

Peer-to-Peer (P2P) Electricity Markets for Low Voltage Networks

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Carnegie Mellon Electricity Industry Center (CEIC)
Advisory Committee Meeting

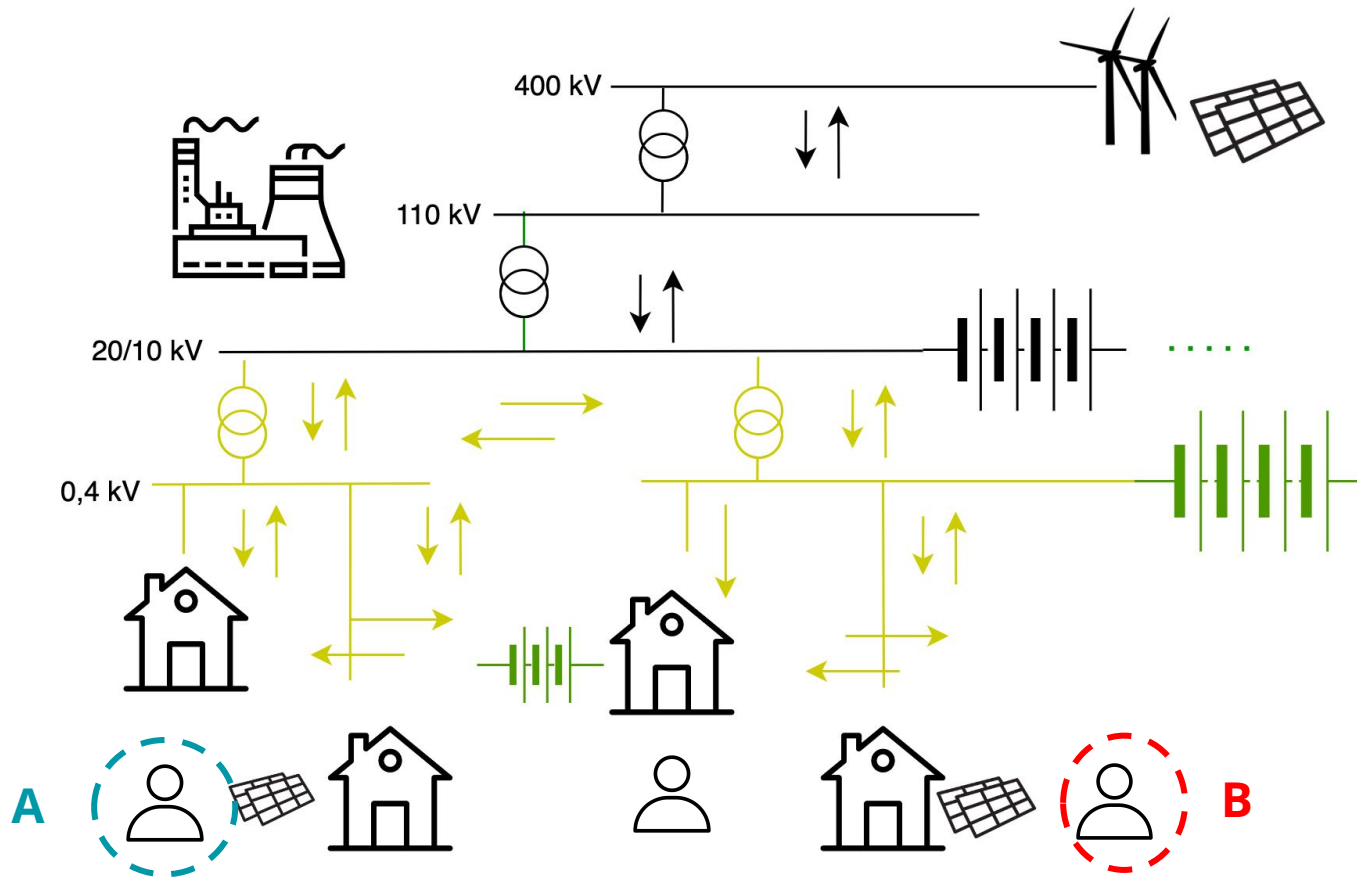
October 16, 2024

Main Contribution

Formalize and develop a novel model for a peer-to-peer clearance and settlement problem:

- ❑ for integrating peer-to-peer energy trading in low-voltage;
- ❑ incorporating network constraints;
- ❑ multiple and concurrent autonomous users;
- ❑ different time windows.

Challenges (Set-up)



low voltage radial networks

Users can be producers and consumers (prosumers)

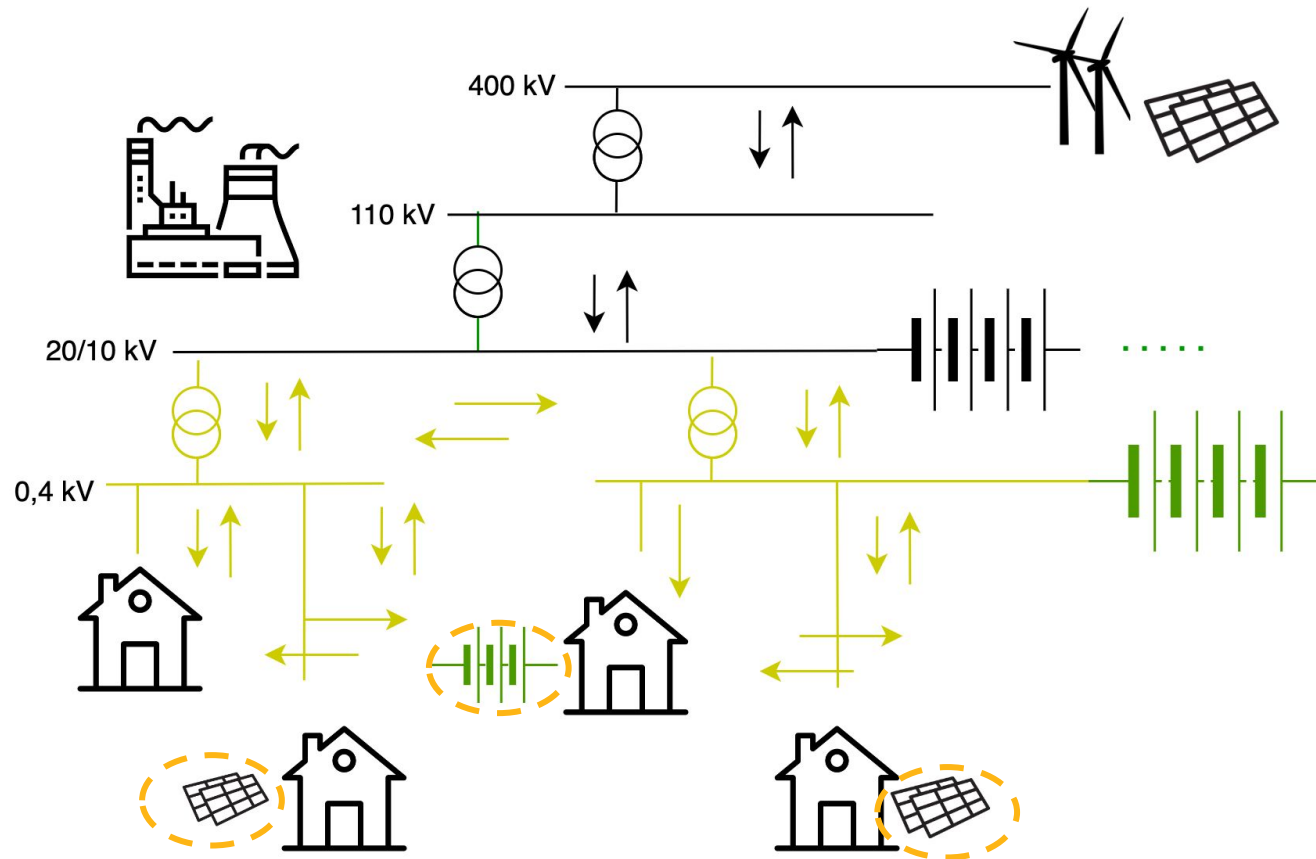
Distributed unused capacity

Mostly Renewable Energy Sources (RES)

Example:

- **A** has excess electricity and can provide to whomever needed
- **B** needs electricity

Challenges



Sets of challenges:

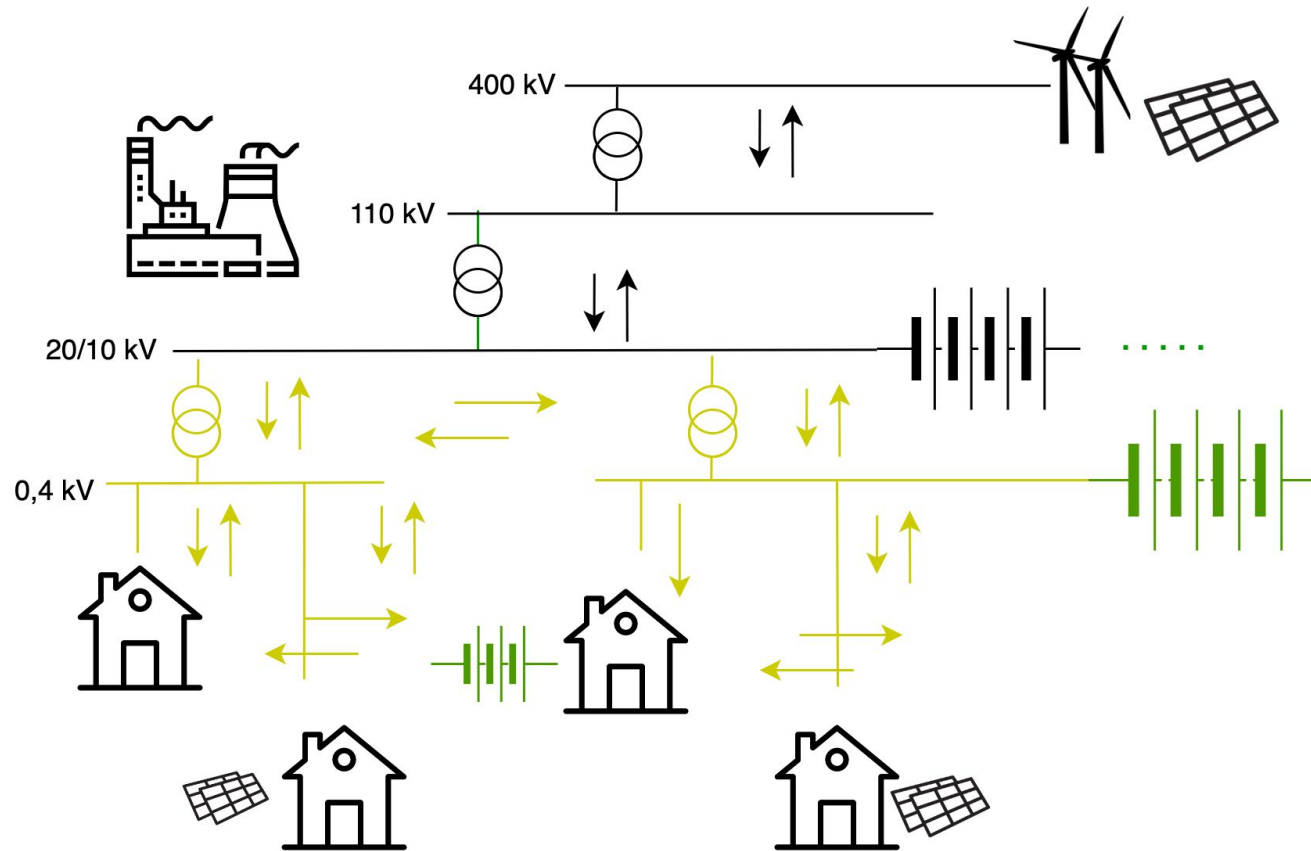
Market:

- **How to use available power capacity closer to consumption?**

Network constraints

low voltage radial networks

Challenges



low voltage radial networks

Sets of challenges:

Market:

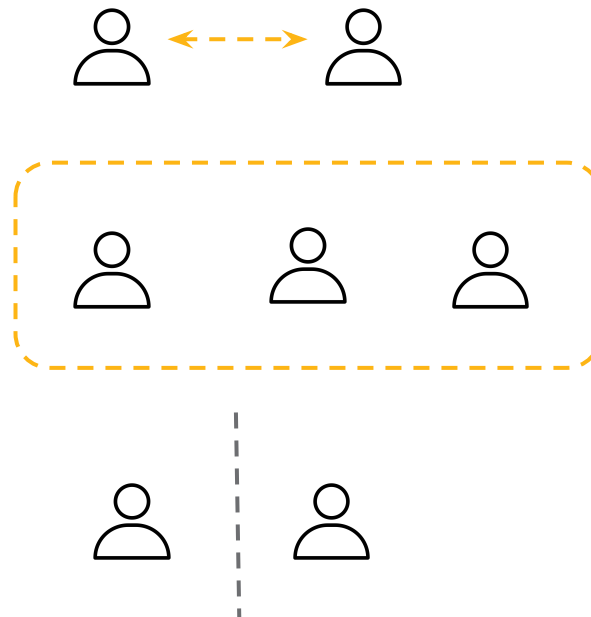
- How to use available power capacity closer to consumption?
- **How to set up (facilitate) markets**

Network constraints

Challenges

New market structures

- peer-to-peer (P2P)
- “energy communities”
- “local energy markets”



P2P is just a micro Power Purchase Agreement (PPA)

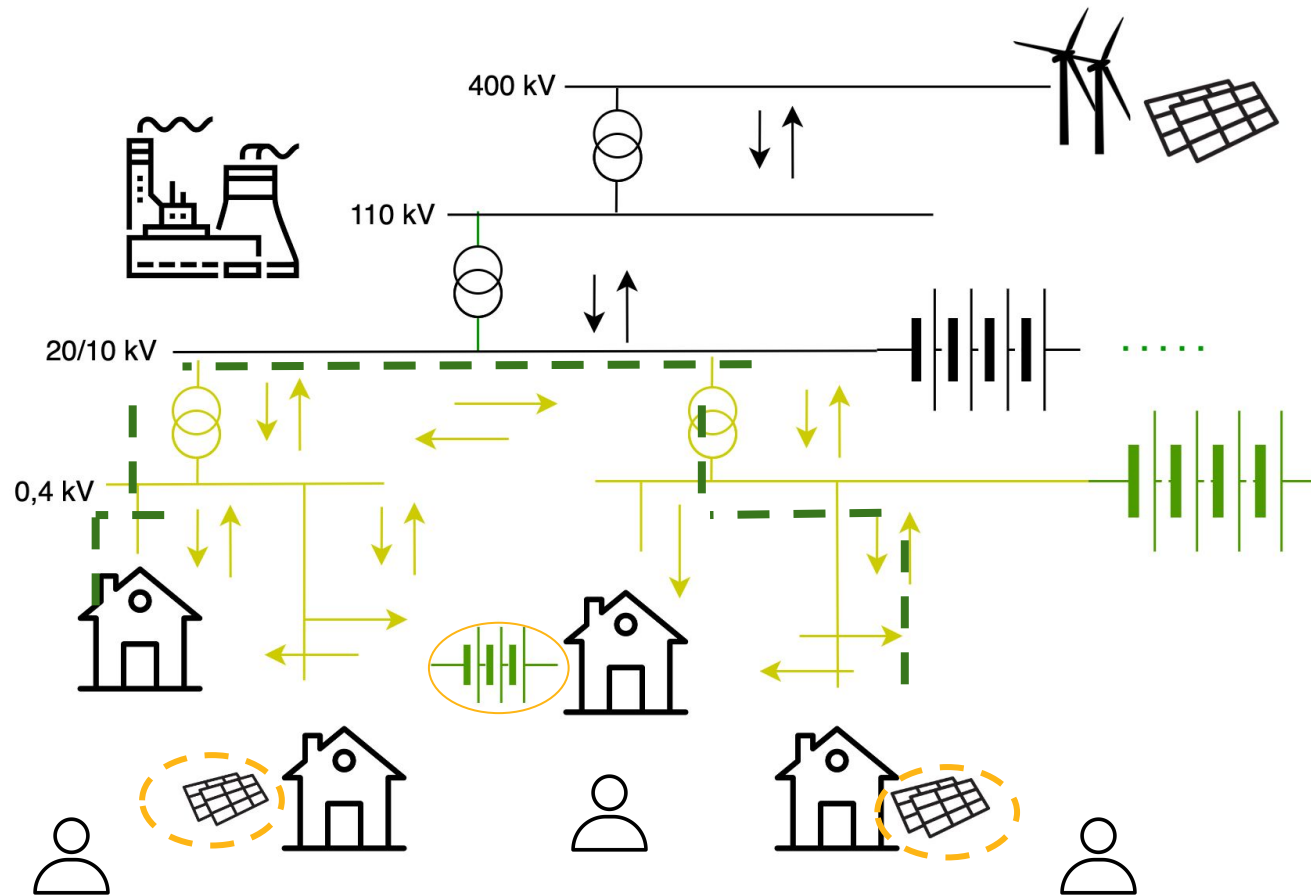
shared P across n users for a given radius

if they don't know each other?
there are market structures for that,
e.g. setting a simple auction for the
matching of inverse offers

Regardless on
how contracts are
formed:

All need a
clearance and
settlement
systems to sell or
buy electricity (in
LV)

Challenges



low voltage radial networks

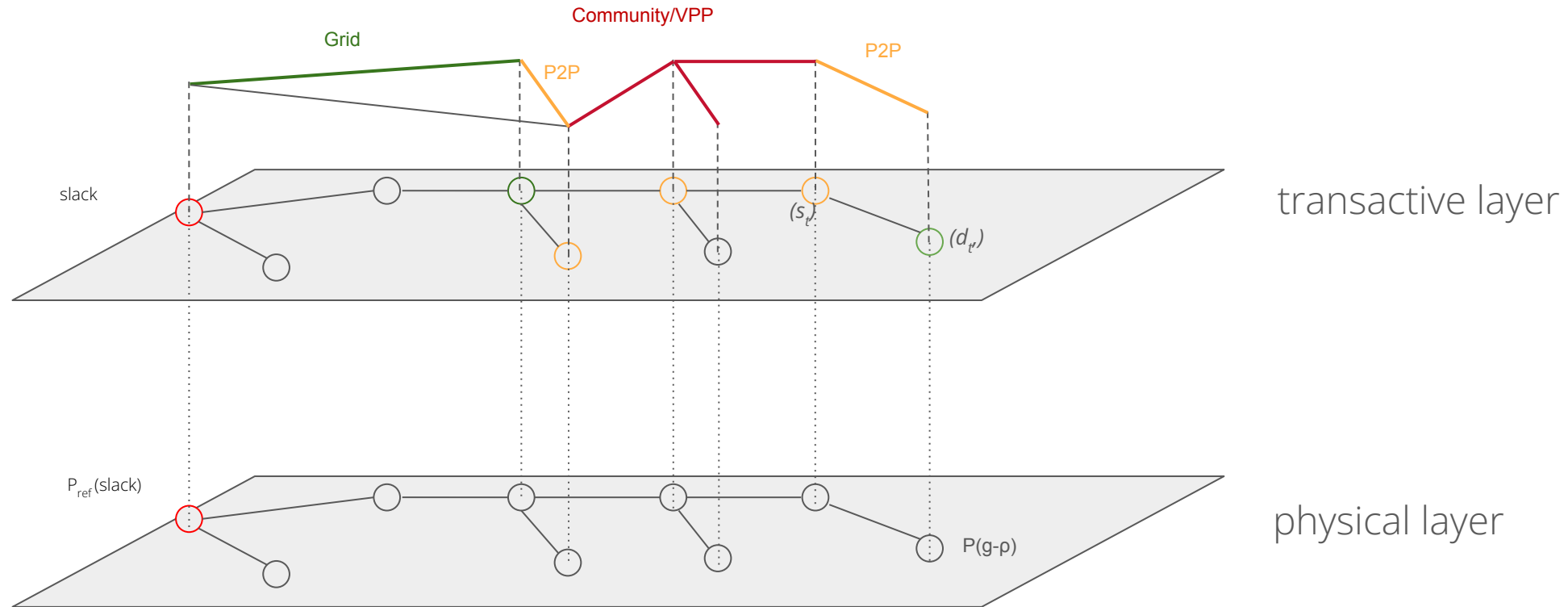
Sets of challenges:

Market:

Network constraints

- physical limitations of the grid when used by multiple parties (shared resource)
- active participation of end users (can inject and receive electricity (not just receptor) across the grid (not just close microgrid structure))

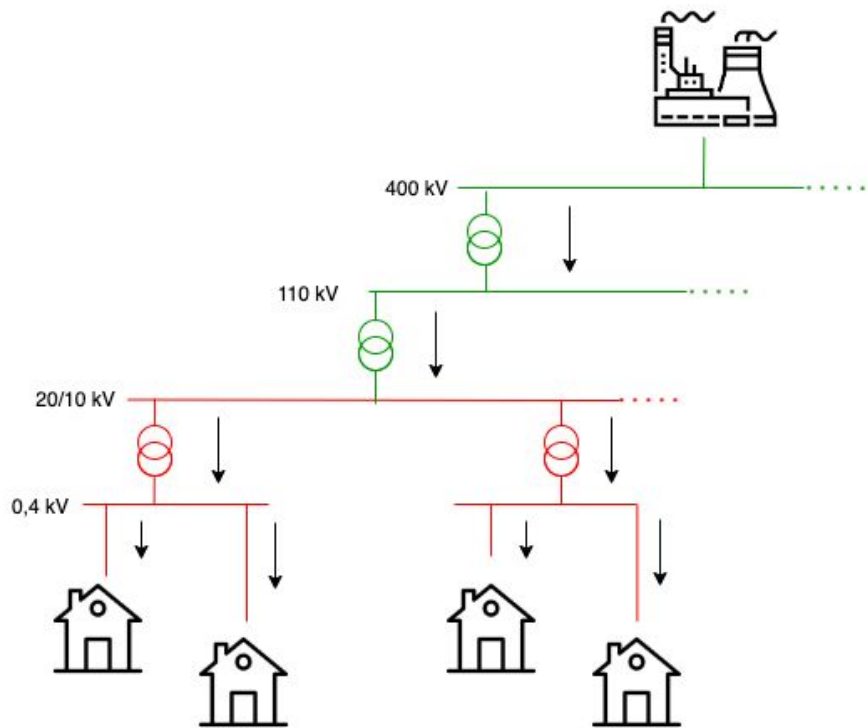
Challenges (integration transactive and physical layers)



- ❑ Network access: cannot discriminate over price (common use)
- ❑ Integration PF with other schemes: market, PPA, Feed-in-Tariff (HV) to P2P, EC/VPP, etc. (LV)
- ❑ Dispatch and operations \neq market (transactive layer)

Why hasn't this been addressed? Transitioning from 'Is it important?' to 'How do we achieve it?'

downstream unidirectional energy flow



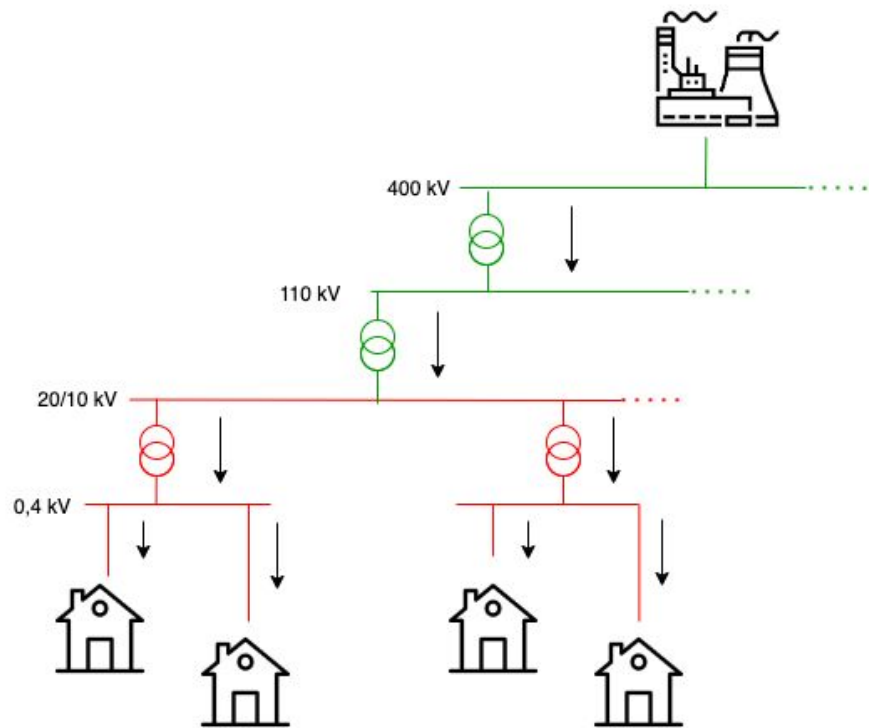
—DSO —TSO

In the past (and still currently):

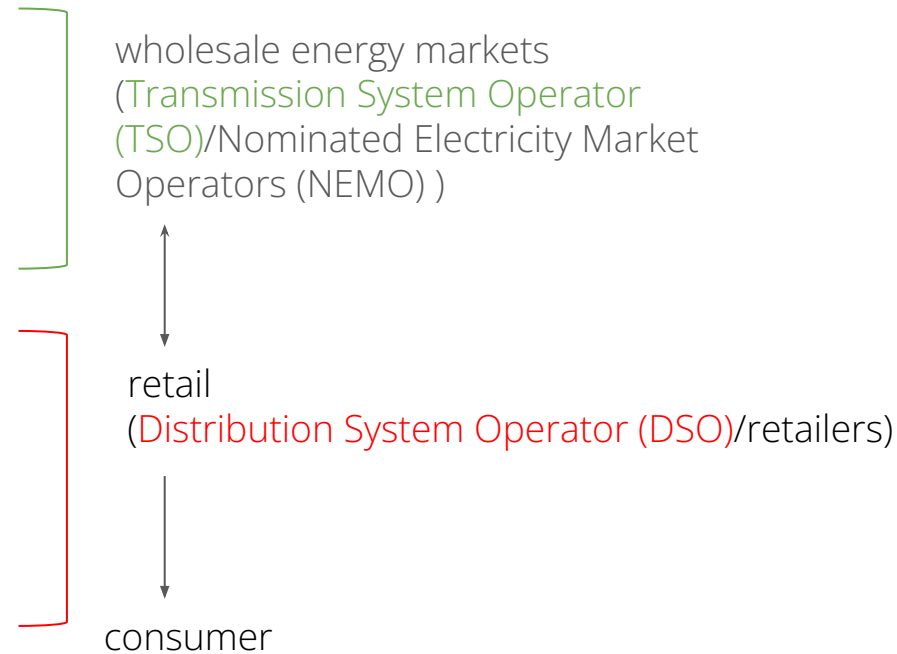
- the flow of electricity followed a unidirectional path
- originating from generators injecting power into high-voltage (HV) systems and
- cascading downstream towards consumers.

Why hasn't this been addressed? Transitioning from 'Is it important?' to 'How do we achieve it?'

downstream unidirectional energy flow

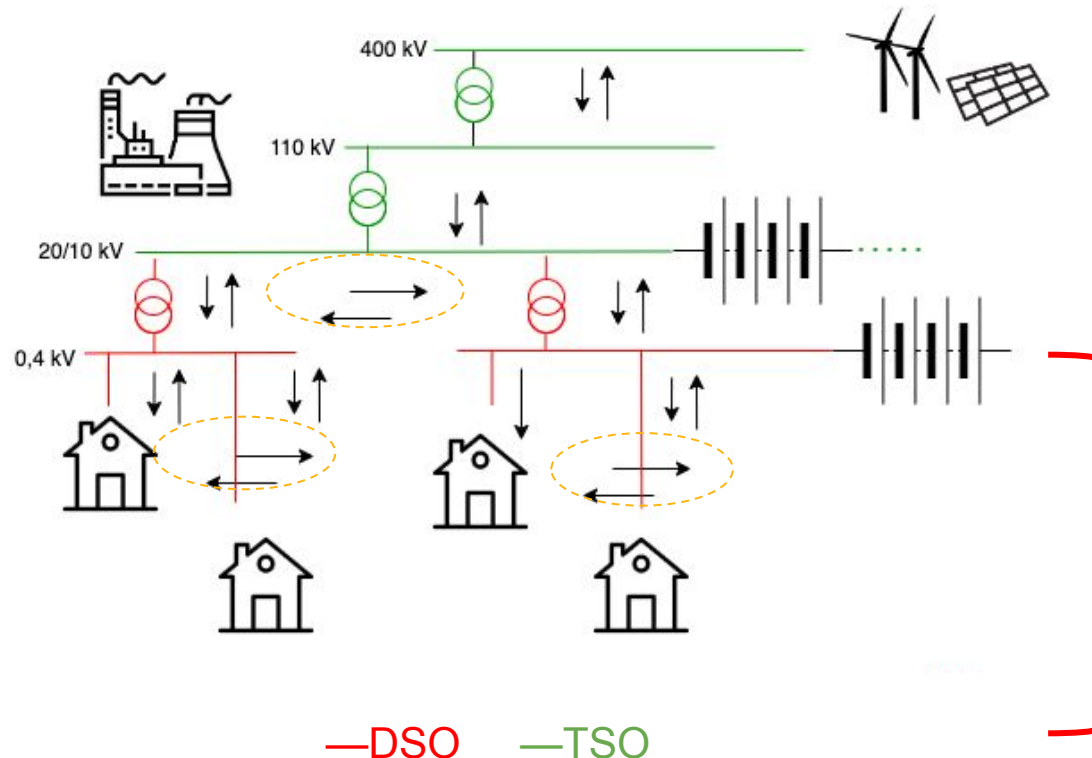


—DSO —TSO



Why hasn't this been addressed? Transitioning from 'Is it important?' to 'How do we achieve it?'

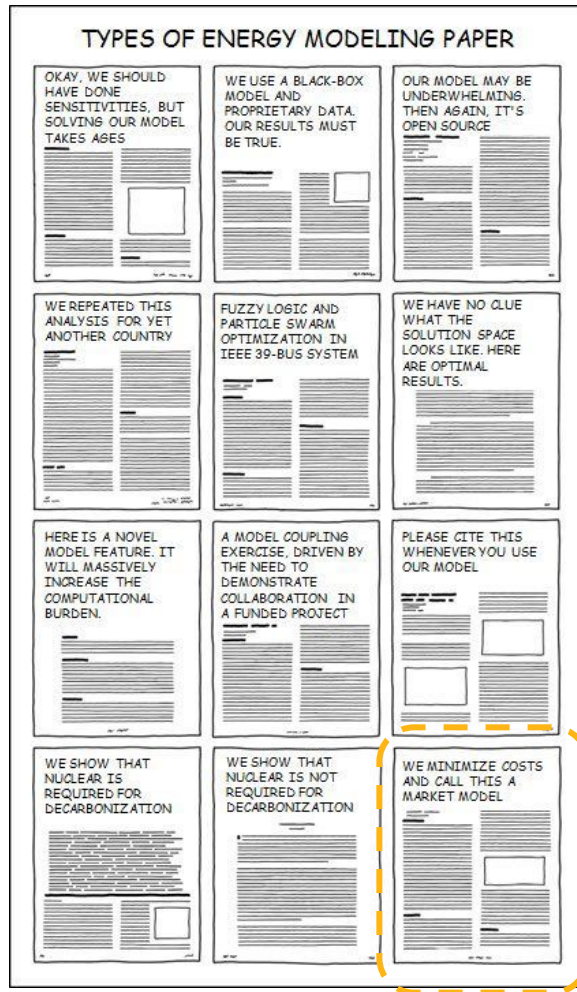
distributed bidirectional energy flow



In future electric systems:

- Focus is **downstream** (low voltage)
- **Bidirectional flows**
- Assume active participation of consumers
- Distributed generation units closer to consumption

Prior works and Literature Review



- ❑ The (centralized) (ED) model presumes an aggregated load (or demand);
- ❑ Its primary objective is to minimize the costs associated with electricity generation and distribution given the (aggregated) power output (and generation cost) of each generator.

≠
P2P

Typical ED formulation (a,b and c are coefficients for each generator(P_g))

$$\text{Minimize } C(\mathbf{P}_g) = \sum_{i \in \mathcal{G}} a_i P_{g_i}^2 + b_i P_{g_i} + c_i$$

Typical P2P formulation (from Sousa et al, 2018)

$$\text{Minimize } D = \sum_{n \in \Omega} C_n(p_n, q_n, \alpha_n, \beta_n) + G(q_{\text{imp}}, q_{\text{exp}})$$

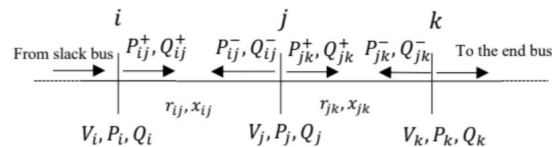
Prior works and Literature Review

(uni)direction assumption (BFM) and bidirectional flows

Using Voltage Sensitivity Coefficients (VSCs) and Power Transfer Distribution Factors (PTDFs) (Guerrero et al., 2019)

Limitation: The current approach for calculating VSCs and PTDFs assumes a linear response to a single change in power injection at a time.

Bidirectional Distflow for Local Energy and Flexibility Markets (Talari et al., 2024)



Limitation: forcing and expanding BFM to allocate bidirectionally (does not seem to solve “directionally” problem)

Research Question

How can we establish an efficient and secure peer-to-peer (P2P) electricity market in low-voltage networks that respect network constraints and support decentralized power generation?

Mathematical Formulation:

The logic behind the system

A P2P trade $t \in T$ is represented as $t = (s_t, d_t, q_t)$ where s_t is the source, d_t the destination, q_t the quantity of trade t .

N is the set of all buses i in the system, and T be the set of all proposed peer-to-peer (P2P) trades in the network.

s.t.

Objective Function:

$$\underset{x_t, \theta_i, P_{i_{\text{ref}}}}{\text{maximize}} \quad \sum_{t \in T} x_t \cdot q_t$$

Execution of Trades

$$\forall t \in T : 0 \leq x_t \leq 1$$

Load and Generation Adjustments Due to Trades

$$\forall i \in N \setminus \{i_{\text{ref}}\}, \forall t \in T :$$

$$g_i = g_i^0 + \sum_{t \in T} x_t q_t \mathbf{1}_{\{s_t=i\}},$$

$$\rho_i = \rho_i^0 + \sum_{t \in T} x_t q_t \mathbf{1}_{\{d_t=i\}},$$

Generation and load bounds

$$\underline{g}_i \leq g_i \leq \bar{g}_i,$$

$$\underline{\rho}_i \leq \rho_i \leq \bar{\rho}_i,$$

Nodal Power Balance

$$g_i - \rho_i = \sum_{j \in \text{Adj}(i)} B_{ij}(\theta_i - \theta_j),$$

Line Capacity Constraints

$$\forall i, j \in N :$$

$$|B_{ij}(\theta_i - \theta_j)| \leq \bar{P}_{ij},$$

Voltage Angle Differences

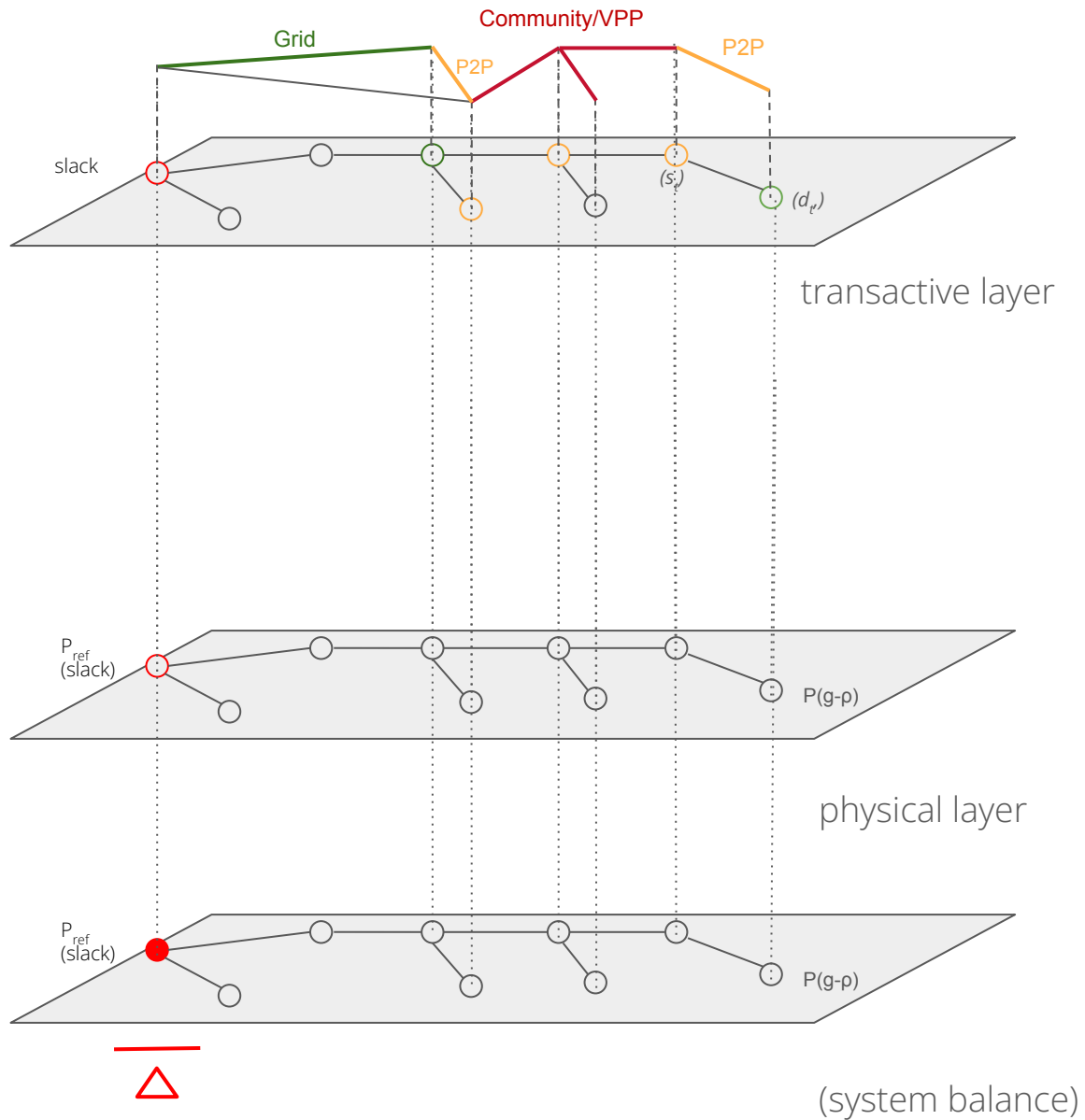
$$|\theta_i - \theta_j| \leq \frac{\pi}{6},$$

$$\theta_{i_{\text{ref}}} = 0,$$

Slack Power Mismatch

$$P_{i_{\text{ref}}} = \sum_{i \in N \setminus \{i_{\text{ref}}\}} (g_i - \rho_i),$$

$$|P_{i_{\text{ref}}}| \leq P^{\text{ref max}}$$



Objective Function:

s.t.

Execution of Trades

Load and Generation Adjustments Due to Trades

Generation and load bounds

Nodal Power Balance

Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

Maximize the total quantity across all trades

(the vector q_t represents the quantity of each trade, and x is a binary variable representing whether a trade is performed.)

s.t.

Objective Function:

$$\underset{x_t, \theta_i, P_{i_{\text{ref}}}}{\text{maximize}} \quad \sum_{t \in T} x_t \cdot q_t$$

Execution of Trades

Load and Generation Adjustments Due to Trades

Generation and load bounds

Nodal Power Balance

Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

Which trades are included?



s.t.

Objective Function:

$$\underset{x_t, \theta_i, P_{i_{\text{ref}}}}{\text{maximize}} \quad \sum_{t \in T} x_t \cdot q_t$$

Execution of Trades

$$\forall t \in T : 0 \leq x_t \leq 1$$

Load and Generation Adjustments Due to Trades

Generation and load bounds

Nodal Power Balance

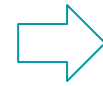
Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

The Combinatorial complexity: Binary Decisions - To Switch or Not to Switch?

This problem of selecting an optimal subset of transactions from a larger set, subject to certain constraints brings the combinatorial aspect.



**Continuous relaxation
(allows trades to take
values between 0 and 1)**

Mapping T to N

Indicates adjustments in load or generation at a bus i due to P2P trades, affected by the decision variable x_t , which sets the execution of trades.

Each trade t will change i in N

Indicator functions indicate whether a specific bus i is the source (s_t) or destination (d_t) for a trade t .

s.t.

Objective Function:

Execution of Trades

Load and Generation Adjustments Due to Trades

$\forall i \in N \setminus \{i_{\text{ref}}\}, \forall t \in T :$

$$g_i = g_i^0 + \sum_{t \in T} x_t q_t \mathbf{1}_{\{s_t=i\}},$$

$$\rho_i = \rho_i^0 + \sum_{t \in T} x_t q_t \mathbf{1}_{\{d_t=i\}},$$

Generation and load bounds

Nodal Power Balance

Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

Objective Function:

s.t.

Execution of Trades

Load and Generation Adjustments Due to Trades

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Generation and load bounds

Nodal Power Balance

Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

g_i^0 invariable generation,

$\sum_{t \in T} x_t q_t \mathbf{1}_{\{s_t=i\}}$, (generation) adjustments due to P2P trades.

g_i is the actual generation at bus i

ρ_i^0 invariable load,

$\sum_{t \in T} x_t q_t \mathbf{1}_{\{d_t=i\}}$, (load) adjustments due to P2P trades.

ρ_i is the actual load at bus i

Objective Function:

s.t.

Execution of Trades

Load and Generation Adjustments Due to Trades

Generation and load bounds

$$\underline{g}_i \leq g_i \leq \bar{g}_i,$$

$$\underline{\rho}_i \leq \rho_i \leq \bar{\rho}_i,$$


Nodal Power Balance

Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

Upper and lower bounds of
generation (g) and load (ρ) of each
bus i



Objective Function:

s.t.

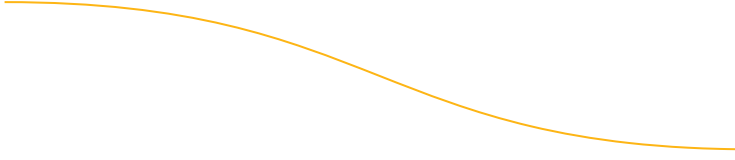
Execution of Trades

Load and Generation Adjustments Due to Trades

Generation and load bounds

Power flow

Nodal Power Balance


$$g_i - \rho_i = \sum_{j \in Adj(i)} B_{ij}(\theta_i - \theta_j),$$

Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

The Non-Linear Hurdle: Riding the Non-Convexity of waves of AC Power Flow. Stretching the line by approximation

An important challenge is integrating the AC Power Flow (PF), which introduces nonlinear, non-convex, and coupled constraints.

AC Power Flow Equations:

$$P_{ij} = V_i^2 G_{ij} - V_i V_j (G_{ij} \cos(\theta_{ij}) + B_{ij} \sin(\theta_{ij}))$$

$$Q_{ij} = -V_i^2 B_{ij} - V_i V_j (G_{ij} \sin(\theta_{ij}) - B_{ij} \cos(\theta_{ij}))$$



Linearization using Distflow (would yield quadratic terms (V^2))

Convex relaxation (W matrix has to be rank 1 and there is need to recover V from W)

DC approximation (simpler and faster)

Objective Function:

s.t.

Execution of Trades

Load and Generation Adjustments Due to Trades

Nodal Power Balance (adjusted to P2P trades)

$(g_i - \rho_i)$ is the net power injection at bus i , accounting for the initial load g_i^0 and adjustments due to trades (both as a source and destination) which can be positive (generation) or negative (consumption).



Generation and load bounds

Nodal Power Balance

$$g_i - \rho_i = \sum_{j \in Adj(i)} B_{ij}(\theta_i - \theta_j),$$

Line Capacity Constraints

Voltage Angle Differences

Slack Power Mismatch

The impact of trade execution x_t is directly integrated into the bus's power balance, affecting the voltage angles and subsequent power flows.

Objective Function:

s.t.

Execution of Trades

Load and Generation Adjustments Due to Trades

Generation and load bounds

Nodal Power Balance

Line Capacity Constraints

$\forall i, j \in N :$

$$|B_{ij}(\theta_i - \theta_j)| \leq \bar{P}_{ij},$$

Voltage Angle Differences

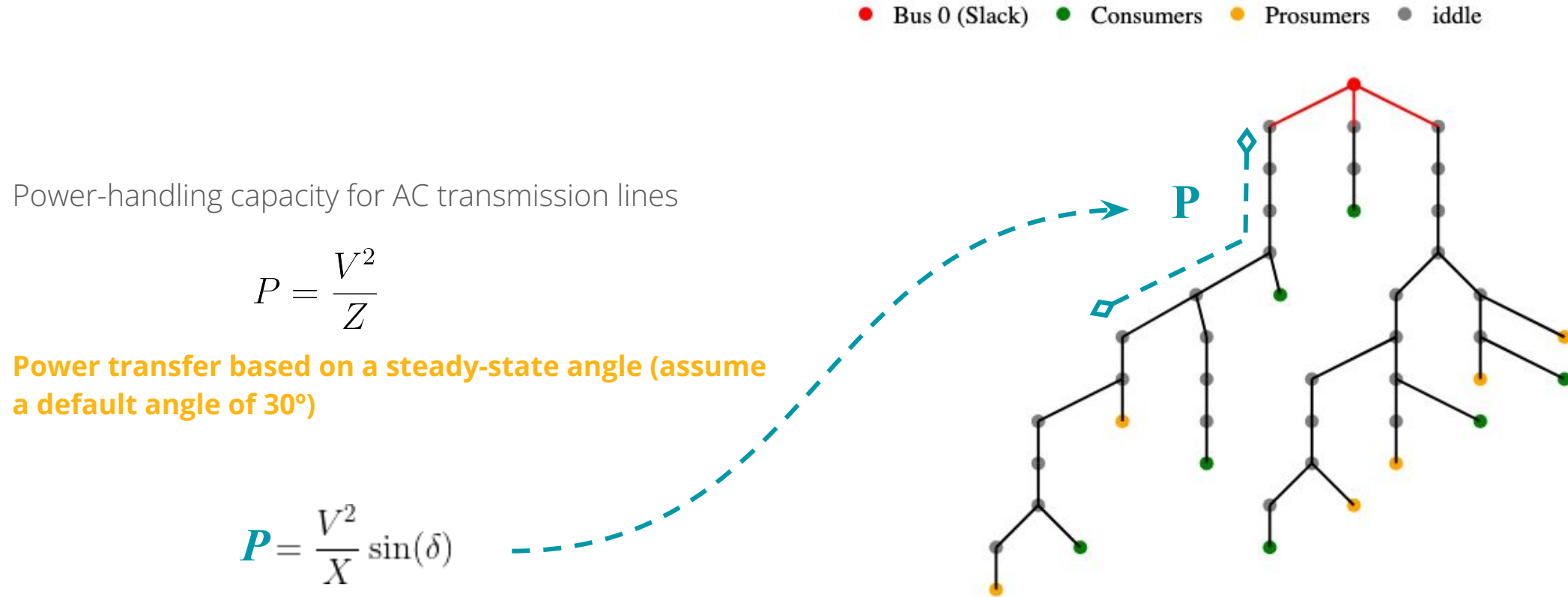
Slack Power Mismatch

setting capacity constraints on
each line, per each P2P trade



Maximum Capacity for Distribution Power Lines

Maximum power capacity for different elements within a power network, based on the electrical parameters of each component.



Objective Function:

s.t.

Execution of Trades

Load and Generation Adjustments Due to Trades

Generation and load bounds

Nodal Power Balance

Line Capacity Constraints

Voltage Angle Differences

$$|\theta_i - \theta_j| \leq \frac{\pi}{6},$$
$$\theta_{i_{\text{ref}}} = 0,$$

Slack Power Mismatch

“safety valve”

Set as $\theta^{\text{ref}} = 0$

Maintain angle differences within acceptable bounds

Objective Function:

s.t.

Execution of Trades

Load and Generation Adjustments Due to Trades

Generation and load bounds

Nodal Power Balance

Line Capacity Constraints

Voltage Angle Differences

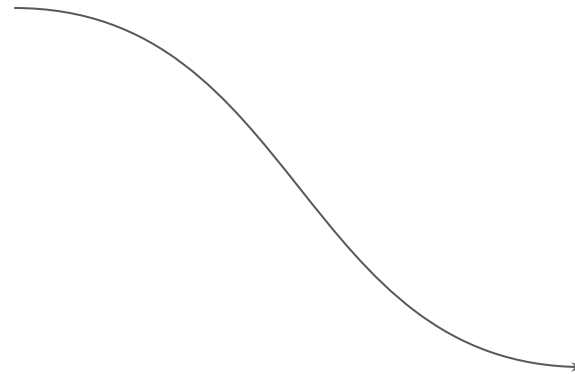
Slack Power Mismatch

$$P_{i_{\text{ref}}} = \sum_{i \in N \setminus \{i_{\text{ref}}\}} (g_i - \rho_i),$$

$$|P_{i_{\text{ref}}}| \leq P^{\text{ref max}}$$

Accounting for mismatches in
generation and demand

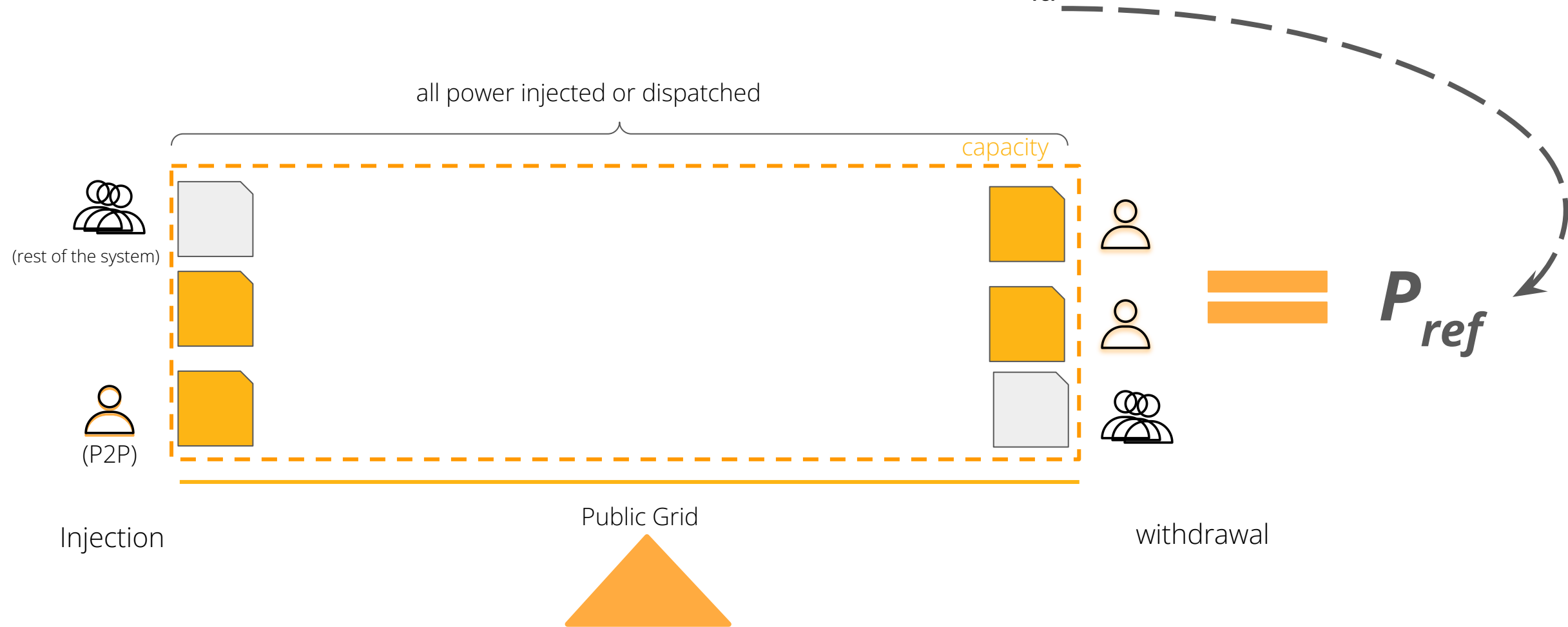
(imports and export to the
grid/slack bus)



Balancing the system

Balancing via **slack bus (imp./exp. from downstream to upstream)**, compensates for mismatches in generation and demand.

Represents the system balance, or the total power imported and exported through **P_{ref} (slack)** (or collectively, the “Grid”).



s.t.

Objective Function:

$$\underset{x_t, \theta_i, P_{i_{\text{ref}}}}{\text{maximize}} \quad \sum_{t \in T} x_t \cdot q_t$$

Execution of Trades

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Load and Generation Adjustments Due to Trades

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Nodal Power Balance

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Voltage Angle Differences

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Slack Power Mismatch

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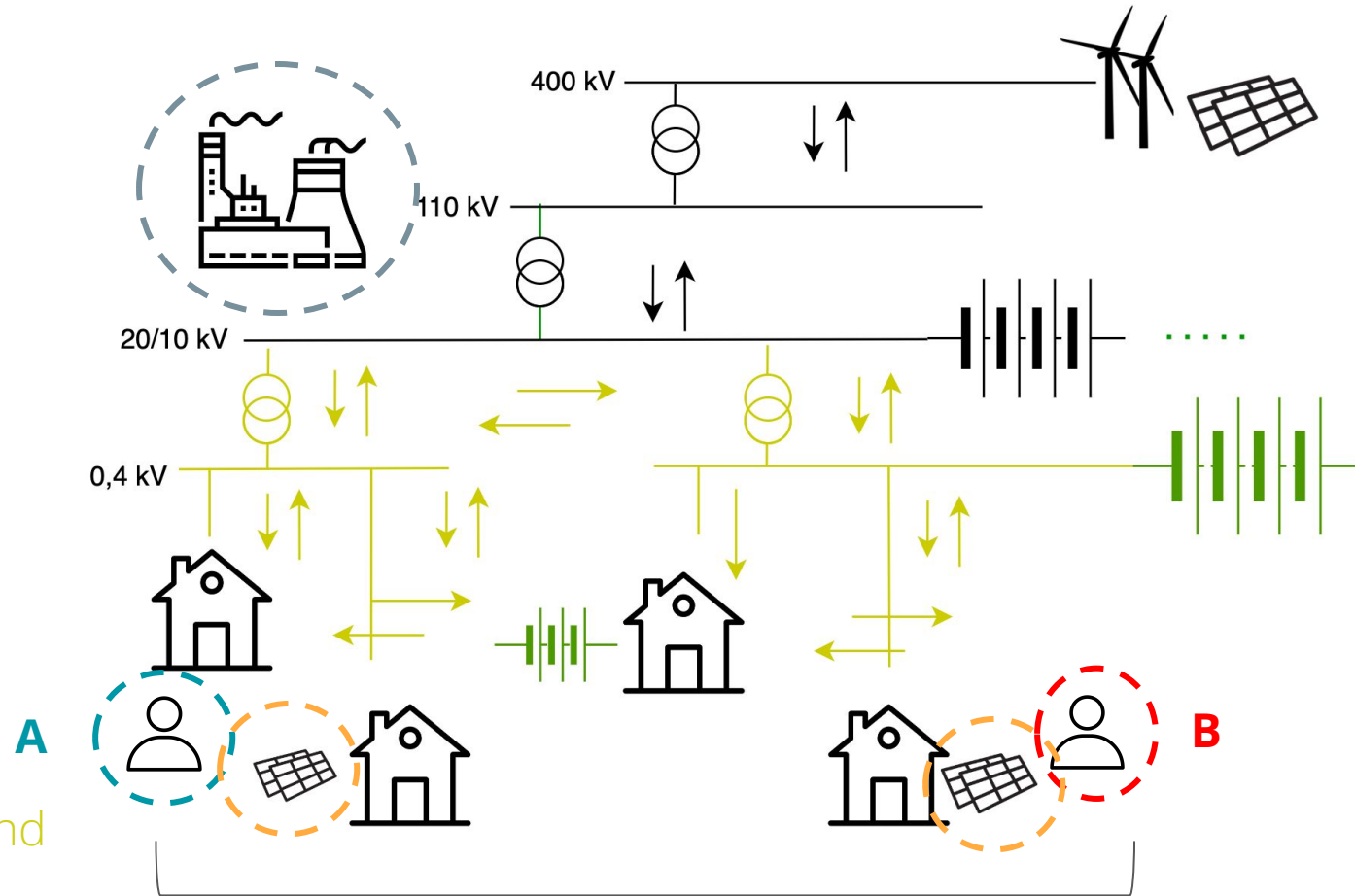
N is the set of all buses i in the system, and T be the set of all proposed peer-to-peer (P2P) trades in the network.

Implementation

Objectives

Integration (and coordination):

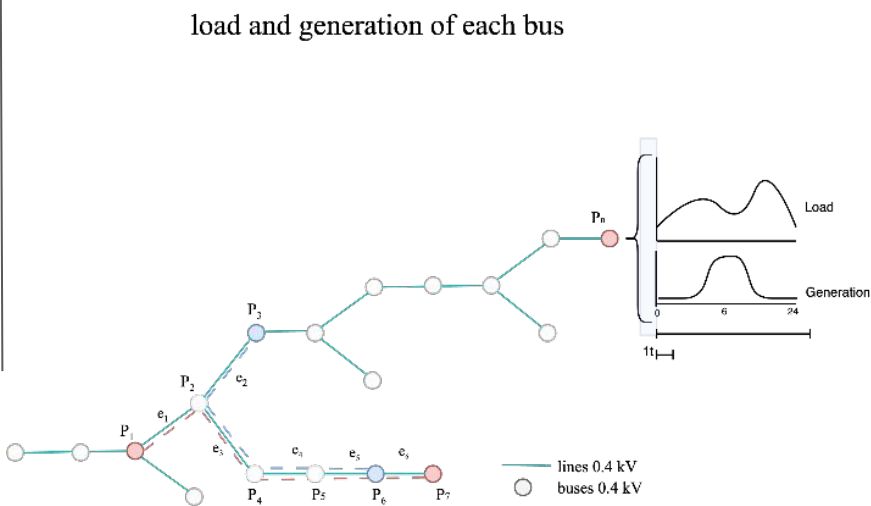
- Distributed unused **capacity** (closer to consumption/low voltage)
- Mostly **Renewable Energy Sources (RES)**
- (Less **upstream** units or "the Grid")
- **merging** these resources **with existing market and operational mechanisms**, with multiple users.



How to settle multiple P2P transactions across the grid?

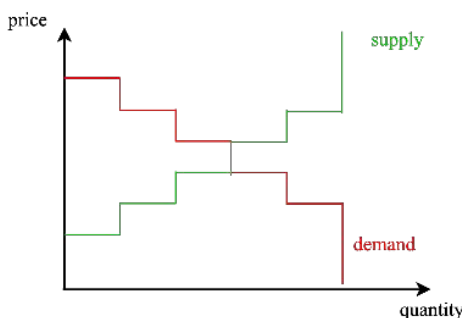
Implementation

1) build use case (simulation)



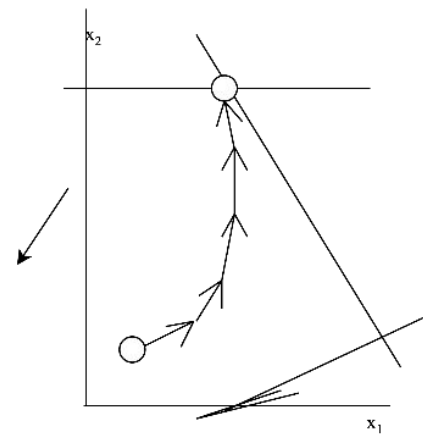
simulation

matching (double auction)



2) apply the proposed model

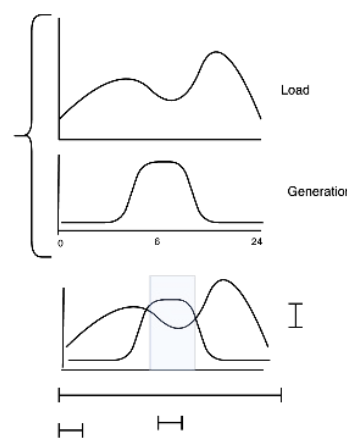
optimization model



model formulation

3) analysis

results and metrics



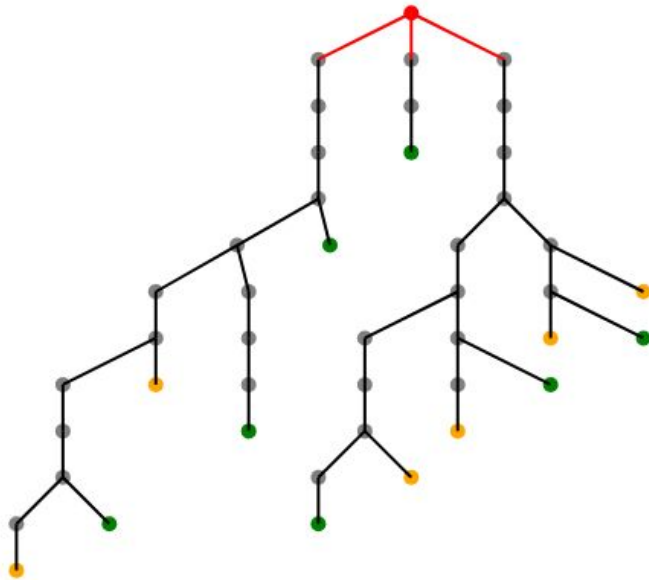
analysis

Simulation with no master node

PF with trading (i.e. adding generation), connected to the external grid

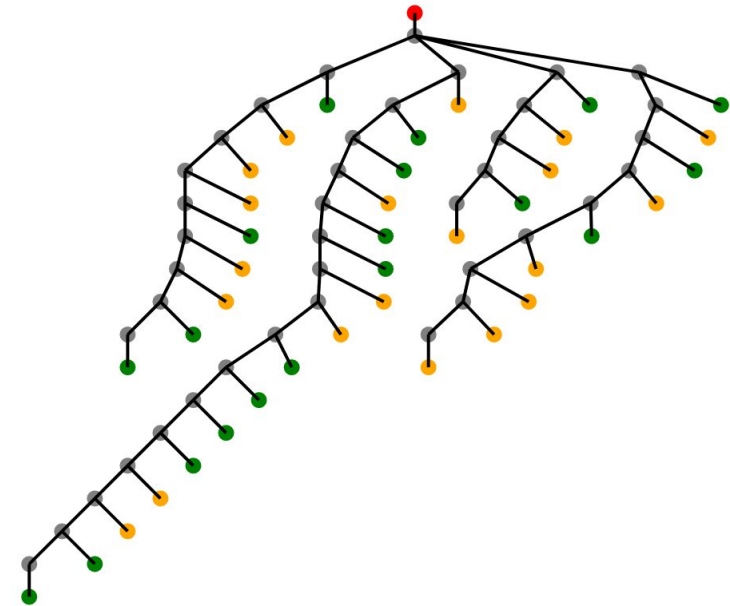
50% of overall consumers (loads) are associated with local solar generation on the same bus.

● Bus 0 (Slack) ● Consumers ● Prosumers ● iddle



Adapted CIGRE low voltage radial distribution network (44 bus system) with users' profile

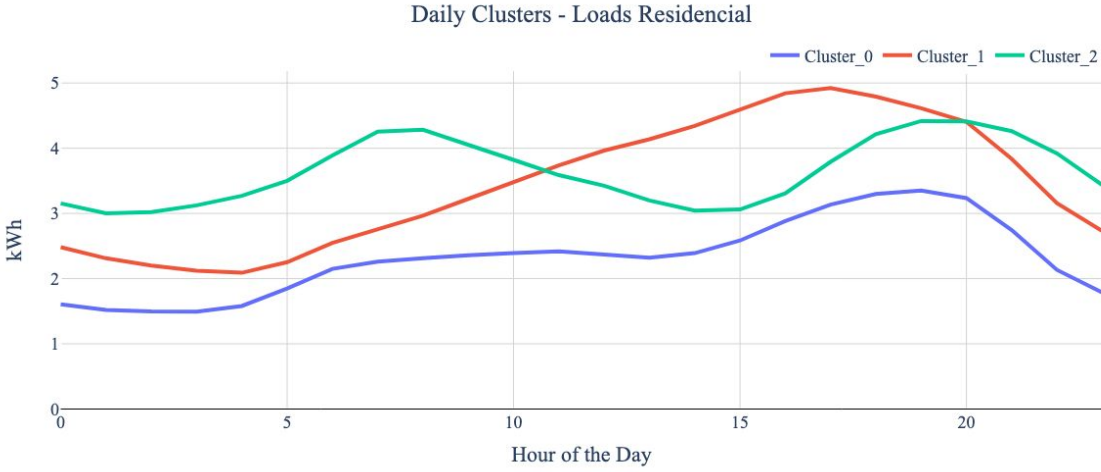
● Bus 0 (Slack) ● Consumers ● Prosumers ● iddle



Adapted Synthetic Voltage Control LV Networks "Village" (80 bus system) with users' profile

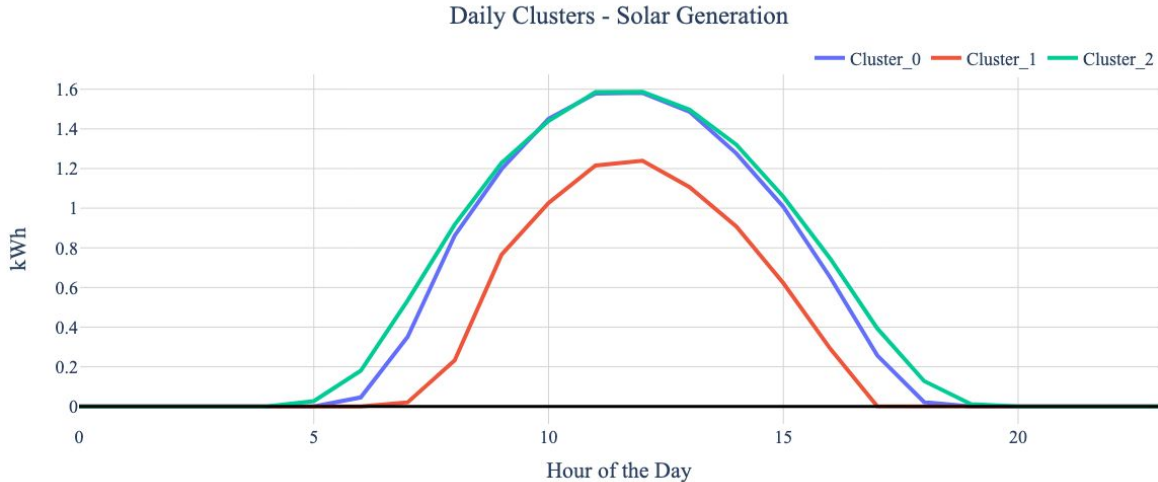
Simulation with no master node

residential (single house, PJM,NREL)



demand (load profiles)

solar (3 kWp, PA, Solar Atlas)

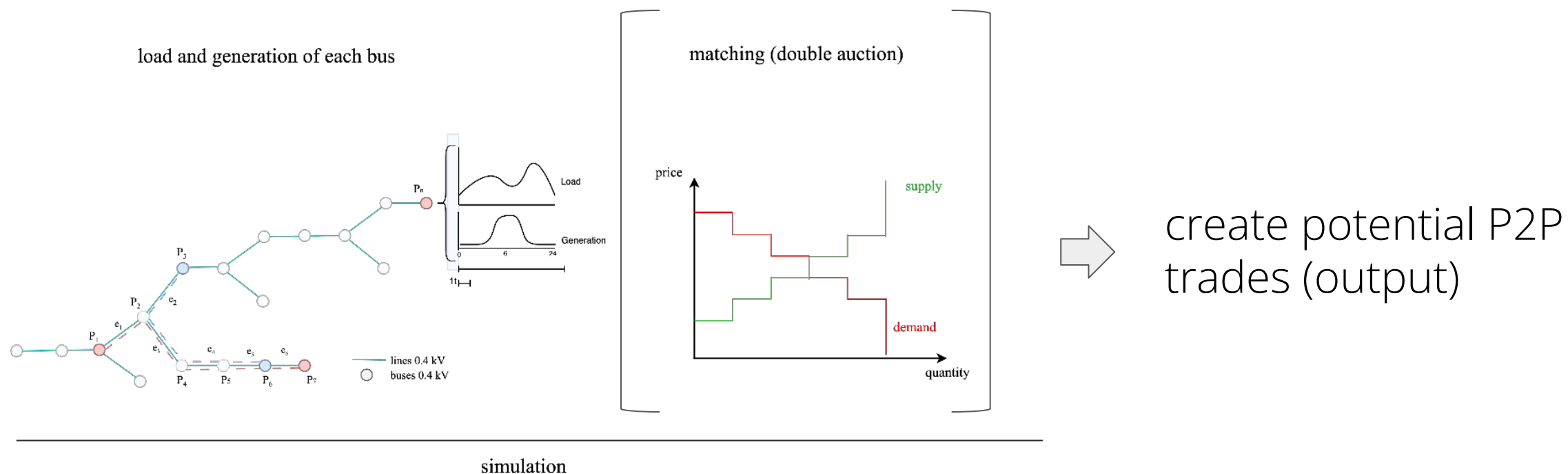


supply (local generation profiles)

Hourly time series, using K-means clustering (hourly) to get profiles for each bus.

Simulation with no master node

Double auction based of load and local general profiles (selling excess)



multiple (node/bus i):

- buyers (load, **demand**) and
- sellers (generation, **supply**)

Greedy approach for simulated double auction (matching)

Putting all together

For each timeblock (hour):

1) Simulation

- a) load and generation (demand and supply) for each bus
- b) create potential P2P trades ($t \in T$, as $t = (s_t, d_t, q_t)$ where s_t is the source, d_t the destination, q_t the quantity of trade t) - output - through a double auction (matching)

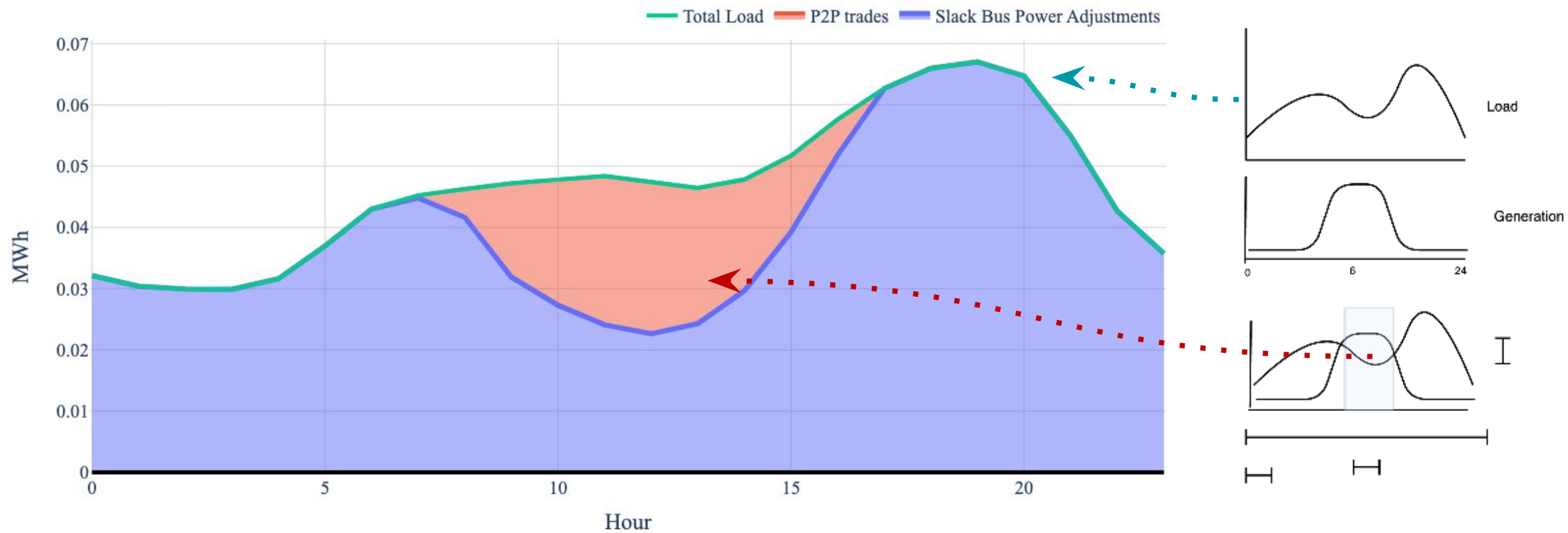
2) Then:

- a) apply optimization model, where
$$\underset{x_t, \theta_i, P_{i_{\text{ref}}}}{\text{maximize}} \quad \sum_{t \in T} x_t \cdot q_t$$

s.t. network constraints

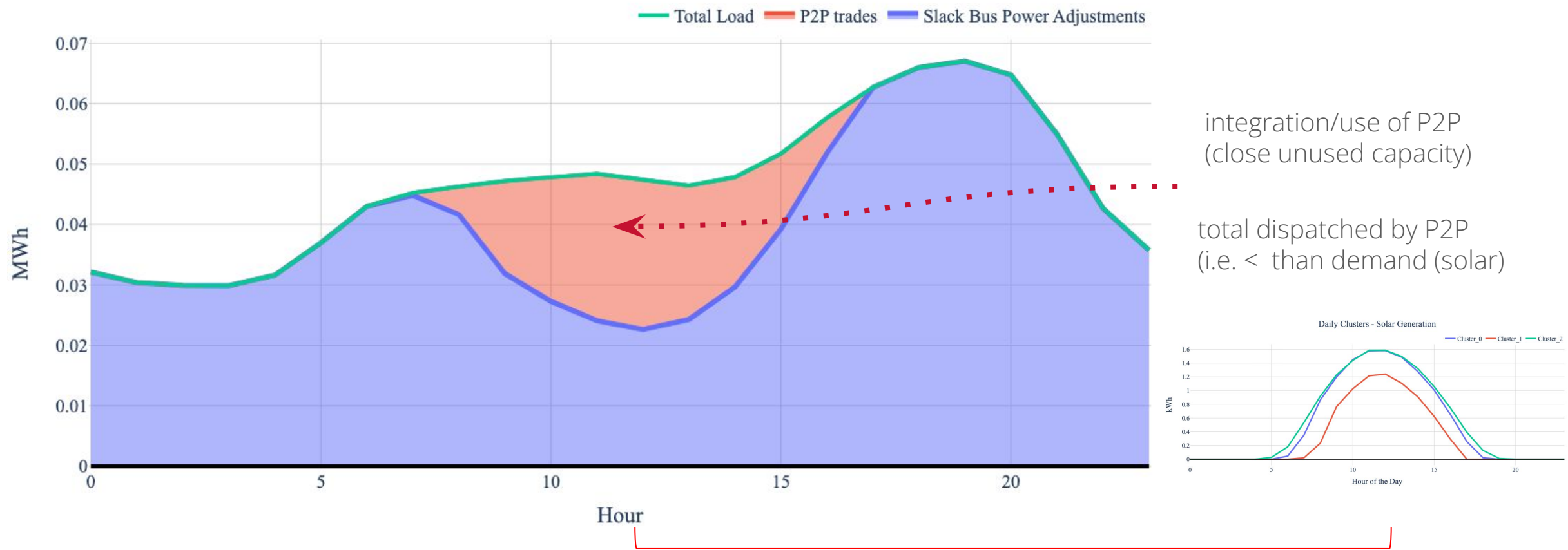
Experimentation: Capacitive Findings

Total P2P traded and slack adjustments per hour - 80 bus system



Experimentation: Capacitive Findings

Total P2P traded and slack adjustments per hour - 80 bus system

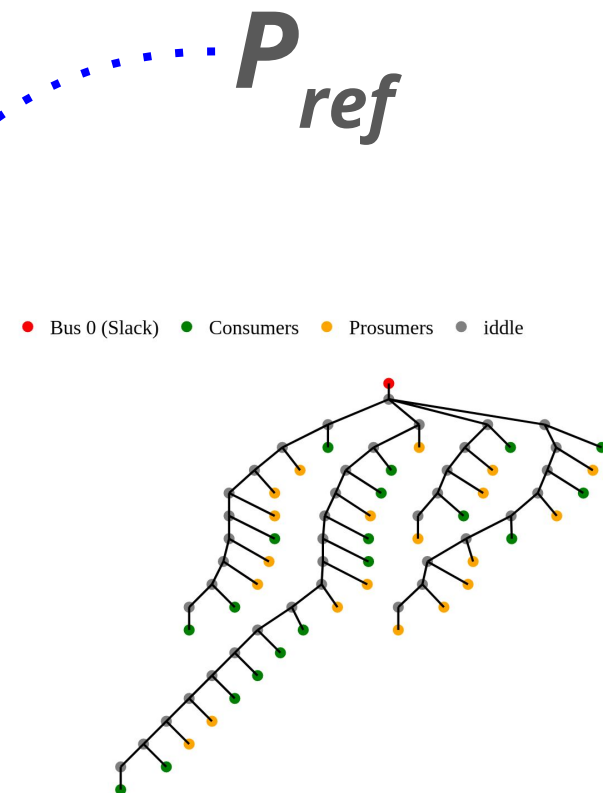
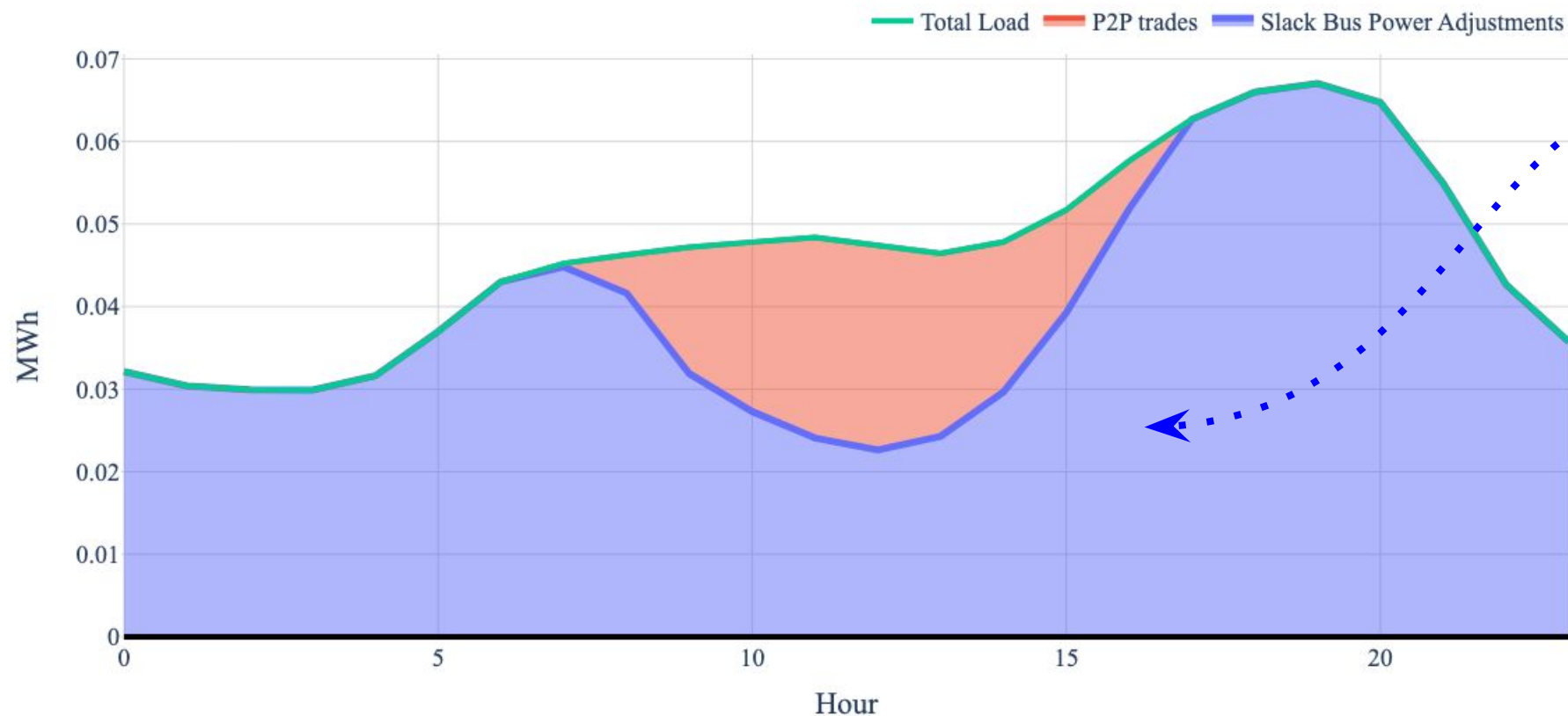


recall solar profile ("bell" shape, peak at 11am)

Experimentation: Capacitive Findings

Total P2P traded and slack adjustments per hour - 80 bus system

total power imported and exported through P_{ref} (slack) (or collectively, the "Grid") to meet demand of all buses within sub system/feeder



Experimentation: Capacitive Findings

Extreme P2P Trades - 44 Bus System

Hour	Seller (Bus)	x_t	Power (kWh)
11	Seller 0 (Bus 16)	0.534534	64.946
11	Seller 1 (Bus 18)	0.524626	63.742
11	Seller 2 (Bus 22)	0.009908	1.204
11	Seller 3 (Bus 36)	0.344749	41.887
11	Seller 4 (Bus 40)	0.344749	41.887
11	Seller 5 (Bus 42)	0.344749	41.887
	Total Load		262.776
	Total P2P Generation		255.5528890
	Slack I/E		-7.223

x_t - relaxed binary variable (% fulfillment of trades)

The extreme case was set at 100 of baseline (121.50 kWh each bus), to evaluate the ability to accommodate extreme values and their (expected) behavior.

Balancing (imp./exp. from downstream to upstream), compensates for mismatches in generation and demand.

Experimentation: Capacitive Findings

The **quantity available for trade is determined by the output** of the generators (or sellers), specifically those using solar generation.

This forward-looking approach not only **maximizes unused local capacity (from P2P) but also ensures that the electrical system remains stable** and within its operational boundaries.

The model is designed **to secure the fulfillment of non-dispatchable loads, effectively merging these resources with existing market mechanisms**, in a decentralized and open manner, with multiple users.

Contributions

- ❑ Fills a research gap by integrating distributed power generation of small, heterogeneous, and autonomous agents within Peer-to-Peer (P2P) trading mechanisms;
- ❑ Facilitates the execution of viable P2P energy exchanges, alongside traditional electricity retailers, merging these resources with existing market and operations mechanisms, in an open manner, with multiple users and transactions;
- ❑ Promotes the evolution of novel market configurations - agnostic to contracting mechanisms, and does not assume prior relationships between participants - facilitating the engagement of "prosumers" in energy trading while ensuring power flow remains within permissible bounds;

Contributions

- ❑ Power flow equations are explicitly integrated where network topology is considered (to reflect the network state from several concurrent transactions);
- ❑ Allows clearance and integration with existing market structures and outputs a pre-operational dispatch schedule (assuming a centralized DSO that receives all potential trades from independent participants);
- ❑ Preliminary testing on various trading scenarios and networks demonstrate our model can facilitate and integrate P2P power flow within acceptable network constraints, even in extreme cases.

Diana Vieira Fernandes, Nicolas Christin, Soummya Kar, *Peer-to-Peer (P2P) Electricity Markets for Low Voltage Networks*. To appear in *IEEE SmartGridComm'24 - 2024 IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm)*.

Pre-print (arXiv): <https://arxiv.org/abs/2407.21403>

