Momentum and Impulse

Momentum

- Momentum describes “mass in motion”.
- An moving object that has mass also has momentum
- $\overrightarrow{momentum} = mass * \overrightarrow{velocity}$

  (or in symbolic form) $\vec{p} = m * \vec{v}$
- Momentum is a **vector** quantity
- *Units*: kg.m.s^{-1} or N.s

Examples

- Find the momentum of a car (1200 kg) moving north with speed 30 m/s
- Does a sleeping elephant have momentum?
- Does a moving electron have momentum?
- Does an x-ray have momentum?

Momentum and Newton's 2nd Law

- Newton's 2nd law of motion (F=m.a) was originally stated as: the sum of the forces acting on a body is equal to the rate of change of its momentum.
- $$\Sigma F = \frac{\Delta(mv)}{\Delta t},$$
- when m is constant,
- $$\Sigma F = \frac{\Delta(mv)}{\Delta t} = m\frac{\Delta v}{\Delta t} = ma$$

Impulse and momentum

- Let's rearrange the previous equation *(we use F instead of ΣF but we mean the sum of forces (net force)*
- $F = \dfrac{\Delta(mv)}{\Delta t}$ can be re-written as: $F\,.\,\Delta t = \Delta(mv)$
- The quantity $F\,.\,\Delta t$ is known as **Impulse** (it is a chunk of momentum)
- $F\,.\,\Delta t = \Delta(mv) = mv_2 - mv_1$
- **impulse** is equal to the **change of momentum**
- *impulse is the same as momentum but we reserve its use to mean a chunk of momentum's caused by a striking force or a collision*

Dynamics from a momentum perspective

- A car of 1000 kg travelling at 20 m/s is stopped by a braking force in 4 s. Find the average force:

- (i) (using $F=ma$) deceleration = 20/4 = 5 ms^{-2}. So $F=ma$ = 1000. 5 = 5000 N

- (ii) (using $Ft=\Delta mv$)

  change in momentum= 20000 Ns.

  So F.4 = 20000 giving F = 5000 N

Why do we need $Ft=\Delta(mv)$

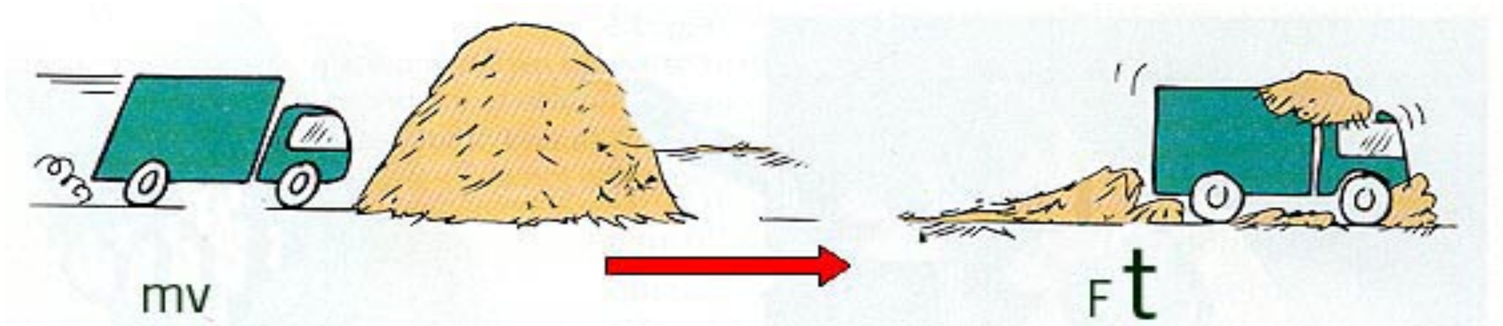
when we have $F = ma$

- Here are two problems that we cannot solve using F= m.a

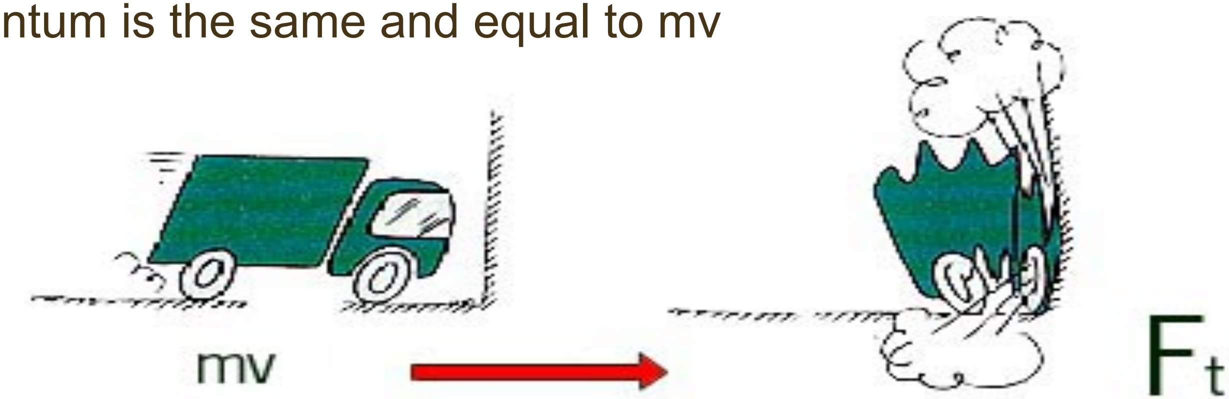
- A conveyor belt carrying sand runs at constant speed 2 m/s. Sand is poured on it at a rate of 5 kg per second. What is the force needed to maintain this speed?

- A boat’s propeller pushes 10 tonnes of water back per second with a speed of 10m/s. What is the force it needs to achieve this? How is this action propelling the boat forward?

Why do we need Ft=Δ(mv)
when we have F = ma



in both cases m.v becomes zero so the change of momentum is the same and equal to mv



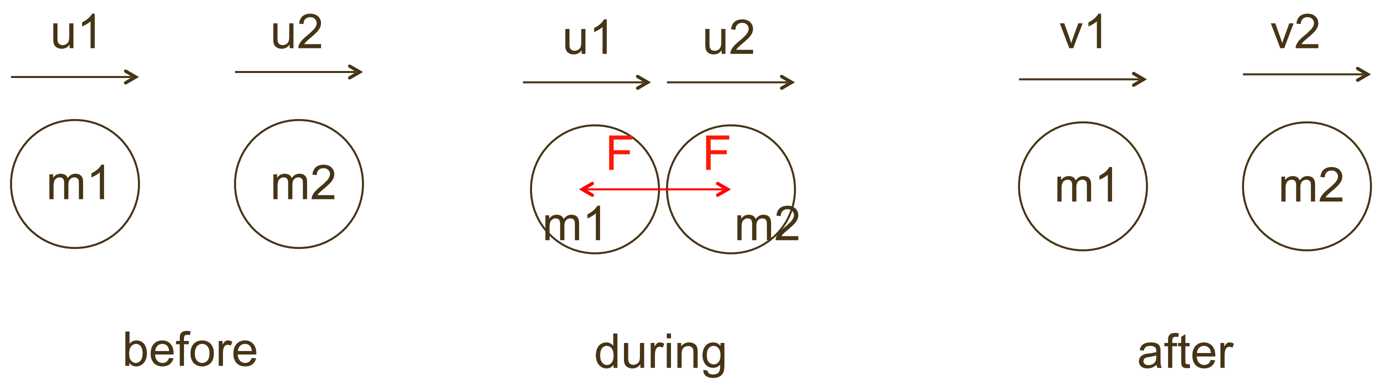
in the top instance we have a small F and a large t, in the second case we have a large F and a small t.

Conservation of Momentum

- Because or Action-Reaction (Newton's 3^{rd} law) when two objects collide the impulse that one gives to the other is equal and opposite to the impulse that the other gives to the one.
- So in collisions **total momentum is conserved**.
- *provided no external force acts on the system*

Conservation of Momentum

Consider a collision:



Consider momentum for m1: $m1.u1 - F.\Delta t = m1.v1$

Consider momentum for m2: $m2.u2 + F.\Delta t = m2.v2$

ADD these two

$$m1.u1 + m2.u2 = m1.v1 + m2.v2$$

Momentum and Energy

- Momentum and energy are different physcal quantities. All bodies that have momentum also have energy (kinetic) but not all bodies that have energy also have momentum. Eg a brick 2m above the ground has potential energy but no momentum. A photon has energy E=hf but no momentum.
- Here is useful formula that connects kinetic energy to momentum:
- $E_k = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$, learn this !

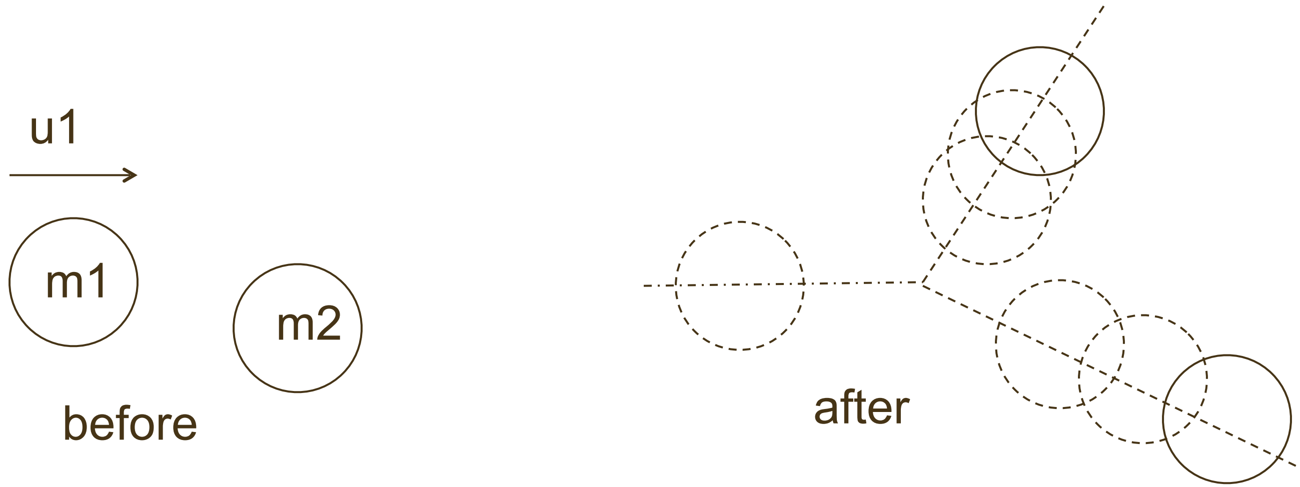
Elastic and Inelastic Collisions

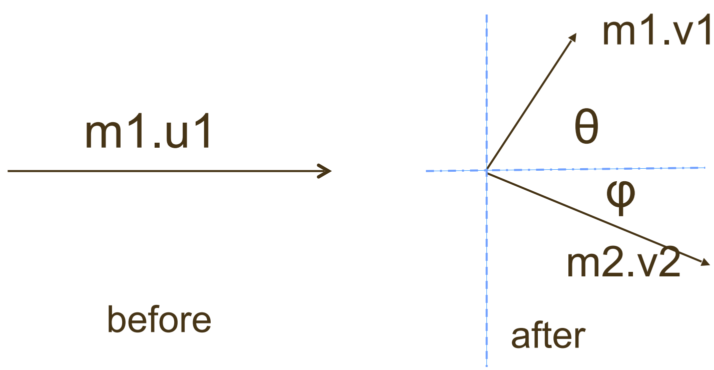
- While **momentum is conse**rved in collisions *(provided no external forces …   )* kinetic energy is <u>not always</u> conserved

- When kinetic energy is conserved in collisions we call these elastic collisions *(in practice perfectly elastic collisions occur only in particle physics, nearly elastic collisions occur between billiard balls or between steel balls)*

Collisions in 2-dimensions

Conservation of momentum applies in 2 and 3 dimensional collisions

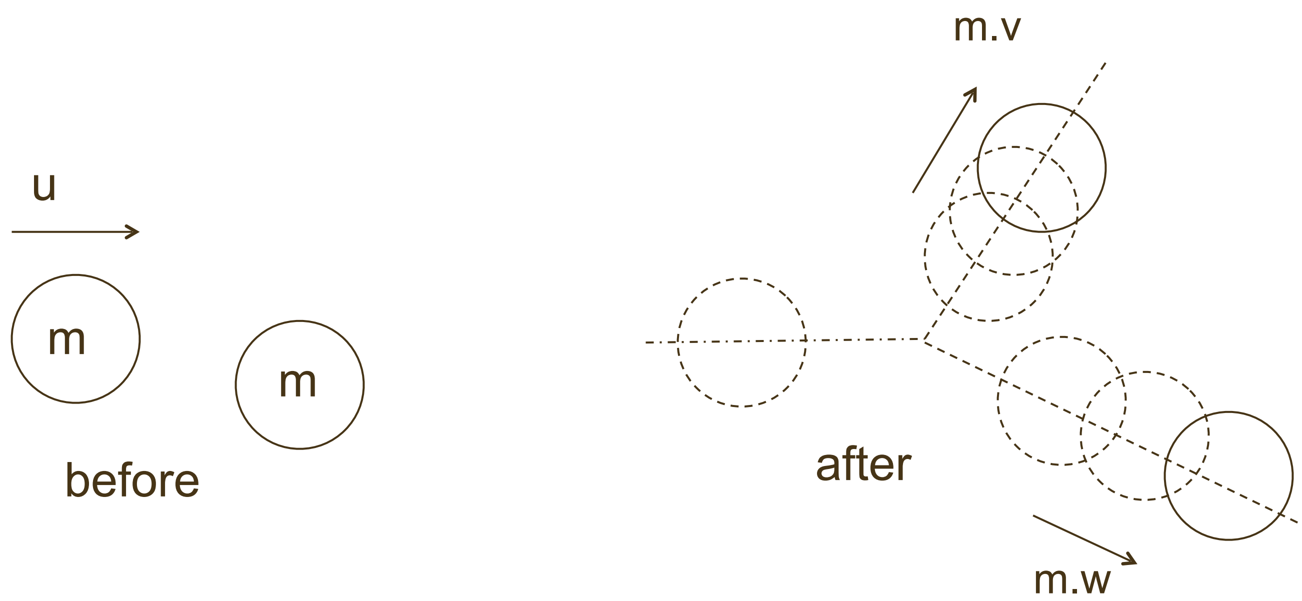




Conservation of momentum means: $m1.\overrightarrow{u1} = m1.\overrightarrow{v1} + m2.\overrightarrow{v2}$

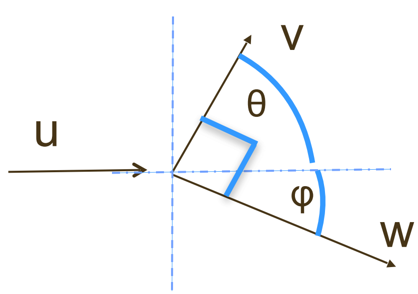
(which can be written as
m1.u1 = m1.v1.cosθ+m2.v2.cosφ
and m1.v1.sinθ = m2.v2.sinφ *)*

special case of elastic collision in 2-d



Conservation of momentum implies that $m\,.\,\vec{u} = m\,.\,\vec{v} + m\,.\,\vec{w}$

Conservation of kinetic energy implies that $\frac{1}{2}m\,.\,u^2 = \frac{1}{2}m\,.\,v^2 + \frac{1}{2}m\,.\,w^2$



the only that both $\vec{u} = \vec{v} + \vec{w}$ and $u^2 = v^2 + w^2$ are satisfied at the same time is if the angle between v and w is 90 °

Summary

- Momentum is the vector m.v
- Impulse is momentum (expressed as F.t)
- Force is rate of change of momentum
- Momentum is conserved (when no external forces are acting)