Leave blank

3. The function f is defined by

$$f : x \to \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $f(x) = \frac{2}{x-1}, \quad x > 1.$

(4)

(b) Find $f^{-1}(x)$.

(3)

The function g is defined by

$$g : x \to x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $fg(x) = \frac{1}{4}$.

(3)

blank

8. The functions f and g are defined by

$$\mathrm{f}: x \mapsto 1 - 2x^3, \ \ x \in \mathbb{R}$$

$$\mathrm{g}: x \mapsto \frac{3}{x} - 4, \ \ x > 0, \ \ x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$\mathrm{gf}: x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $\mathrm{gf}(x) = 0$.

(2)

4. The function f is defined by

$$\mathrm{f} : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \qquad x > 3.$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{x+1}, \quad x > 3.$

(4)

(b) Find the range of f.

(2)

(c) Find $\mathrm{f}^{-1}(x)$. State the domain of this inverse function.

(3)

The function g is defined by

$$\mathrm{g} : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $\mathrm{fg}(x) = \dfrac{1}{8}$.

(3)

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5.

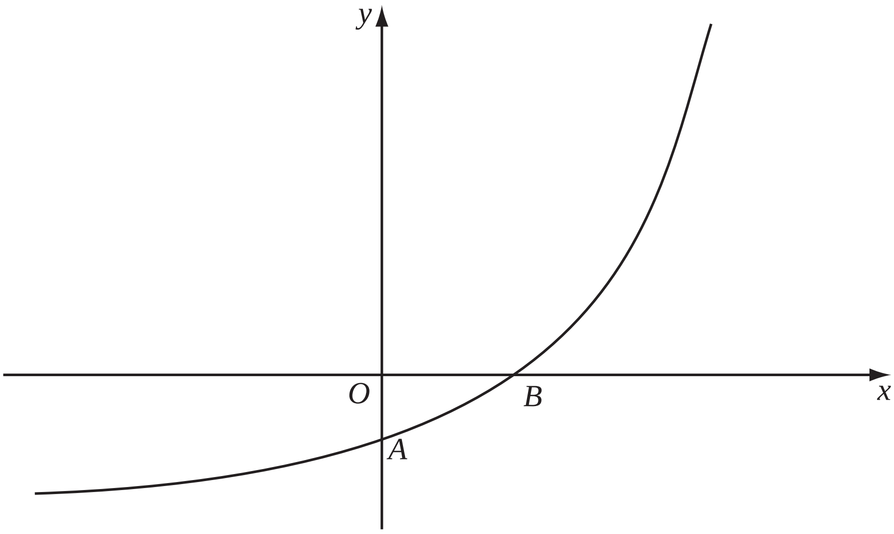

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$, where k is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

(b) $y = \mathrm{f}^{-1}(x)$.

(2)

Show on each sketch the coordinates, in terms of k, of each point at which the curve meets or cuts the axes.

6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \; x \neq 5$$

(a) Find $f^{-1}(x)$.

(3)

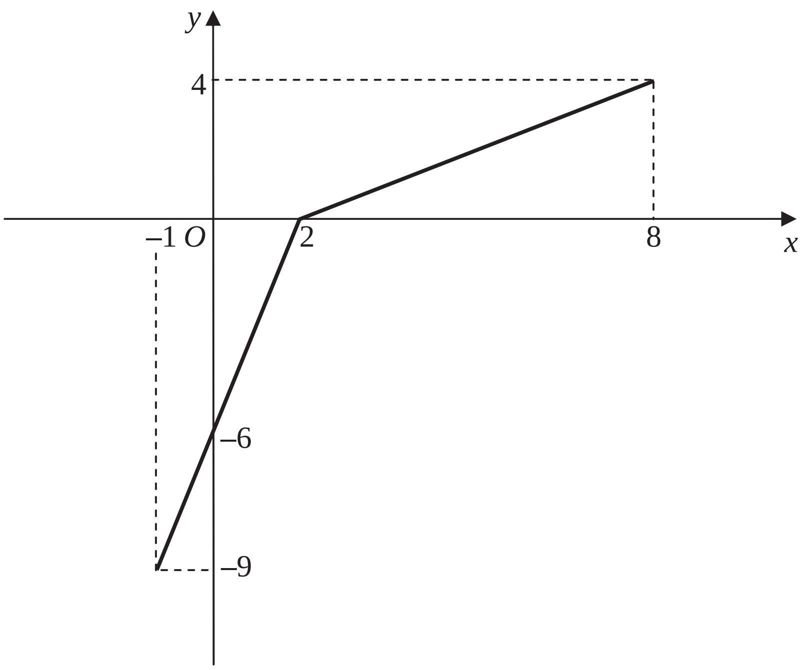

Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from (–1, –9) to (2, 0) and from (2, 0) to (8, 4). Figure 2 shows a sketch of the graph of $y = g(x)$.

(b) Write down the range of g.

(1)

(c) Find gg(2).

(2)

(d) Find fg(8).

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1}.

(1)

Leave blank

4. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(a) State the range of f.

(2)

(b) Find fg(1).

(2)

(c) Find g^{-1}, the inverse function of g.

(2)

(d) Solve the equation

$$\mathrm{gg}(x) + [\mathrm{g}(x)]^2 = 0$$

(5)

7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

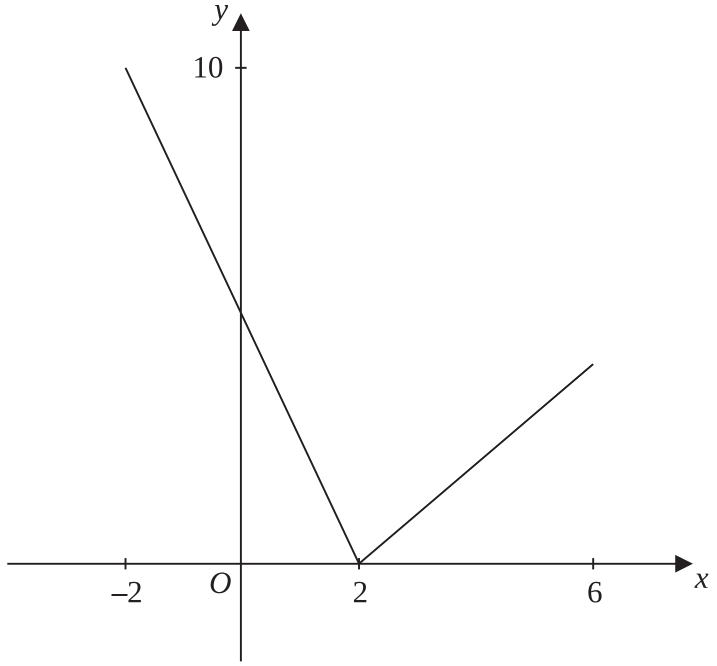

Figure 1

(a) Write down the range of f.
(1)

(b) Find ff(0).
(2)

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4+3x}{5-x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$
(3)

(d) Solve the equation $\mathrm{gf}(x) = 16$
(5)