1. Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\frac{dy}{dx}$

(3)

(b) $\frac{d^2y}{dx^2}$

(2)

2. Given that

$$a = \frac{1}{64}x^2 \qquad b = \frac{16}{\sqrt{x}}$$

express each of the following in the form kx^n where k and n are simplified constants.

(a) $a^{\frac{1}{2}}$

(1)

(b) $\dfrac{16}{b^3}$

(1)

(c) $\left(\dfrac{ab}{2}\right)^{-\frac{4}{3}}$

(2)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Write $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ in the form $a\sqrt{3}+b\sqrt{5}$ where a and b are integers to be found.

(3)

(b) Hence, or otherwise, solve

$$\left(x+5\sqrt{3}\right)\sqrt{5} = 40 - 2x\sqrt{3}$$

giving your answer in simplest form.

(3)

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4.

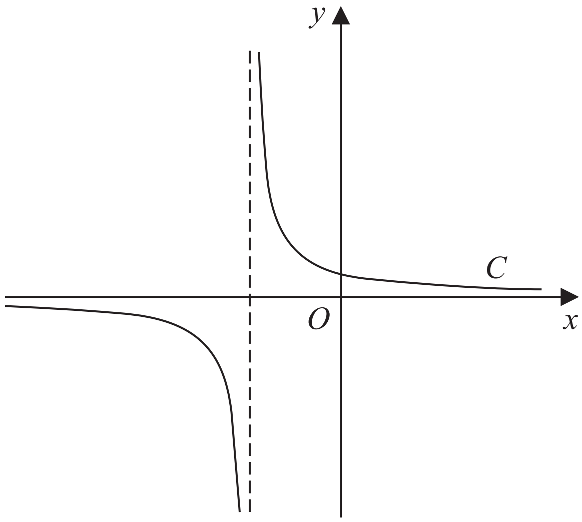


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \dfrac{1}{x+2}$

(a) State the equation of the asymptote of C that is parallel to the y-axis.

(1)

(b) Factorise fully $x^3 + 4x^2 + 4x$

(2)

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes.

(3)

(d) Hence state the number of real solutions of the equation

$$(x + 2)(x^3 + 4x^2 + 4x) = 1$$

giving a reason for your answer.

(1)

Question 4 continued

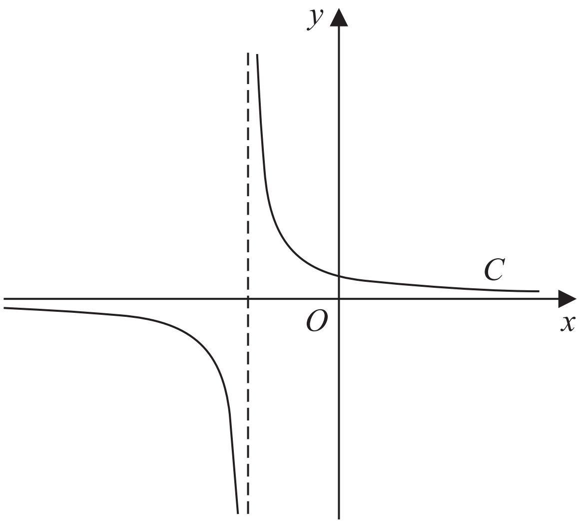
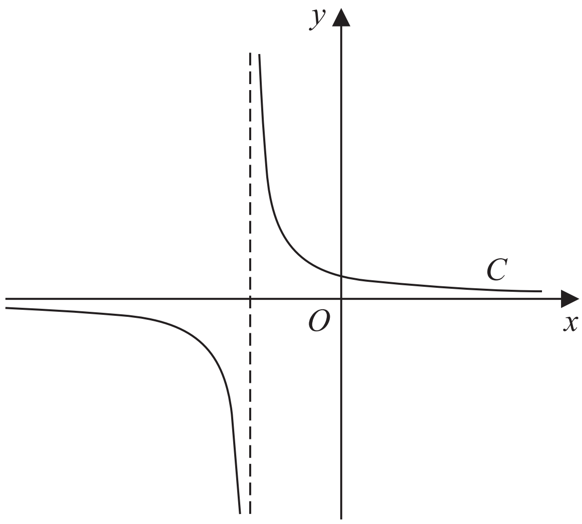


Diagram 1



copy of Diagram 1

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

(Total for Question 4 is 7 marks)

5.

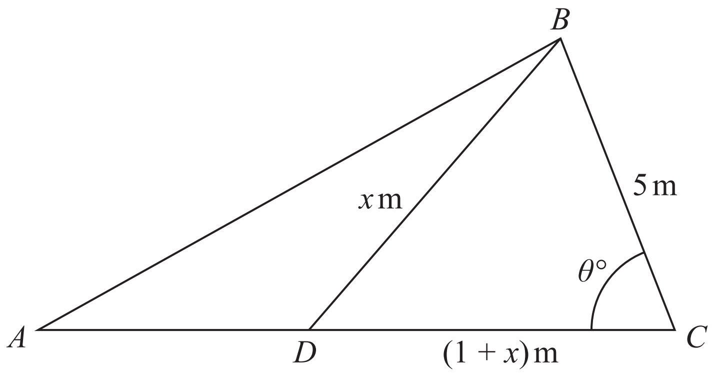


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle ABD joined to triangle BCD.

Given that

- $BD = x\,\text{m}$
- $CD = (1 + x)\,\text{m}$
- $BC = 5\,\text{m}$
- angle $BCD = \theta°$

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$

(2)

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- ADC is a straight line

(b) find the area of triangle ABC, giving your answer, in m^2, to one decimal place.

(5)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The equation

$$4(p - 2x) = \frac{12 + 15p}{x + p} \qquad x \neq -p$$

where p is a constant, has two distinct real roots.

(a) Show that

$$3p^2 - 10p - 8 > 0$$

(3)

(b) Hence, using algebra, find the range of possible values of p

(3)

7. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$
- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $\mathrm{f}(x)$.

(6)

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8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$xy = \frac{15}{2} - 5x \qquad x \neq 0$$

The curve C_2 has equation

$$y = x^3 - \frac{7}{2}x - 5$$

(a) Show that C_1 and C_2 meet when

$$2x^4 - 7x^2 - 15 = 0$$

(2)

Given that C_1 and C_2 meet at points P and Q

(b) find, using algebra, the exact distance PQ

(5)

9.

Diagram **NOT** accurately drawn

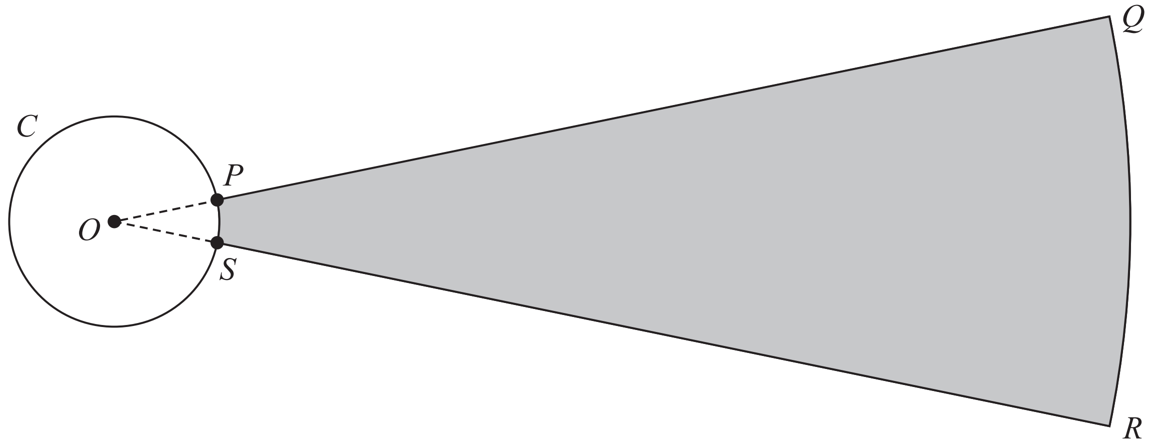

Figure 3

Figure 3 shows the plan view of the area being used for a ball-throwing competition.

Competitors must stand within the circle C and throw a ball as far as possible into the target area, $PQRS$, shown shaded in Figure 3.

Given that

- circle C has centre O
- P and S are points on C
- $OPQRSO$ is a sector of a circle with centre O
- the length of arc PS is 0.72 m
- the size of angle POS is 0.6 radians

(a) show that $OP = 1.2$ m

(1)

Given also that

- the target area, $PQRS$, is $90\,\text{m}^2$
- length $PQ = x$ metres

(b) show that

$$5x^2 + 12x - 1500 = 0$$

(3)

(c) Hence calculate the total perimeter of the target area, $PQRS$, giving your answer to the nearest metre.

(3)

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10.

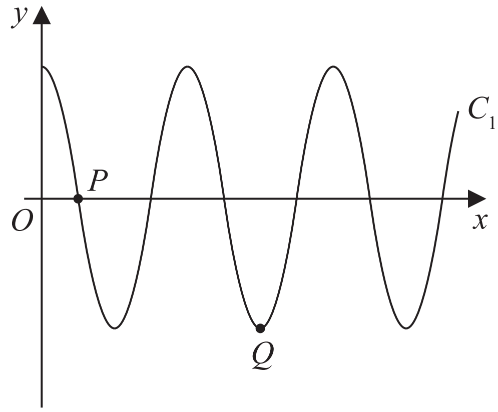

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 3\cos\left(\frac{x}{n}\right)^\circ \qquad x \geqslant 0$$

where n is a constant.

The curve C_1 cuts the positive x-axis for the first time at point $P(270, 0)$, as shown in Figure 4.

(a) (i) State the value of n

(ii) State the period of C_1

(2)

The point Q, shown in Figure 4, is a minimum point of C_1

(b) State the coordinates of Q.

(2)

The curve C_2 has equation $y = 2\sin x^\circ + k$, where k is a constant.

The point $R\left(a, \frac{12}{5}\right)$ and the point $S\left(-a, -\frac{3}{5}\right)$, both lie on C_2

Given that a is a constant less than 90

(c) find the value of k.

(2)

11.

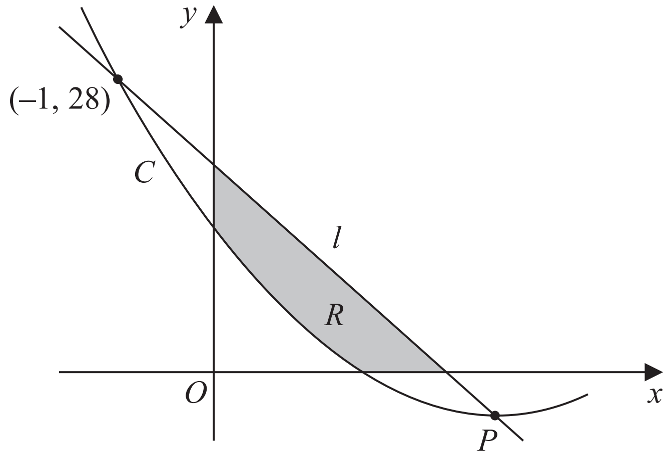

Figure 5

Figure 5 shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)