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Exercise Sheet Problem of the week IV

# Algorithms Lab

## Exercise 1 – *Fluid Borders*

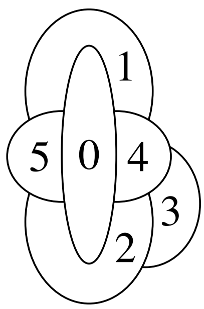
The distant planet Li-Kuid III hosts life – but not as we know it! Li-Kuid III is inhabited by gelatinous spheres, so-called "blobs" (beings that link to organized basins). Multiple blobs form communities by fusing together into a big gelatinous meta-blob. The surface of Li-Kuid III is covered by many such meta-blobs of various sizes. Meta-blobs are like earth countries and just like on earth it is very important for a meta-blob who its neighbors (other meta-blobs sharing a nonzero length portion of the meta-blob's boundary) are. However, meta-blobs are much less rigid than earth countries – in fact, once in a Li-Kuid III year, every meta-blob has a big election in which every individual blob votes for a preferred neighbor. Depending on the outcome of the different elections, the planets surface is completely redistributed amongst the meta-blobs in order to satisfy the wishes of the individual blobs as much as possible. After the reformation, the surface of the planet usually looks completely different as blobs are quickly fed up with their current neighbors – however, no blob ever changes the meta-blob it lives in.

Due to increasing population, in recent years the process of deciding how to reform the borders has become more and more involved. Your help is required!

**Problem** Given the election outcomes of every meta-blob, determine how good the votes can be implemented. The goal is to find the maximum threshold $T$ such that the $T$ most requested neighbors of every meta-blob can be established. Evey blob can move around arbitrarily, however once the new borders are established, all inhabitants of a meta-blob must be connected, i.e. no meta-blob may be split. In addition, different meta-blobs may not overlap.

The following table is one example for election outcomes, the figure next to it gives a realizing layout for $T = 2$.

| meta- | election oucome | | | | |
|---|---|---|---|---|---|
| blob | 1st | 2nd | 3rd | 4th | 5th |
| 0 | 5 | 4 | 3 | 1 | 2 |
| 1 | 0 | 4 | 5 | 3 | 2 |
| 2 | 4 | 0 | 3 | 1 | 5 |
| 3 | 2 | 4 | 1 | 0 | 5 |
| 4 | 3 | 2 | 0 | 1 | 5 |
| 5 | 0 | 1 | 2 | 4 | 3 |




**Input** The first line of input contains the number of test cases $n$. Each test case begins with a line containing the number $0 < m \leq 200$ of meta-blobs; $m$ lines describing the election results of the different meta-blobs follow. The $i$-th line contains $m - 1$ zero based, space separated indices $0 \leq p_{ij} < m$ ordered descending by the number of votes each other meta-blob got in the election of meta-blob $i$ (i.e. $p_{i0}$ is the index of the meta-blob that got the most votes).

**Output** For every test case output a single line containing the maximum value $T \leq m-1$ such that there is a layout of the planet in which for all $i \in [0, m-1]$, $j \in [0, T-1]$ the meta-blobs $i$ and $p_{ij}$ are neighbors.

**Sample Input**

```
2
6
5 4 3 1 2
0 4 5 3 2
4 0 3 1 5
2 4 1 0 5
3 2 0 1 5
0 1 2 4 3
4
3 1 2
2 0 3
0 1 3
0 2 1
```

**Sample Output**

```
2
3
```

**Points** There are four testsets, worth 30, 30, 30 and 10 points respectively, for a total of 100 points.

1. For the first set, worth 30 points, you may assume $m \leq 10$.
2. For the second set, worth 30 points, you may assume $m \leq 50$.
3. There are no additional restrictions for the third set, worth 30 points.
4. The fourth set contains bordercases worth 10 points.