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1. Given that

$$(3pq^2)^4 \times 2p\sqrt{q^8} \equiv ap^bq^c$$

find the values of the constants a, b and c.

(3)

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2. $f(x) = 3 + 12x - 2x^2$

(a) Express $f(x)$ in the form

$$a - b(x + c)^2$$

where a, b and c are integers to be found.

(3)

The curve with equation $y = f(x) - 7$ crosses the x-axis at the points P and Q and crosses the y-axis at the point R.

(b) Find the area of the triangle PQR, giving your answer in the form $m\sqrt{n}$ where m and n are integers to be found.

(4)

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3.

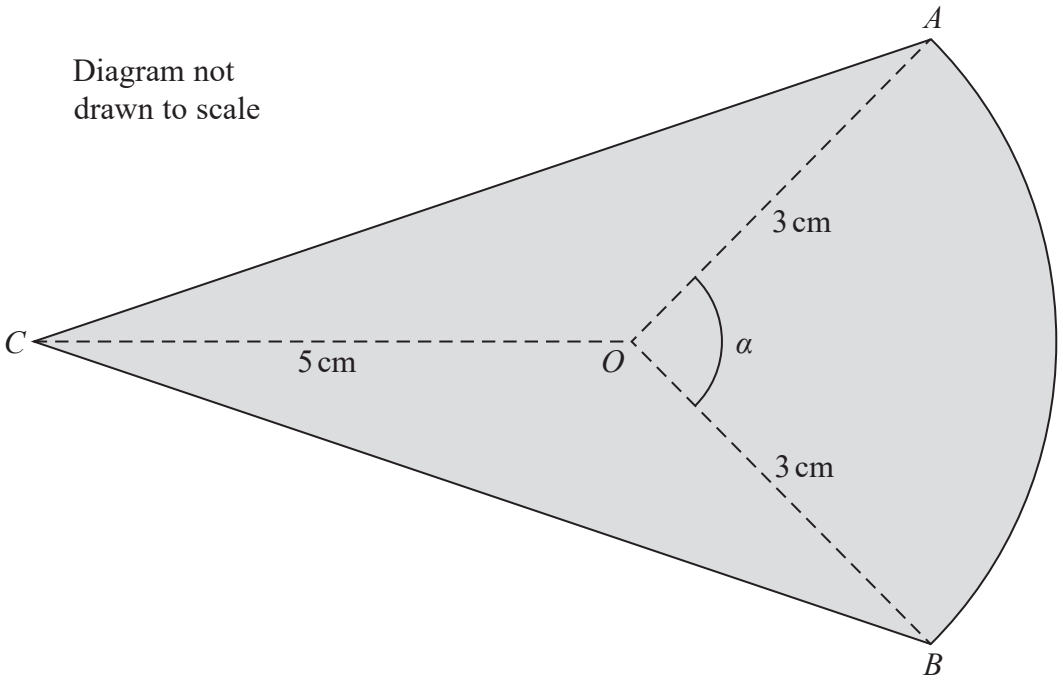

Figure 1

Figure 1 shows the design for a badge.

The design consists of two congruent triangles, AOC and BOC, joined to a sector AOB of a circle centre O.

- Angle $AOB = \alpha$
- $AO = OB = 3\,\text{cm}$
- $OC = 5\,\text{cm}$

Given that the area of sector AOB is $7.2\,\text{cm}^2$

(a) show that $\alpha = 1.6$ radians.

(2)

(b) Hence find

(i) the area of the badge, giving your answer in cm^2 to 2 significant figures,

(ii) the perimeter of the badge, giving your answer in cm to one decimal place.

(8)

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4. Use algebra to solve the simultaneous equations

$$y - 3x = 4$$
$$x^2 + y^2 + 6x - 4y = 4$$

You must show all stages of your working.

(7)

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5. (i)

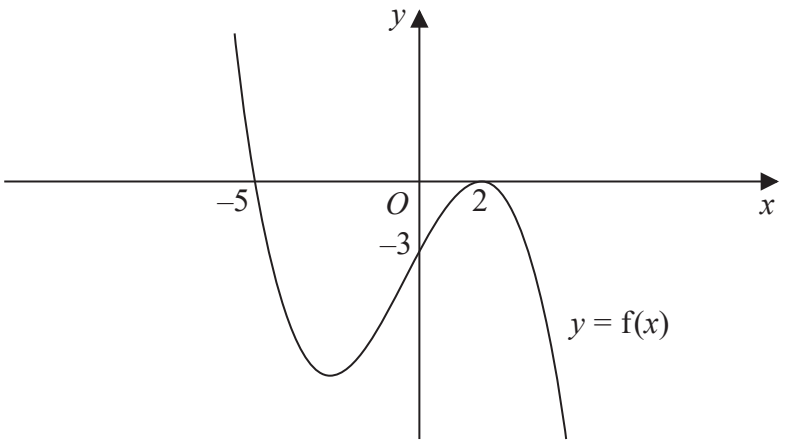

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the points $(-5, 0)$ and $(0, -3)$ and touches the x-axis at the point $(2, 0)$.

On separate diagrams sketch the curve with equation

(a) $y = f(x + 2)$

(b) $y = f(-x)$

On each diagram, show clearly the coordinates of all the points where the curve cuts or touches the coordinate axes.

(6)

(ii)

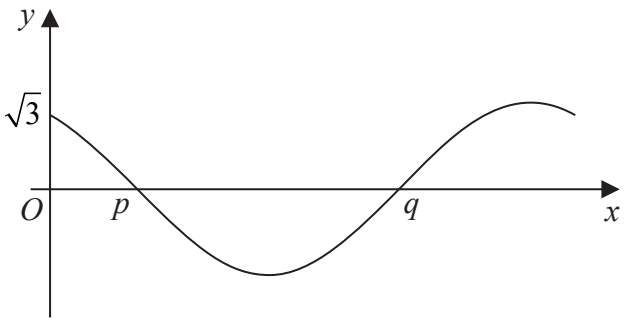

Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = k\cos\left(x + \frac{\pi}{6}\right) \qquad 0 \leqslant x \leqslant 2\pi$$

where k is a constant.

The curve meets the y-axis at the point $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

Find

(a) the value of k,

(b) the exact value of p and the exact value of q.

(3)

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Question 8 continued

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6. The point A has coordinates $(-4, 11)$ and the point B has coordinates $(8, 2)$.

(a) Find the gradient of the line AB, giving your answer as a fully simplified fraction.

(2)

The point M is the midpoint of AB. The line l passes through M and is perpendicular to AB.

(b) Find an equation for l, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

(4)

The point C lies on l such that the area of triangle ABC is 37.5 square units.

(c) Find the two possible pairs of coordinates of point C.

(5)

Leave blank

7. The curve C has equation

$$y = \frac{1}{2 - x}$$

(a) Sketch the graph of C. On your sketch you should show the coordinates of any points of intersection with the coordinate axes and state clearly the equations of any asymptotes.

(3)

The line l has equation $y = 4x + k$, where k is a constant.

Given that l meets C at two distinct points,

(b) show that

$$k^2 + 16k + 48 > 0$$

(4)

(c) Hence find the range of possible values for k.

(4)

Leave blank

8. The curve C has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

(4)

The line l_1 is the tangent to C at the point where $x = 6$

(b) Find the equation of l_1, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

The line l_2 is the tangent to C at the point where $x = \alpha$

Given that l_1 and l_2 are parallel and distinct,

(c) find the value of α

(3)

Leave blank

9. A curve with equation $y = f(x)$ passes through the point (9, 10).

Given that

$$f'(x) = 27x^2 - \frac{21x^3 - 5x}{2\sqrt{x}} \qquad x > 0$$

find $f(x)$, fully simplifying each term.

(6)