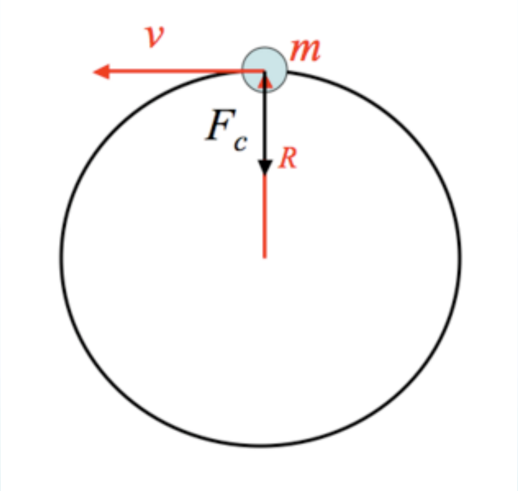
Circular Motion



What you need to know

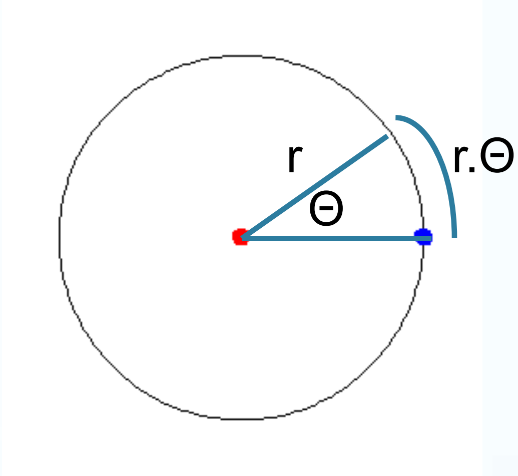
Objectives

- Explain the characteristics of uniform circular motion
- Derive the equation for centripetal acceleration of an object moving in a circle at constant speed
- Understand the role of centripetal force and understand what provides it.
- Understand that centrifugal force does not play a role in circular motion
- Explain and apply the relationship between the angular velocity and tangenial velocity.

Essential Questions

- What are the applications of circular motion?
- What is the difference between centripetal and centrifugal force and is centrifugal force real ?
- What forces keep satellites in orbit?
- What evidence is there that a falling apple and an orbiting planet are identical situations?
- How does apparent weight vary during circular motion?

Angular Displacement θ and Angular Velocity ω



In **uniform** circular motion an object covers equal angles θ in equal times. We say it travels with constant **angular velocity** $\omega = \frac{\theta}{t}$

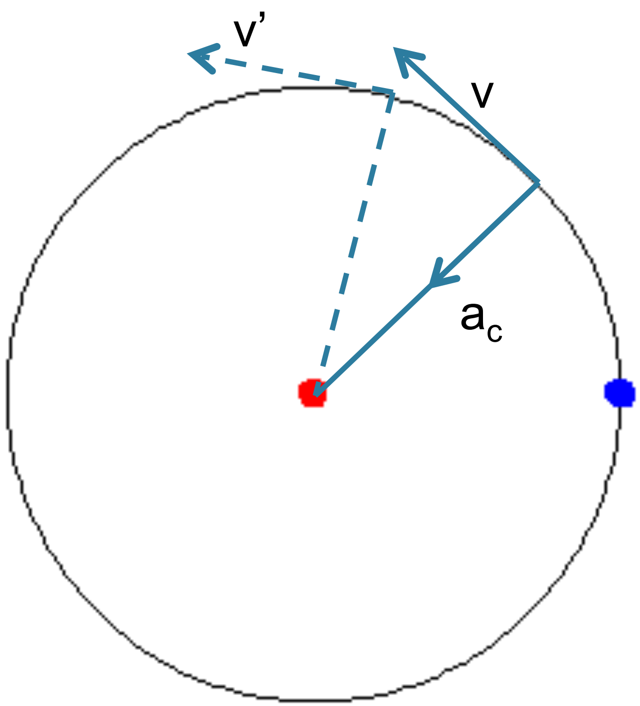
The object also has a (tangenial) velocity v. It covers an arc of distance r.θ (θ in radians) in time t. So $v = \frac{r\theta}{t} = \omega.r$

$$\boldsymbol{v = \omega.r}$$

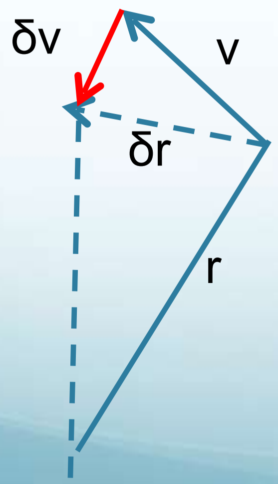
Definitions:
Period (T) – The time for **ONE** revolution
Frequency (f) : Number or complete revolutions per second

Centripetal Acceleration

In Uniform Circular Motion,
speed is constant but **velocity is not!**



Direction of Velocity is that of the **TANGENT** at every point along the circle.
So because velocity is changing (in direction) the body accelerates continuously.
It accelerates toward the centre of the circle.



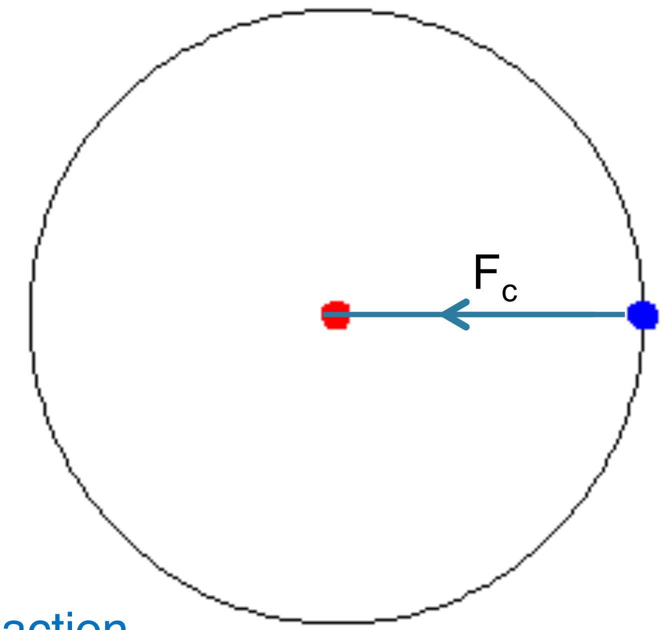
Because of similar triangles, $\frac{\delta v}{v} = \frac{\delta r}{r} \therefore \delta v = \delta r.\frac{v}{r}$

So $a = \frac{\delta v}{\delta t} = \frac{\delta r}{\delta t}\frac{v}{r} = v.\frac{v}{r} = \frac{v^2}{r} = \omega^2 r$

Centripetal Force

The centripetal acceleration is provided by the **centripetal force.**

$$F_c = m\omega^2 r \quad \text{or} \quad F_c = \frac{mv^2}{r}$$



Who provides the centripetal force?

Particle rotated by a string: tension in the string
Car turning: friction with the road surface
Planet rotating around the sun: Gravitational attraction

Is there a centrifugal force?

Yes but it plays no role in circular motion.
It is merely **the reaction** to the centripetal force.

More Useful Formulas

Remember that:

T is the period and

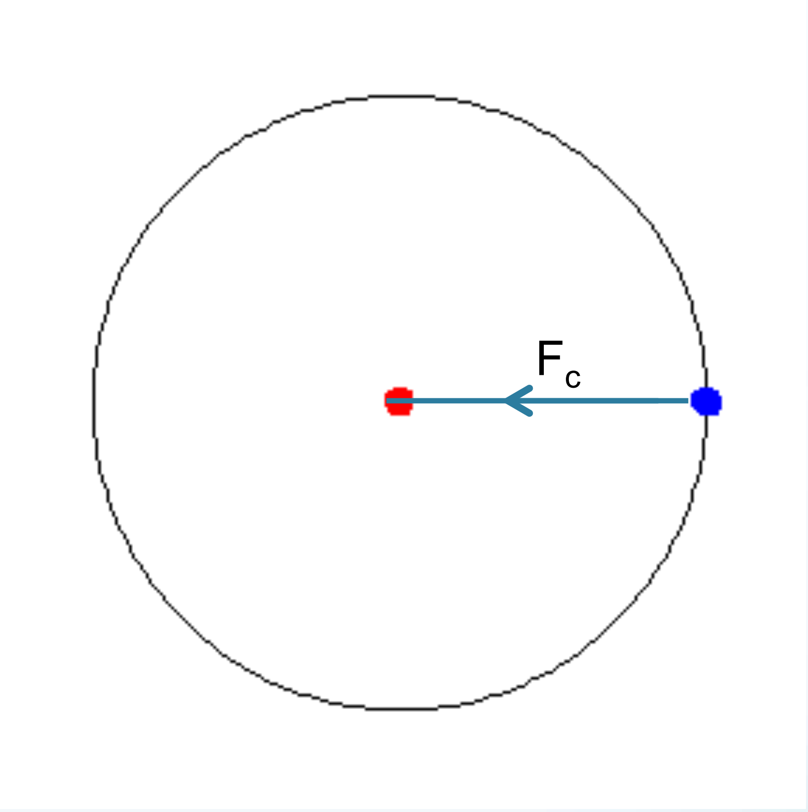
f is the frequency.

So, because f is the number of full revolutions in 1 sec

$$f = \frac{1}{T}$$

also note that $\omega = \frac{2\pi}{T}$

then you can see that $\omega = 2\pi f$



Example

A Ferris wheel with a diameter of 18.0 meters rotates 4 times in 1 minute. a) Calculate the velocity of the Ferris wheel. b) Calculate the centripetal acceleration of the Ferris wheel at a point along the outside. c) Calculate the centripetal force a 40 kg child experiences.

$$v_c = \frac{2\pi r}{T} = \frac{2(3.14)9}{15} =$$ **3.77 m/s**

$$a_c = \frac{v^2}{r} \rightarrow \frac{v^2}{9} =$$ **1.58 m/s/s**

$$F_c = \frac{mv^2}{r} \rightarrow \frac{(40)v^2}{9} =$$ **63.17 N**

$$or \quad F_c = ma_c \rightarrow (40)(a_c) =$$ **63.17 N**

Centripetal Force and Turning

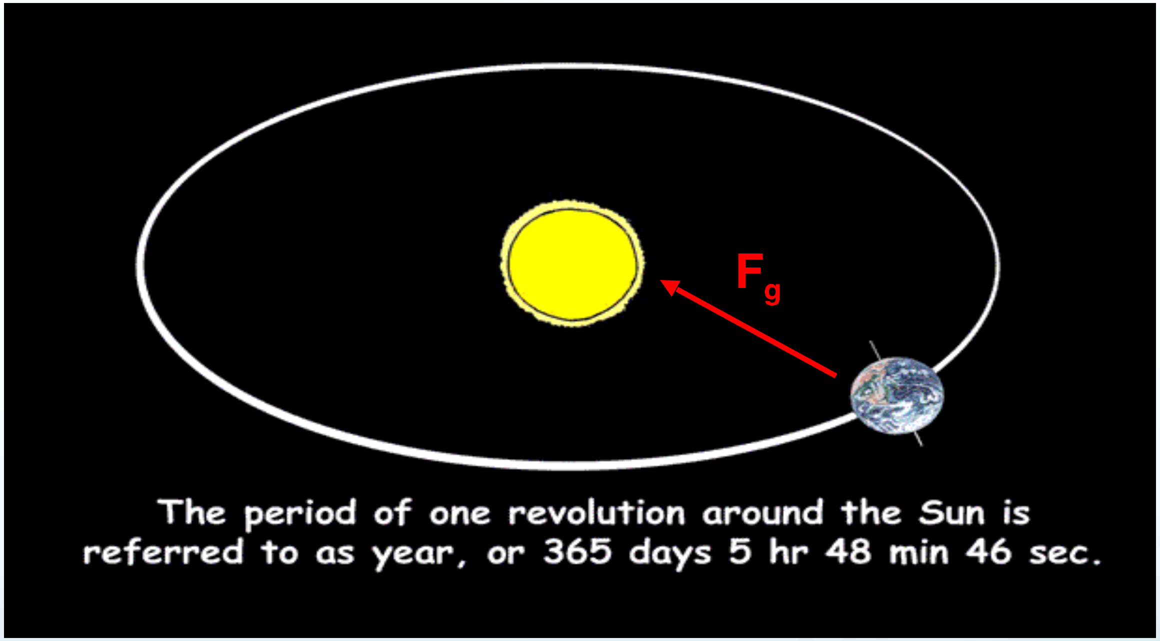
Rounding a curve

Let’s draw an FBD.

What is the F_c?

Centripetal Force and orbits

The earth in orbit around the sun

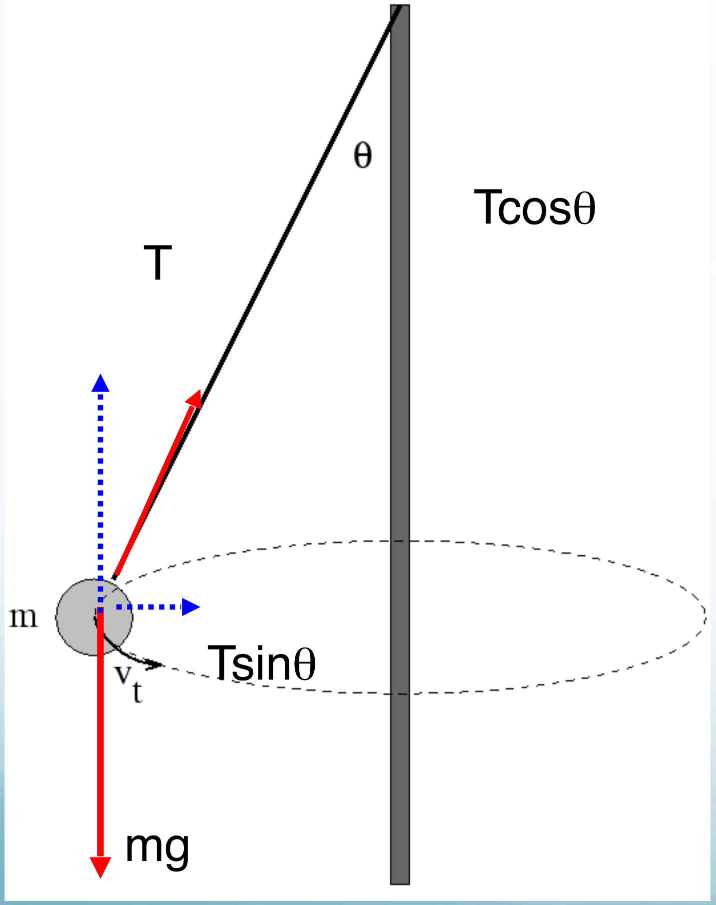


What is the F_c?

It is force of attraction between earth and sun (gravitational force)

$$F_c = G\frac{mM}{r^2}$$

Example of circular motion



What is the F_c? **Tsinθ**