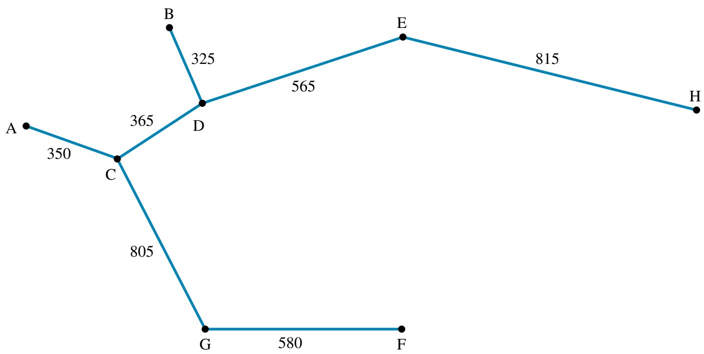
d.



The total distance is 9012 km.

2. a. i. 3805 m

ii.

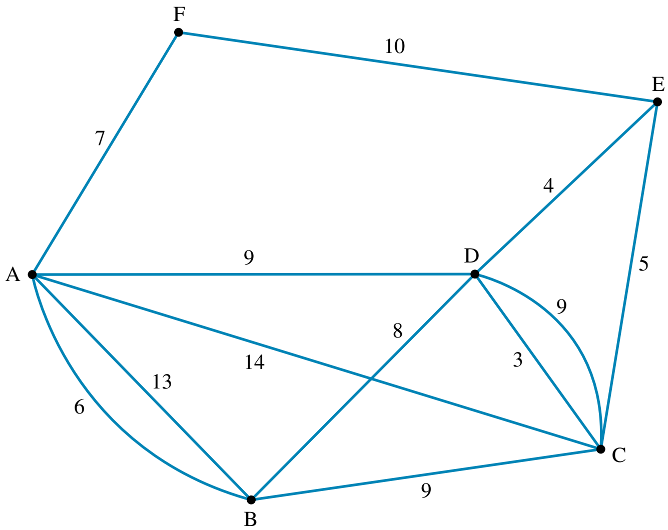


b. HEDA, 1860 m

c. 4905 m, DFGCABEH

d. DEHFGCABD, 5260 m

3. a.

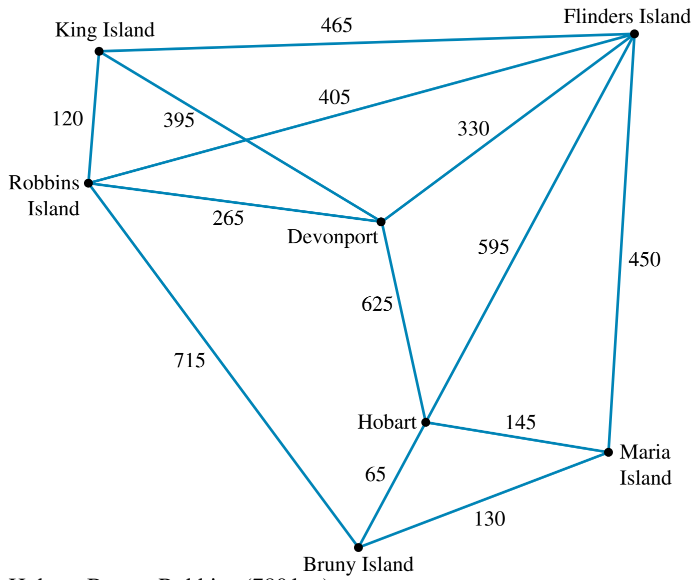


b. $\begin{bmatrix} 0 & 2 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

c. No, as there are more than two vertices of odd degree.

d. AFEDCBA(39)

4. a.



b. Hobart–Bruny–Robbins (780 km)

c. Hobart–Bruny–Robbins–King–Devonport–Flinders–Maria (2075 km)

d. King–Devonport–Flinders–Maria–Hobart–Bruny–Robbins–King (2220 km)