ETH Zurich
Institut für Theoretische Informatik
Prof. Dr. Angelika Steger
Prof. Dr. Emo Welzl
Prof. Dr. Peter Widmayer

HS 2014
Exercise Sheet Exam Preparation Week II

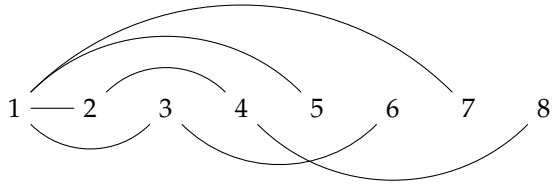
Algorithms Lab

Exercise 1 – *Divisor Distance*

Let n be some positive integer. Let G_n be a graph with the numbers from 1 to n as vertices. Further in G_n there is an (undirected) edge between vertices i and j with $i > j$ if and only if j is the largest proper divisor of i. A proper divisor of a positive integer i is any other positive integer j less than i and such that j divides i without a remainder.

What is the minimal distance of two given vertices in this graph?

As an example here is the graph G_8



Input The first line of the input contains an integer $T \leq 100$ denoting the number of test cases. Then T test cases follow. The first line of test case i contains two integers n_i and c_i. It holds that $1 \leq n_i \leq 10'000'000$ and $1 \leq c_i \leq 100$. The first integer (n_i) denotes that we consider the graph G_{n_i} as defined above. The second integer indicates the number of pairs of vertices for which we wish to know their minimum distance in G_{n_i}. After the line with n_i and c_i, c_i lines follow. Each of these lines contains two integers v_1, and v_2.

Output For each test case i you have to print for each of the c_i pairs of vertices v_1 and v_2 one line containing the minimal distance of v_1 and v_2 in the graph G_{n_i}.

Test sets There are 4 groups of test sets, and solving all of them is worth 100 points.

- The test set named "small", worth *20 points*, contains only cases with $n_i \leq 1'000$ and with $v_1 = 1$.
- The test set named "medium", worth *40 points*, contains only cases with $n_i \leq 10'000$, v_1 arbitrary.
- The test set named "large", worth *20 points*, contains only cases with $n_i \leq 100'000$.
- The test set named "huge", worth *20 points*, contains cases covering the full range of n_i.

Sample input

```
1
8 3
1 2
1 8
1 6
```

Sample output

```
1
3
2
```