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## **Algorithms Lab**

## **Exercise 2** – *Light Patterns*

New Year is just a couple of months away, and the city officials are already starting to plan how to decorate Bahnhofstrasse. This year they want to prepare a big surprise!

It starts, of course, with a lot of light bulbs in a straight line. Here a *lot* means n, and the bulbs are enumerated from 0 up to n-1 according to their unique distance from the Hauptbahnhof.

At the beginning of the night, each bulb is either turned on or off (we refer to this as the *state* of a bulb). Exactly at the midnight, the city officials will start changing the state of the bulbs until for all i,  $0 \le i < n/k$ , all the bulbs numbered from  $i \cdot k$  to  $(i+1) \cdot k - 1$ , in that order, follow a fixed light pattern. A light pattern is a sequence of states, and an ordered set of bulbs follows the pattern if the sequence of the states of the bulbs (in the given order) matches the sequence of the pattern.

There are two ways to change the state of a bulb,

- 1. change the state of a single bulb, or
- 2. change the state of every bulb from 0 up to  $t \cdot k 1$ , for some  $t \leq n/k$ .

Each operation takes one second to perform, and the operation can be started only after the previous one has finished. Given the initial states of the bulbs, the city officials wants to know what is the minimum amount of time needed to produce the desirable patterns.

Input The first line of the input contains  $t \le 20$ , the number of test cases. Each test case starts with one line containing the total number of light bulbs  $1 \le n \le 100000$ , the number of bulbs  $1 \le k \le 16$  that form a light pattern, and the integer x which represents a light pattern as following: for every  $1 \le i \le k$ , the (k-i+1)-th bulb of the pattern should be turned off if the i-th least significant bit of x is 0, and turned on otherwise. Next line of the test case contains n bits, separated by a space, which represent the current state of bulbs (i-th bit is 0 if the bulb numbered with i-1 is initially turned off, and 1 if it is turned on). It is guaranteed that n is a multiple of k and that  $\log_2 x \le k$ .

**Output** For each test case output the smallest possible amount of time needed to reach the desired light patterns.

## Sample Input

1 0 1 0 0 0 1 1

| 2 |   |   |   |   |   |   |   | Sample Output |
|---|---|---|---|---|---|---|---|---------------|
| 8 | 1 | 0 |   |   |   |   |   | Sample Output |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 3             |
| 8 | 2 | 1 |   |   |   |   |   | 3             |

**Explanation** In the first test case, the optimal strategy would be to first change the state of bulbs numbered with 3 and 4, and then change all the states.

The initial state of the second test case is "on off on off off on on" and the desired light pattern is "off on". Thus the optimal strategy is to change the state of the first 4 bulbs, and then of bulbs numbered with 5 and 7.

**Remark** For the easy input, it will always hold that x=0 and k=1. Solving it will give you 50 points.

(100 *Points*)