Handbook of Statistics

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Contents

1 Definitions

1.1 Random variables and expectations

Definition 1.1. Random Experiment: an experiment whose outcomes are determined only by chance factors.

Definition 1.2. Sample Space: the set of all possible outcomes of a random experiment.

Definition 1.3. Event: the collection of none, one, or more than one outcomes from a sample space.

Definition 1.4. Random Variable: a variable whose numerical values are determined by chance factors. It is a function from the sample space to a set of real numbers.

Definition 1.5. Discrete Random Variable: if the set of all possible values of a random variable X is countable, then X is called a *discrete random variable.*

Definition 1.6. Probability Mass Function (pmf): let R be the set of all possible values of a random variable X, and $f(k) = P(X = k)$ for each k in R. Then $f(k)$ is called the *probability mass function* of X.

Definition 1.7. Continuous Random Variable: if the set of all possible values of X an interval or union of two or more nonoverlapping intervals in $\mathbb{R}$, then X is called a *continuous random variable.*

Definition 1.8. Probability Density Function (pdf): any real valued function $f(x)$ that satisfies the following requirements is called a *probability density function*:

$$f(x) \geq 0 \text{ for all } x, \text{ and } \int_{-\infty}^{\infty} f(x)\, dx = 1$$

Definition 1.9. Cumulative Distribution function (pdf):

Definition 1.10.

Definition 1.11.

Definition 1.12.

Definition 1.13.

2 Discrete uniform distribution

2.1 Description

Used to model experimental outcomes which are ”equally likely”.

2.1.1 Probability mass function

$$P(X = k) = \frac{1}{N}, \quad k = 1, \dots, N$$

2.1.2 Cumulative distribution function

$$P(X \leq k) = \frac{k}{N}, \quad k = 1, \dots, N$$

2.2 Plots

2.3 Moments

Mean	$\frac{N+1}{2}$
Variance	$\frac{(N-1)(N+1)}{2}$

3 Binomial distribution

3.1 Description

A binomial experiment involves n independent and identical trial such that each trial can result into one of the two possible outcomes: success of failure. If p is the probability of observing success in each trial, then the number of successes X that can be observed out of these n trials is referred to as the **binomial random variable with n trials and success probability** p, or $B(n, p)$.

3.1.1 Probability mass function

The probability of observing k successes out of n trials is given by the following probability mass function

$$P(X = k \mid n, p) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \dots, n$$

3.1.2 Cumulative distribution function

$$P(X \leq k \mid n, p) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}, \quad k = 0, 1, \ldots, n$$

Binomial distribution is often used to estimate or determine the proportion of individuals with a particular attribute in a large population. Suppose that a random sample of n units is drawn by sampling with replacement from a finite population or by sampling without replacement from a large population. The number of units that contain the attribute of interest in the sample follows a normal distribution.

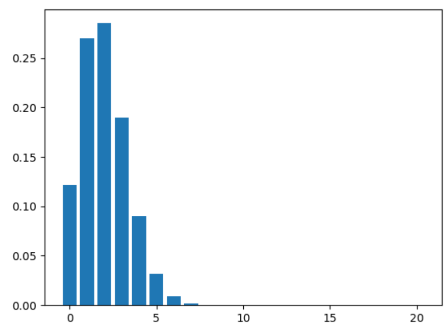
If the sample was drawn without replacement from a small finite population, the hypergeometric distribution should be used instead of the binomial.

3.2 Moments

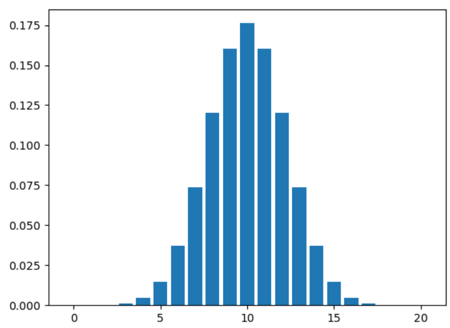
Mean	np
Variance	$np(1-p)$

3.3 Plots

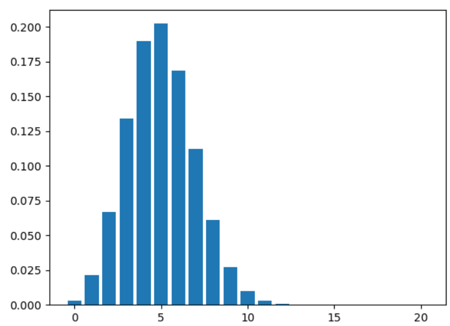
The distribution is right-skewed when $p < 0.5$, left-skewed when $p > 0.5$ and symmetric when $p = 0.5$.



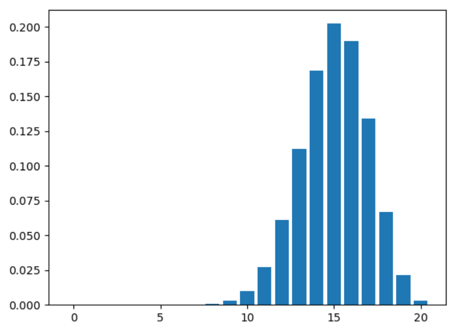
(a) $B(n, p), \ n = 20, p = 0.1$



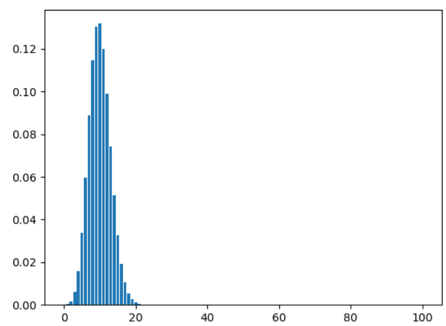
(b) $B(n, p), \ n = 20, p = 0.5$



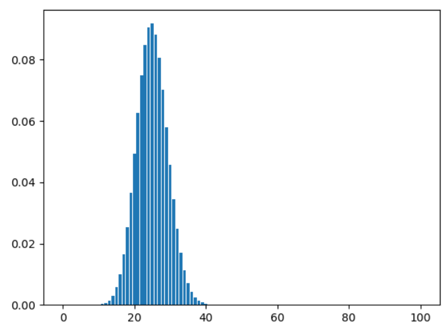
(c) $B(n, p), \ n = 20, p = 0.25$



(d) $B(n, p), \ n = 20, p = 0.75$



(e) $B(n, p), \ n = 100, p = 0.1$



(f) $B(n, p), \ n = 100, p = 0.25$

Figure 1: Binomial distribution

3.4 Properties

1. Let $X_1, \ldots, X_m$ be independent random variables with $X_i \sim B(n_i, p)$, $i = 1, 2, \ldots, m$. Then,
$$\sum_{i=1}^{m} X_i \sim B\left(\sum_{i=1}^{m} n_i, p\right)$$
2. Let $X_1, \ldots, X_m$ be independent Bernoulli(p) random variables with success probability p. That is: $P(X_i = 1) = p$, $P(X_i = 0) = 1 - p$, $i = 1, \ldots, n$. Then,
$$\sum_{i=1}^{m} X_i \sim B\left(n, p\right)$$

3.5 Examples

Example 3.1. *A fair die is rolled n times.*

- *The probability of obtaining exactly one 6 is* $n\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{n-1}$.
- *The probability of obtaining no 6 is* $\left(\frac{5}{6}\right)^{n}$.
- *The probability of obtaining at least one 6 is* $1 - \left(\frac{5}{6}\right)^{n}$.
- *The number of trials needed for the probability of at least one 6 to be* $\geq \frac{1}{2}$ *is given by the smallest integer n such that* $1 - \left(\frac{5}{6}\right)^{n} \geq \frac{1}{2}$, *so that* $n \geq \frac{\log 2}{\log 1.2} \approx 3.8$.