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Exercise Sheet Exam Preparation Week II

Algorithms Lab

Exercise 2 – *Portfolios revisited*

Several of your friends have recently asked you for investment advice. They want you to design their investment portfolio from the (prescribed) set of assets available in your country.

There are n assets called $1, \ldots, n$, where the cost of the asset i is c_i. Imagine you are in a perfect world and return of (one unit of) each asset (per unit of time) is a random variable with given expectation r_i for the i-th asset and covariance v_{ij} between asset i and asset j. The matrix formed by the covariances (v_{ij}) is symmetric and positive semidefinite.

A portfolio of assets $1, \ldots, n$ is a sequence of amounts $\alpha_1, \ldots, \alpha_n$ of each asset bought (where α_i is the amount of asset i in the portfolio). All α_i have to be nonnegative (you cannot sell short) but can attain arbitrary non-integer values (i.e., buying a fraction of a unit of an asset is perfectly acceptable).

The total cost C of a portfolio $\alpha_1, \ldots \alpha_n$ is $C := \sum_{i=1}^{n} \alpha_i c_i$, the total expected return is $R := \sum_{i=1}^{n} \alpha_i r_i$, and the total variance is $V := \sum_{i,j=1}^{n} \alpha_i \alpha_j v_{ij}$ (variance is a measure of risk in this setting).

You have m friends waiting for your advice, having different constraints on their portfolios. The i-th person is willing to invest at most C_i and is willing to have total variance at most V_i of his or her portfolio. Note that they are not allowed to borrow further money for investment, and any non-invested money has return 0.

Your task is to compute, for each of your friends, the maximum possible (under the given conditions) expected portfolio return.

Input The input consists of several test cases. Each of them starts with a line consisting of two integers n and m ($1 \leq n \leq 40$, $1 \leq m \leq 10$), where n is the number of assets and m the number of friends seeking advice. The following n lines consist of 2 integers each. The i-th line consists of two integers c_i and r_i ($1 \leq c_i \leq 10^6, -10^6 \leq r_i \leq 10^6$), separated by a space. The following n lines describe the covariances. The i-th line consists of n integers $v_{i1} \ldots v_{in}$ separated by spaces ($-10^6 \leq v_{ij} \leq 10^6$).

Each of the following m lines describes the individual investors and consists of two integers C_i and V_i ($1 \leq C_i \leq 10^6, 1 \leq V_i \leq 10^8$), separated by a space.

The input is terminated by a line `0 0` (i.e., test case with no assets and no portfolio requests).

There might be empty lines between the test cases.

Output For each test case, the output should consist of m lines, each containing the largest integer r, such that there is a portfolio that has expected return at least r and fulfills the cost and variance constraints.

Sample Input

```
1 1
2000 160
10000
1000 2500
3 4
2000 160
2000 160
2000 60
10000 0 0
0 10000 0
0 0 400
1000 2500
1000 1250
1000 466
1000 64
0 0
```

Sample Output

```
80
80
80
60
30
```

(Points)*100