Notation:

1. k is the BMC bound. All paths have length $k+1$.
2. i, i_1, i_2 are FSM states.
3. v is a vertex of the scenario tree.
4. a is an action, A is an action sequence.

Note: guard formulae f are treated as sub-events.

1 Additional exist-variables and exist-constraints

New variables $z_{i,z,e,f}$: whether there is action z somewhere on a transition from state i for action e and formula f.

Additional constrains:

1. $\bigwedge_{i_1} \bigwedge_{(e,f)} \bigvee_{i_2} y_{i_1,i_2,e,f}$: optional completeness constraint (can influence LTL semantics for wasEvent).
2. $\bigwedge_{i_1} \bigwedge_{a} \bigwedge_{(e,f)} \left(z_{i_1,a,e,f} \to \bigvee_{i_2} y_{i_1,i_2,e,f} \right)$: if there is an action, then there is a transition (this constraint is unnecessary if completeness is enabled).
3. $\bigwedge_{v} \bigvee_{i} \left(x_{v,i} \wedge \bigwedge_{(e,f,A) \in \mathrm{EdgesFrom}(v)} \left(\left(\bigwedge_{a \in A} z_{i,a,e,f} \right) \wedge \left(\bigwedge_{a \notin A} \neg z_{i,a,e,f} \right) \right) \right)$: z-variables correspond to scenarios. This constraint is stronger than the constraint "each node has at least one color".

2 Forall-variables

States of Kripke structure, as well as positions j of the path correspond to the transitions of the FSM.

1. $\sigma_{i,j}$: j-th position of the path is a transition from state i of the FSM.
2. $\epsilon_{e,f,z}$: j-th position of the path is a transition with event e and formula f.
3. $\zeta_{a,j}$: j-th position of the path is a transition with action a (and possibly some other actions).
4. h_α: optional variables for subterms α of the LTL formula. They are generated during formula translation. Without them, the QBF size is exponential of k.

Atomic predicates can be expressed as follows:

1. $\mathrm{wasEvent}(e)_j = \bigvee_{(e,f)} \epsilon_{e,f,j}$.
2. $\mathrm{wasAction}(a)_j = \zeta_{a,j}$.

Note: action order is not captured by these predicates.

3 Reduction formula and forall-constraints

$$\exists \{x_{v,i}\}, \{y_{i_1,i_2,e,f}\}, \{z_{i,a,e,f}\} \quad \forall \{\sigma_{i,j}\}, \{\epsilon_{e,f,z}\}, \{\zeta_{a,j}\}, \{h_\alpha\}$$

$$S \wedge \left(\neg H \vee \neg [[M]]_k \vee \neg \left(\neg L_k \wedge [[g]]_k^0 \vee \bigvee_{l=0}^{k} \left({}_l L_k \wedge {}_l[[g]]_k^0 \right) \right) \right)$$

1. g is the LTL formula to verify, which is negated and converted to negation-normal form (all negations are before atomic predicates).
2. S are the constrains from scenarios (with the ones from Section 1).
3. $H = \bigwedge_\alpha (h_\alpha = \alpha)$ are optional constraints which define subterm variables.
4. $[[M]]_k = \sigma_{0,0} \wedge A_1 \wedge A_2 \wedge B \wedge C \wedge D$: the path is initialized (starts from state 0 of the FSM) and is correct. Correctness constraints A_1, A_2, B, C, D are defined below.

5. $L_k = \bigvee_{l=0}^{k} {}_lL_k$: the path is looping for some l.

6. ${}_lL_k = \bigvee_{i_1} \bigvee_{i_2} \bigvee_{(e,f)} (\sigma_{i_1,k} \wedge \epsilon_{e,f,k} \wedge \sigma_{i_2,l} \wedge y_{i_1,i_2,e,f})$: there exists a loop from the last position of the path to some position l. Note that this looping edge is not included in the path, and thus $y_{i_1,i_2,e,f}$ might not hold if removed from the constraint.

7. $A_1 = \bigwedge_{j=0}^{k} \left(\bigwedge_{\{i_1 \neq i_2\}} \neg (\sigma_{i_1,j} \wedge \sigma_{i_2,j}) \right)$: there is no transition in the path from some two states simultaneously.

8. $A_2 = \bigwedge_{j=0}^{k} \left(\bigwedge_{\{(e_1,f_1) \neq (e_2,f_2)\}} \neg (\epsilon_{e_1,f_1,j} \wedge \epsilon_{e_2,f_2,j}) \right)$: there is no transition in the path for some two (e,f)-pairs simultaneously.

9. $B = \bigwedge_{j=0}^{k} \left(\left(\bigvee_{i} \sigma_{i,j} \right) \wedge \left(\bigvee_{(e,f)} \epsilon_{e,f,j} \right) \right)$: each transition in the path is from some state i and for some (e,f)-pair.

10. $C = \left(\bigwedge_{j=0}^{k-1} \bigwedge_{i_1} \bigwedge_{i_2} \bigwedge_{(e,f)} (\sigma_{i_1,j} \wedge \epsilon_{e,f,j} \wedge \sigma_{i_2,j+1} \rightarrow y_{i_1,i_2,e,f}) \right) \wedge \left(\bigwedge_{i_1} \bigwedge_{(e,f)} \left(\sigma_{i_1,k} \wedge \epsilon_{e,f,k} \rightarrow \bigvee_{i_2} y_{i_1,i_2,e,f} \right) \right)$: transitions in the path correspond to y-variables (the path is indeed a path in the Kripke structure). The part after the topmost AND is not required, if completeness constrain is present (Section 1).

11. $D = \bigwedge_{j=0}^{k} \bigwedge_{i_1} \bigwedge_{a} \bigwedge_{(e,f)} (\sigma_{i_1,j} \wedge \epsilon_{e,f,j} \rightarrow (\zeta_{a,j} = z_{i_1,a,e,f}))$: each edge in the path must have correct (corresponding to z-variables) actions.

12. $[[g]]_k^0$ and ${}_l[[g]]_k^0$ are formula translations (see sections 2.4–2.5 of P. Jackson, D. Sheridan. A compact linear translation for bounded model checking. 2006).