The International No.1 Bestseller

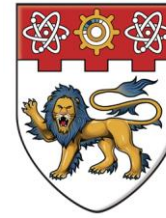The
TIPPING
POINT

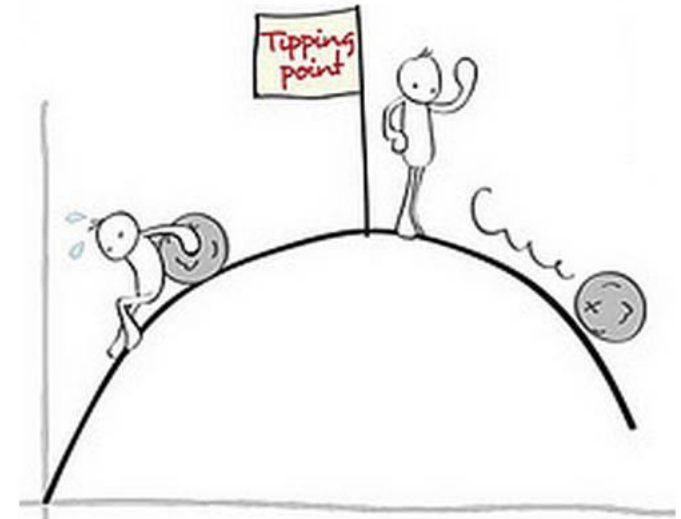HOW LITTLE THINGS CAN MAKE
A BIG DIFFERENCE

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NTU Complexity Winter School
10 Mar 2016

Layman Definitions

Dictionary.com

1. the point at which an issue, idea, product, etc., crosses a certain threshold and gains significant momentum, triggered by some minor factor or change.
2. the point in a situation at which a minor development precipitates a crisis.
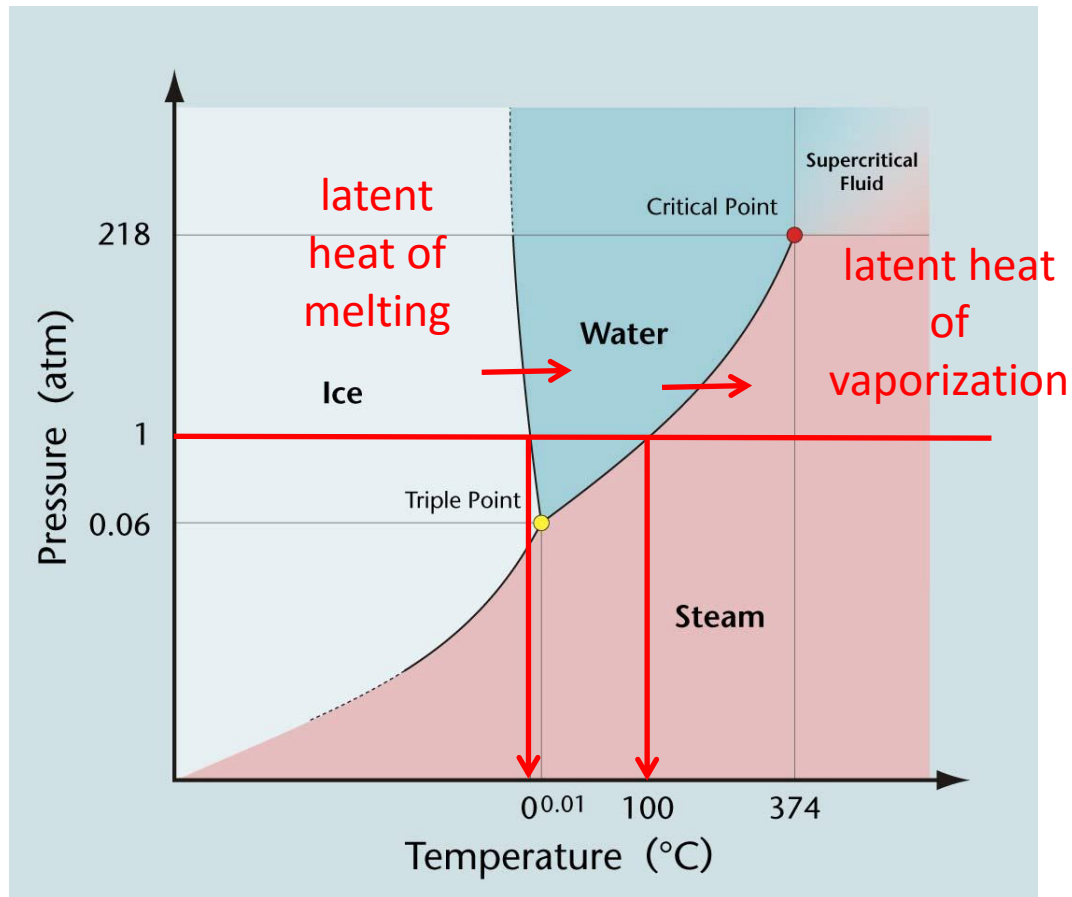
Merriam-Webster

1. the critical point in a situation, process, or system beyond which a significant and often unstoppable effect or change takes place.

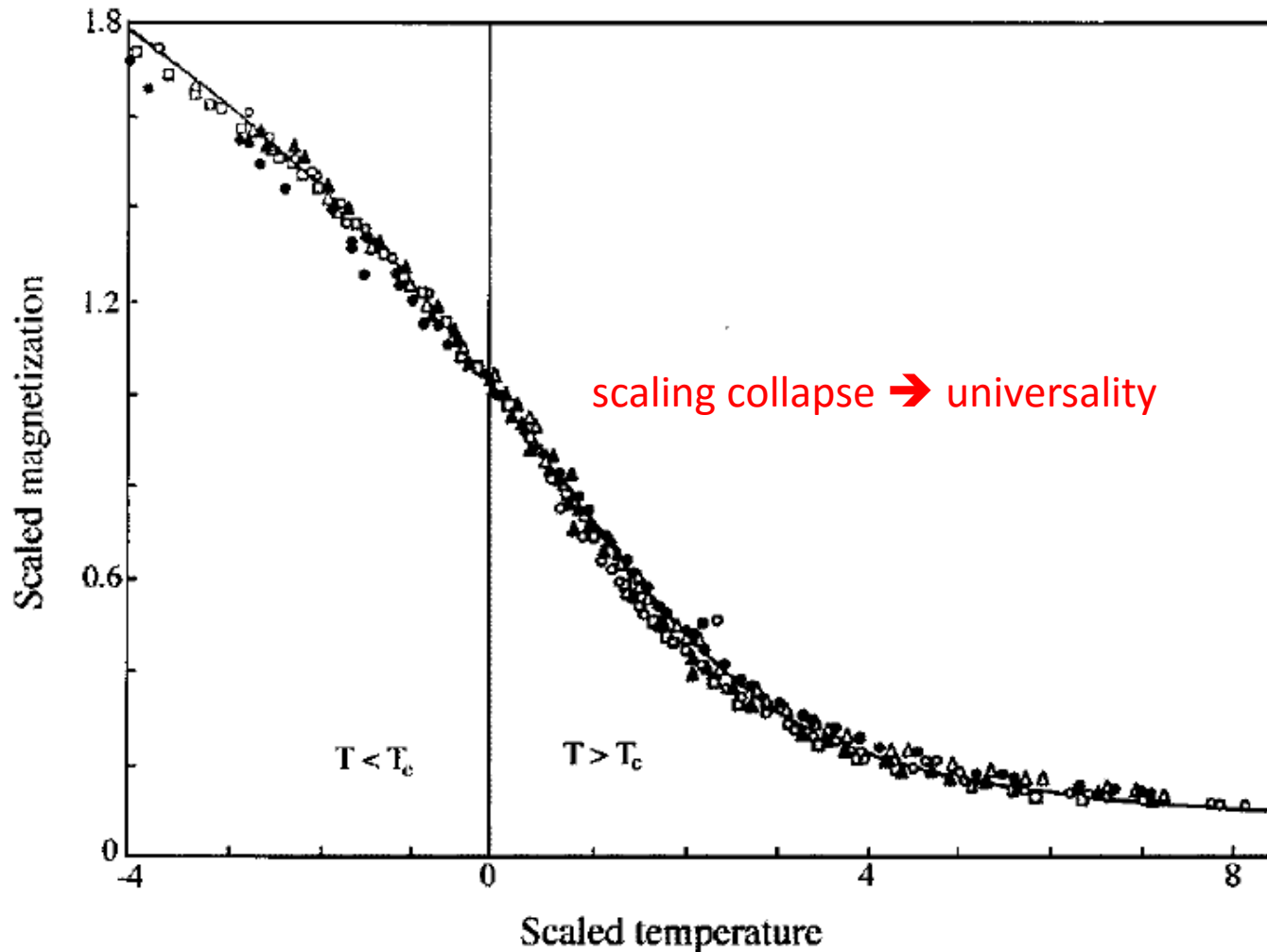**Sudden Change!**

Nomenclature

- **Physical Sciences**
  - Phase Transitions, Critical Transitions

- **Ecological Sciences**
  - Regime Shifts, Critical Transitions

- **Socio-economic Sciences**
  - Regime Shifts, Regime Switches

Phase Transitions

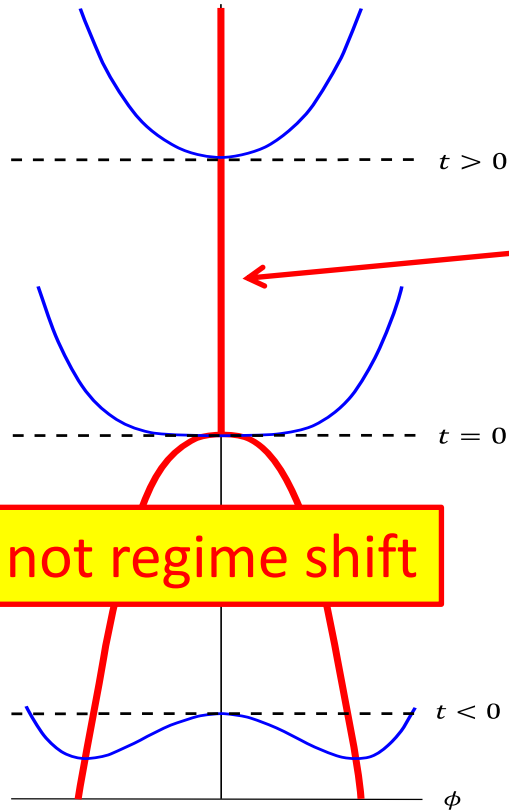# Criticality

diverge a
tempera

scaling collapse ➔ universality

scale
ee

T < T$_c$ T > T$_c$

Scaled magnetization

Scaled temperature

Landau Theory

$$F(\phi) = F_0 + \frac{t}{2!}\phi^2 + \frac{1}{4!}\phi^4$$

Landau free energy

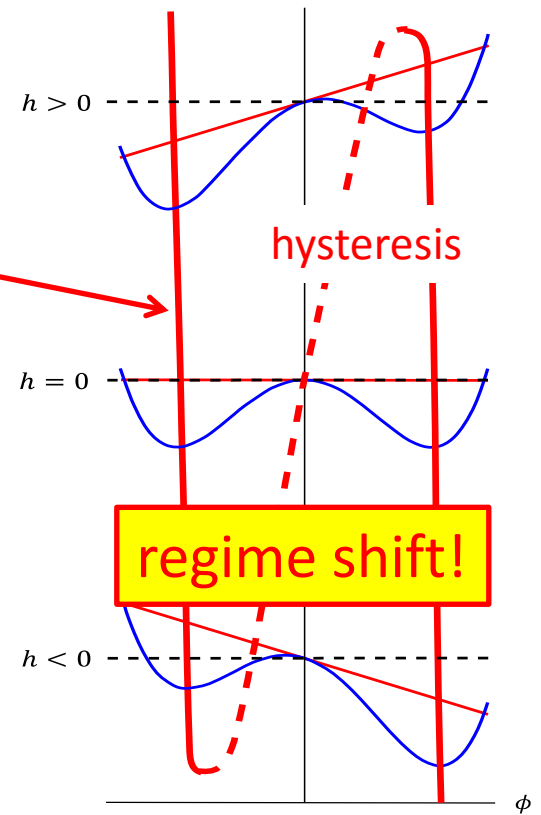$$F(\phi) = F_0 + h\phi + \frac{t}{2!}\phi^2 + \frac{1}{4!}\phi^4$$

fixed points

hysteresis

bifurcation diagram

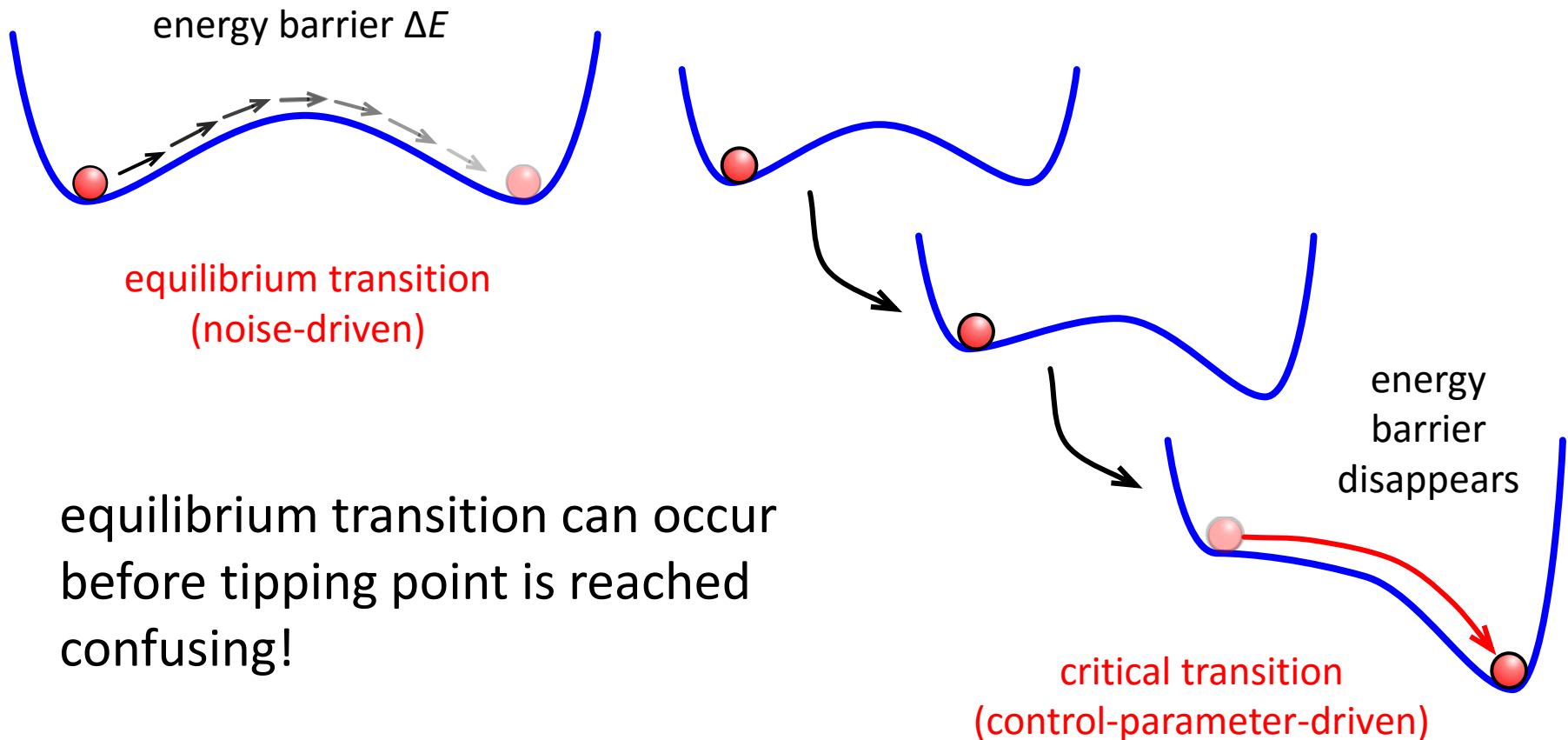not regime shift

regime shift!

Second-order phase transition

First-order phase transition

Terminology

- Control parameter
  - Variable we can change or changes

- Order parameter
  - Variable telling us which regime the system is in

- Tipping point
  - Value of control parameter where critical transition takes place

Equilibrium vs Critical Transitions

energy barrier ΔE

equilibrium transition
(noise-driven)

equilibrium transition can occur
before tipping point is reached
confusing!

energy
barrier
disappears

critical transition
(control-parameter-driven)

Forecasting Regime Shifts

If $\dfrac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}, t)$ known, then can tell how far from tipping point

abrupt, but can be forecasted

ANN

AR models
$$x_{t+1} = \sum_{r=1}^{p} a_r x_{t-r} + \epsilon_t$$
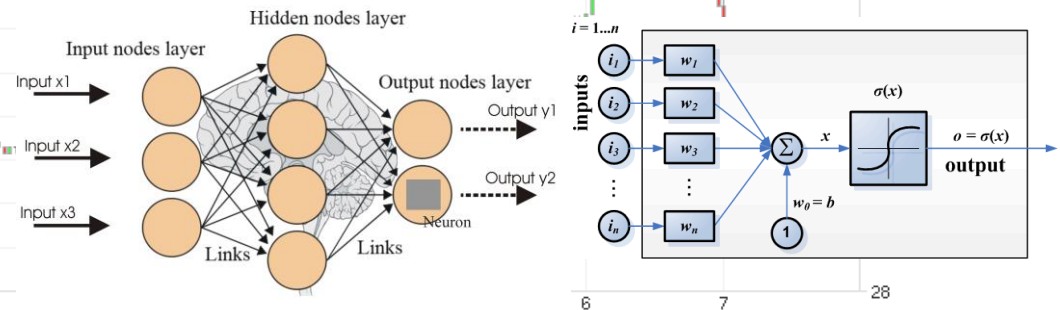
technical trading

Jump diffusion models

not regime-shift-aware!

Universal Early Warning

gaussian
distribution

growing spatial correlations,
diverging length scale
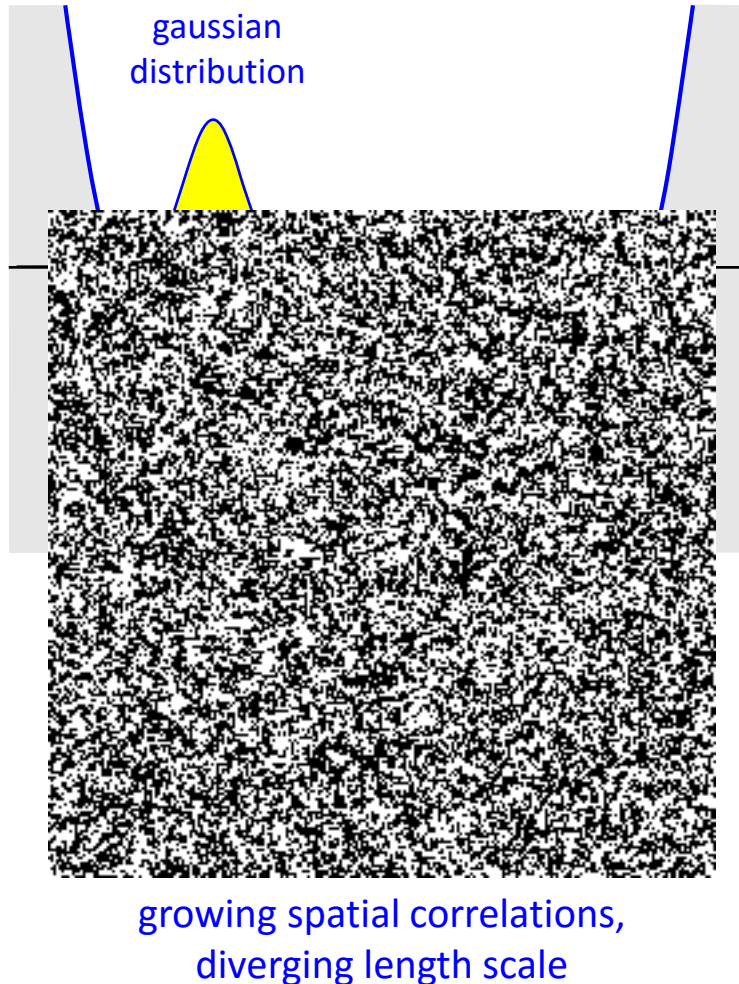
power-law tail at tipping point

variance → ∞

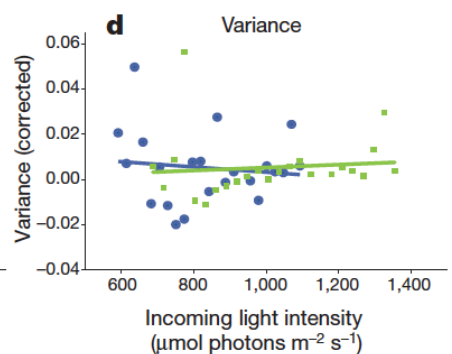critical fluctuations
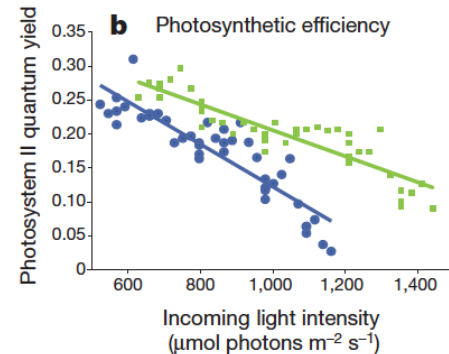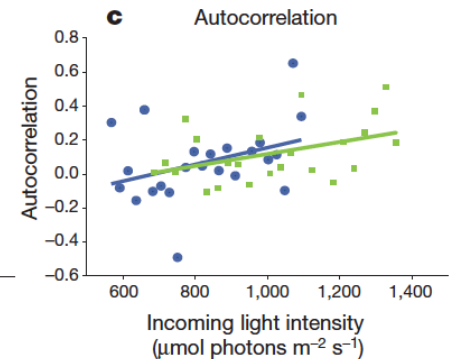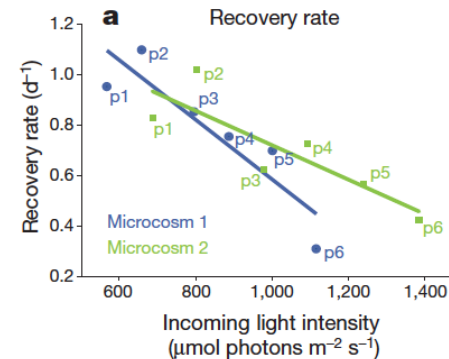
relaxation time → ∞

critical slowing down
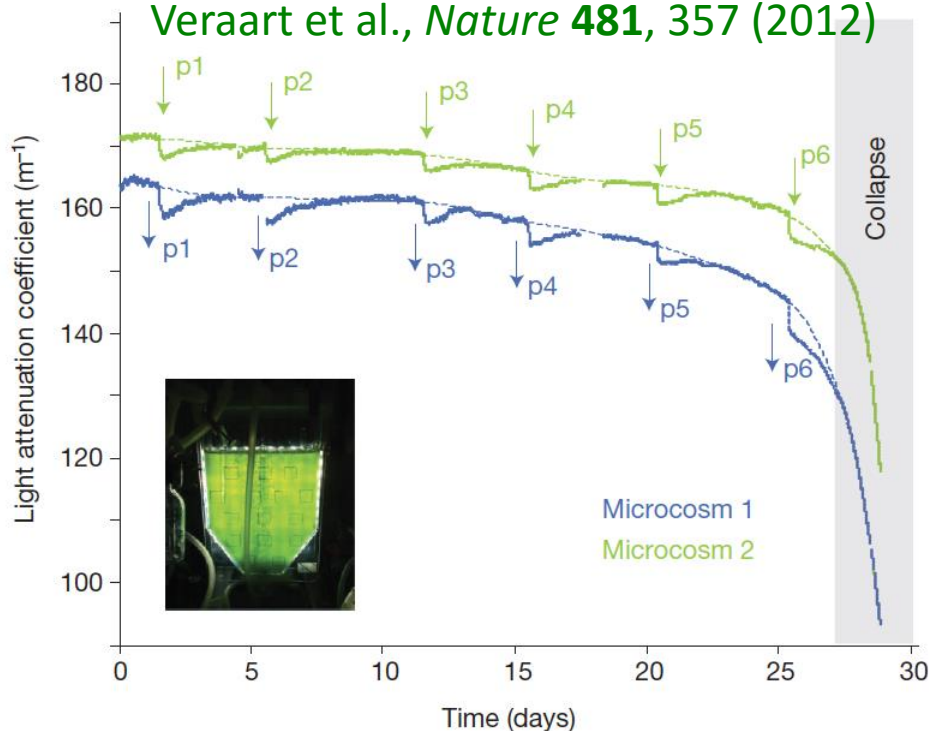
growing autocorrelation,
spectral reddening

US Housing Market

Fourier Transform, Jan 2003 - Dec 2004, Columbus OH

normal

Fourier Transform, Jan 2001 - Dec 2002, Columbus OH

concentration of power in lower frequencies = slowing down of time scale

Homes Sold (Spectrum)
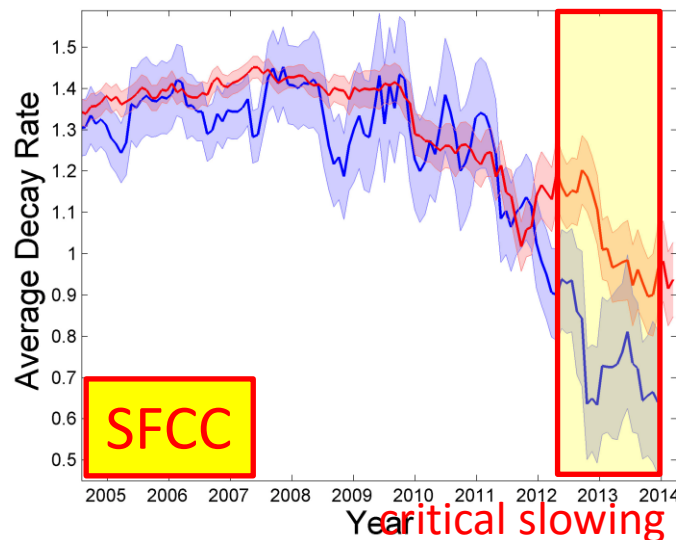
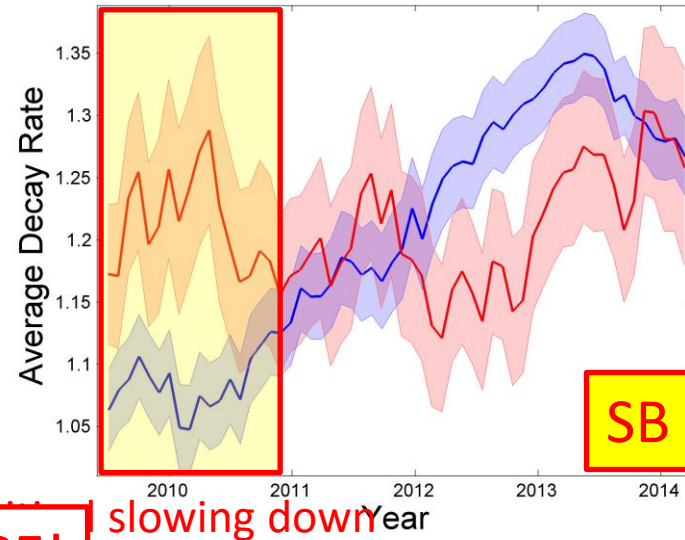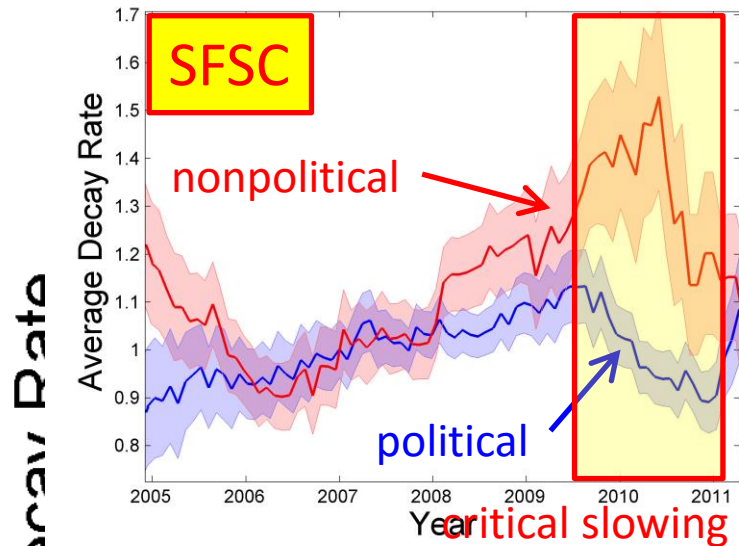Tan & Cheong, *Eur. Phys. J. B* **87**, 38 (2014)
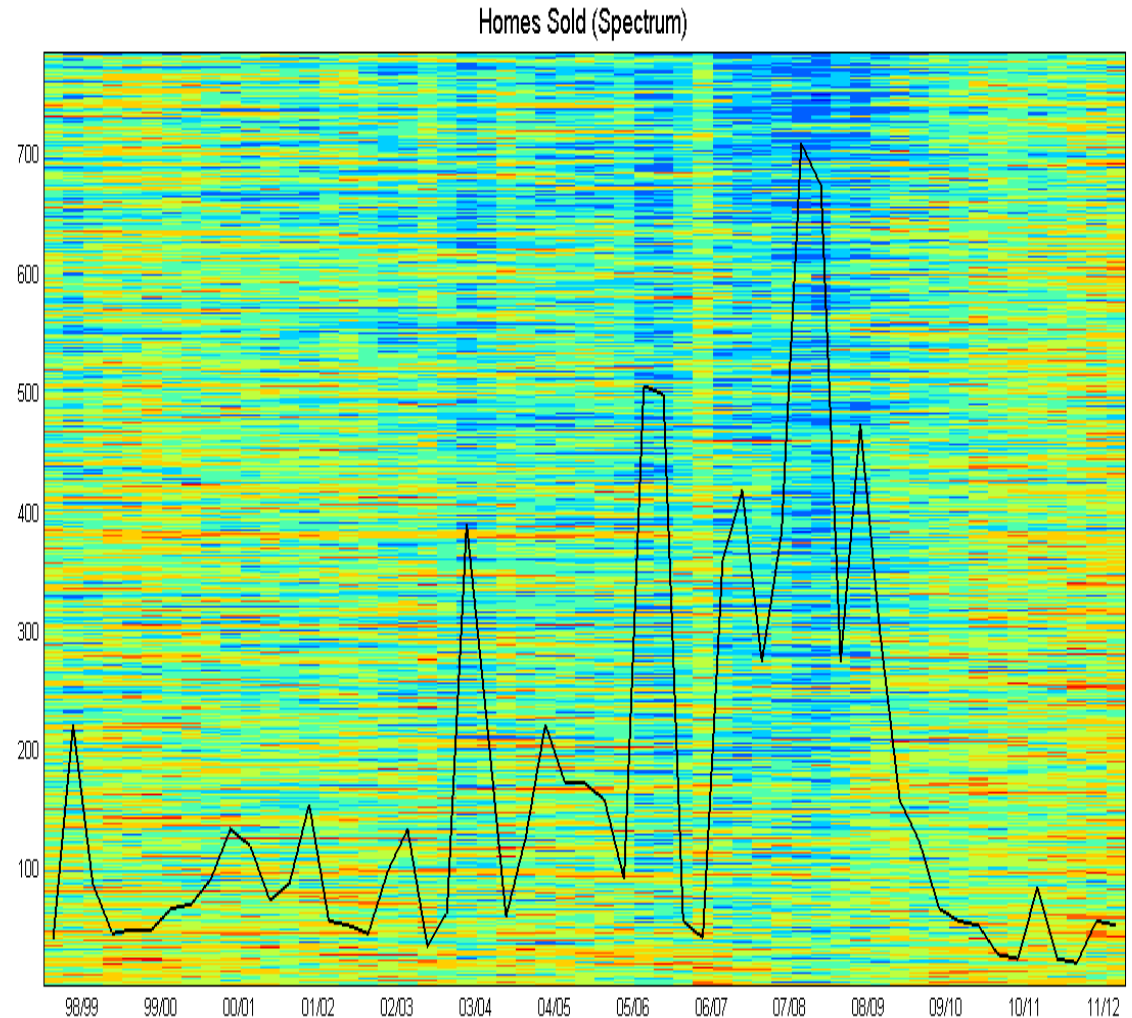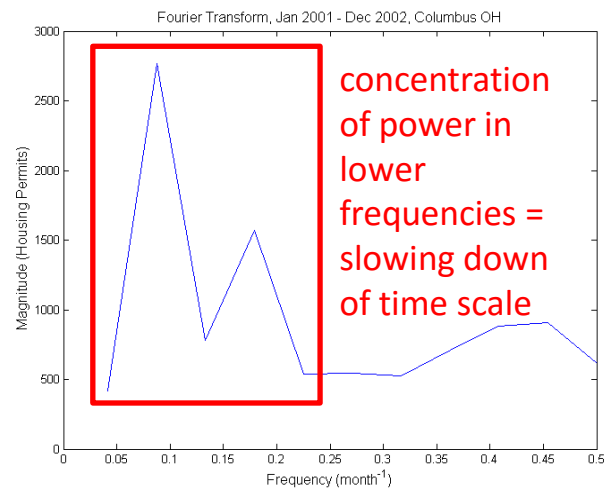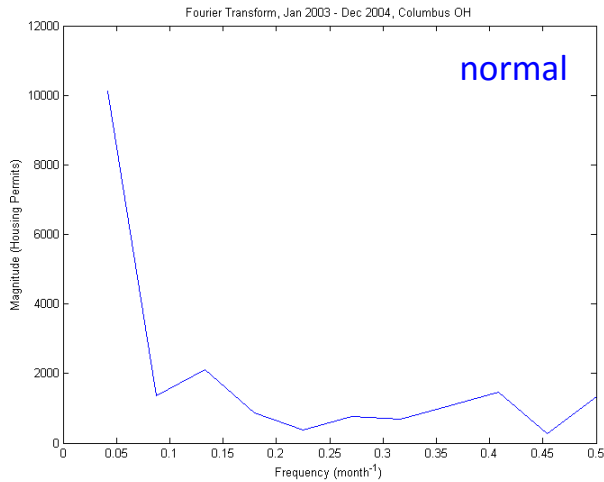
Slow Recovery

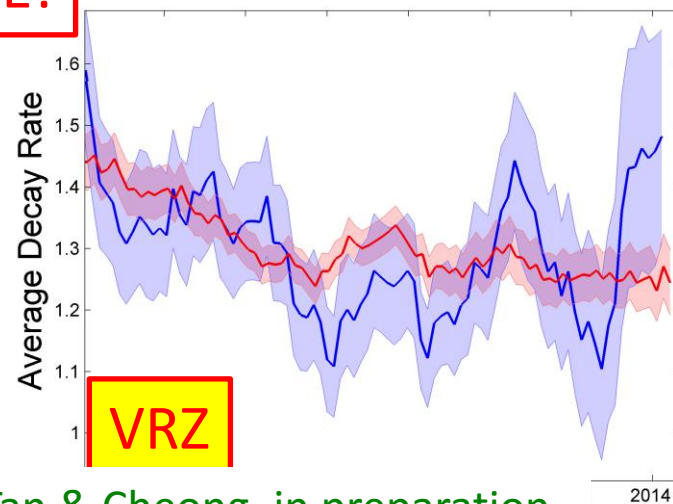Veraart et al., *Nature* **481**, 357 (2012)

- cyanobacteria in controlled chemostat microcosm
- light intensity slowly increased until population collapse
- 10% dilution perturbation every 4-5 days
- measure recovery rate
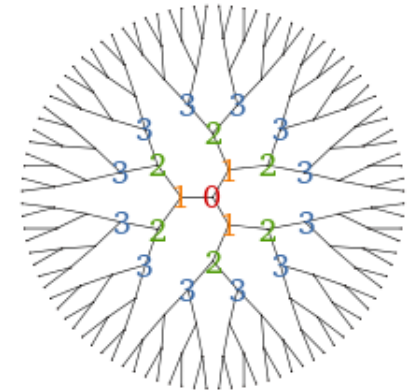
Sociopolitical Regime Shift

Tan & Cheong, in preparation

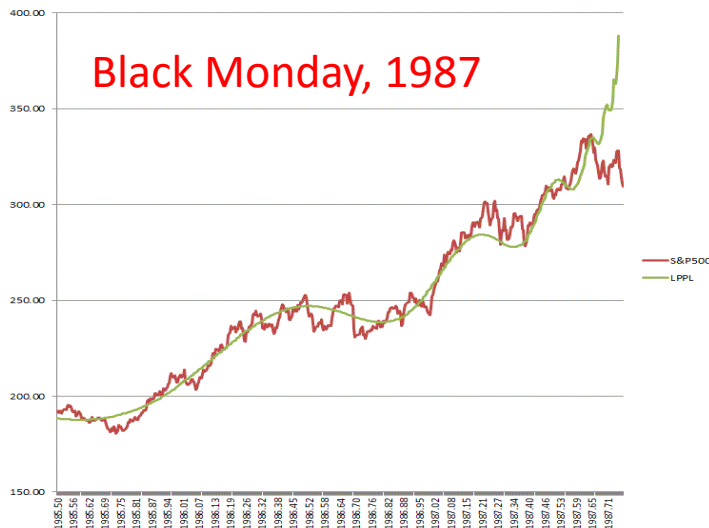13

Log-Periodic Power Law (LPPL)

Prof Didier Sornette, ETH Zurich

Bethe Lattice

Discrete Scale Invariance

$$I(t) = A + B(t_C - t)^\alpha + C(t_C - t)^\alpha \cos\left[\omega(t_C - t) + \Phi\right]$$

Black Monday, 1987

S&P500
LPPL

US Average Unemployment Duration

Soup-of-Groups (SOG) Model

Bohorquez, Gourley, Dixon, Spagat, and Johnson, *Nature* **462**, 911 (2009)
Johnson, Ashkenazi, Zhao, and Quiroga, *AIP Advances* **1**, 012114 (2011)

$$\frac{\partial n_s}{\partial t} = \frac{v_{\text{coal}}}{N^2} \sum_{k=1}^{s-1} k n_k (s-k) n_{s-k} - \frac{v_{\text{frag}} s n_s}{N} - \frac{2 v_{\text{coal}} s n_s}{N^2} \sum_{k=1}^{\infty} k n_k, \quad s \geq 2,$$

$$\frac{\partial n_1}{\partial t} = \frac{v_{\text{frag}}}{N} \sum_{k=2}^{\infty} k^2 n_k - \frac{2 v_{\text{coal}} n_1}{N^2} \sum_{k=1}^{\infty} k n_k,$$
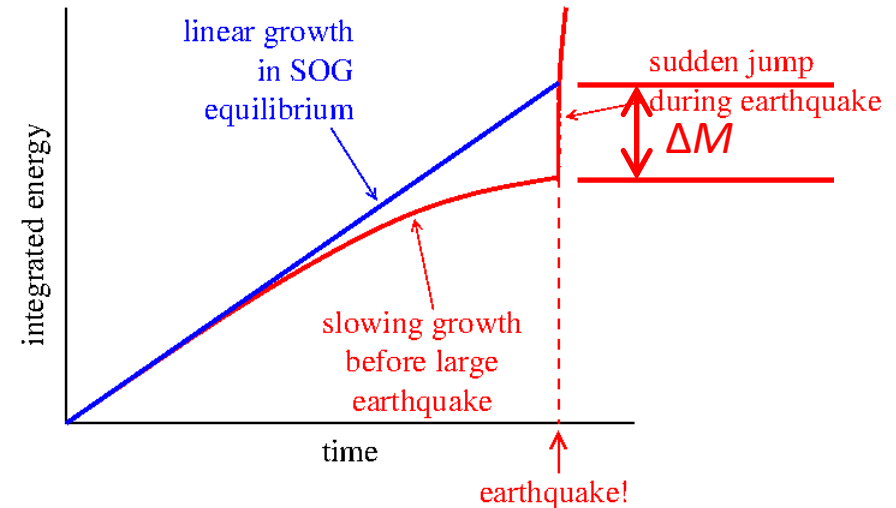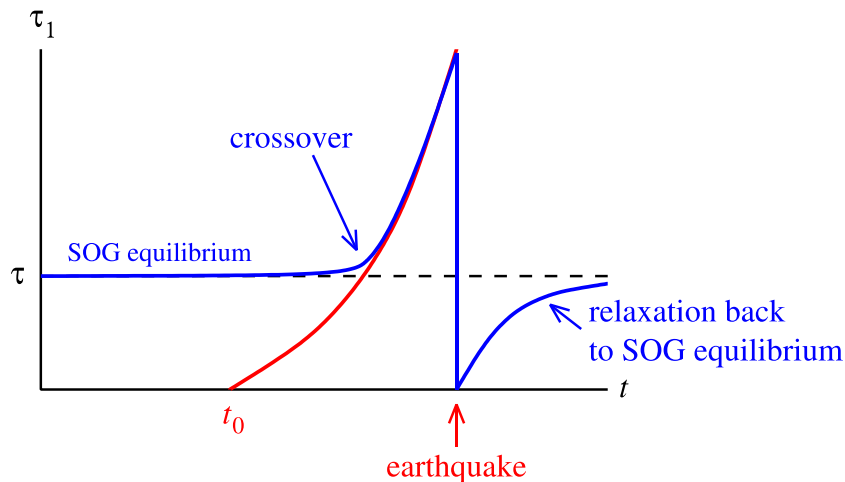
v_{frag}

v_{coal}

$$p(s) \sim s^{-\frac{5}{2}} \longrightarrow \text{2 for } d = 2$$

Gutenberg-Richter Law!

Growth of Giant Cluster

linear growth in SOG equilibrium

sudden jump during earthquake ΔM

slowing growth before large earthquake

earthquake!

crossover

SOG equilibrium

relaxation back to SOG equilibrium

t_0

earthquake

ΔM

Mean Field Theory

linear growth

GR law

$$E(t) = \int_0^t \int_{m_0}^{\log_{10}(S_0 - \gamma t)} 10^m R(m) \, dm \, dt'$$
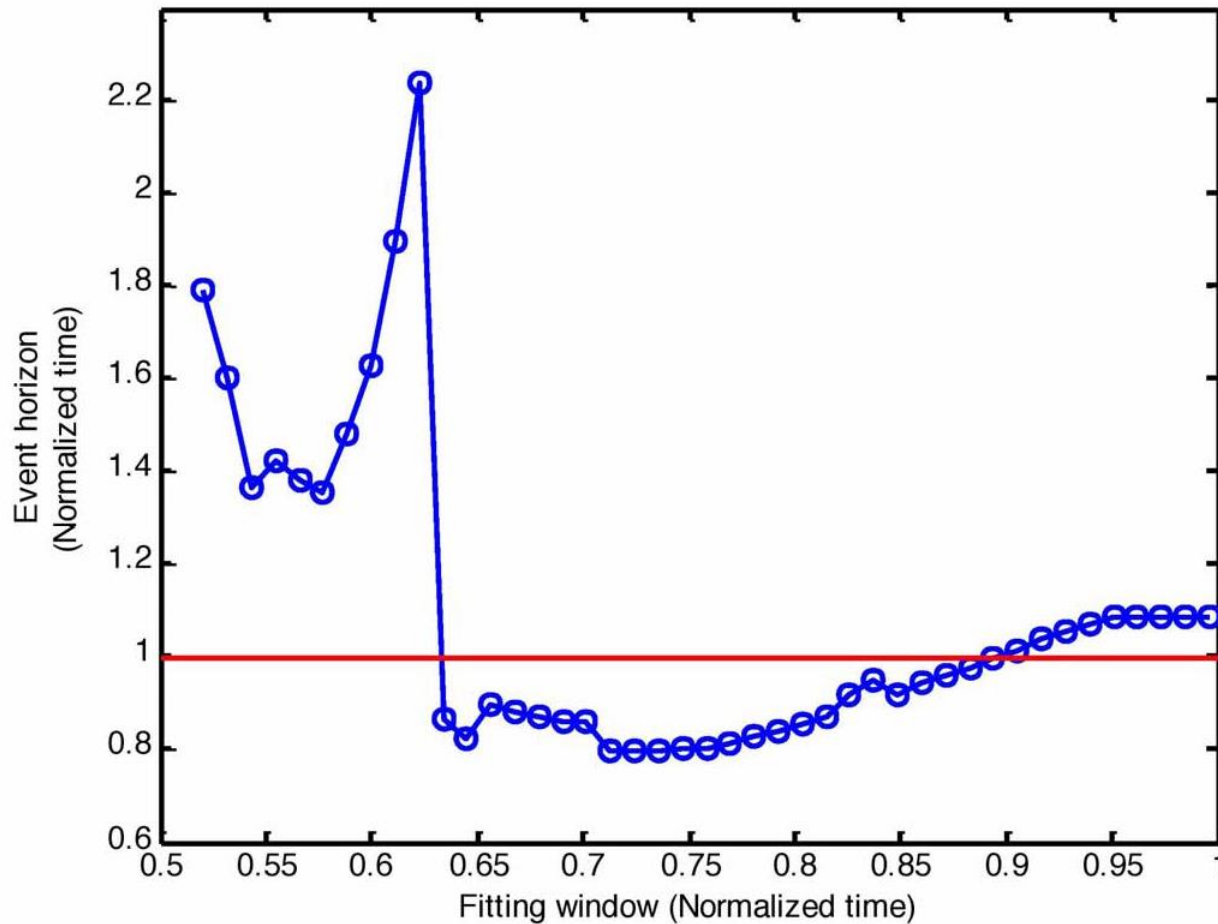
$$= At \left(\log_{10} \frac{S_0 - \gamma t}{s_0} - 1 \right) - \frac{A S_0}{\gamma} \log_{10} \frac{S_0 - \gamma t}{S_0}$$
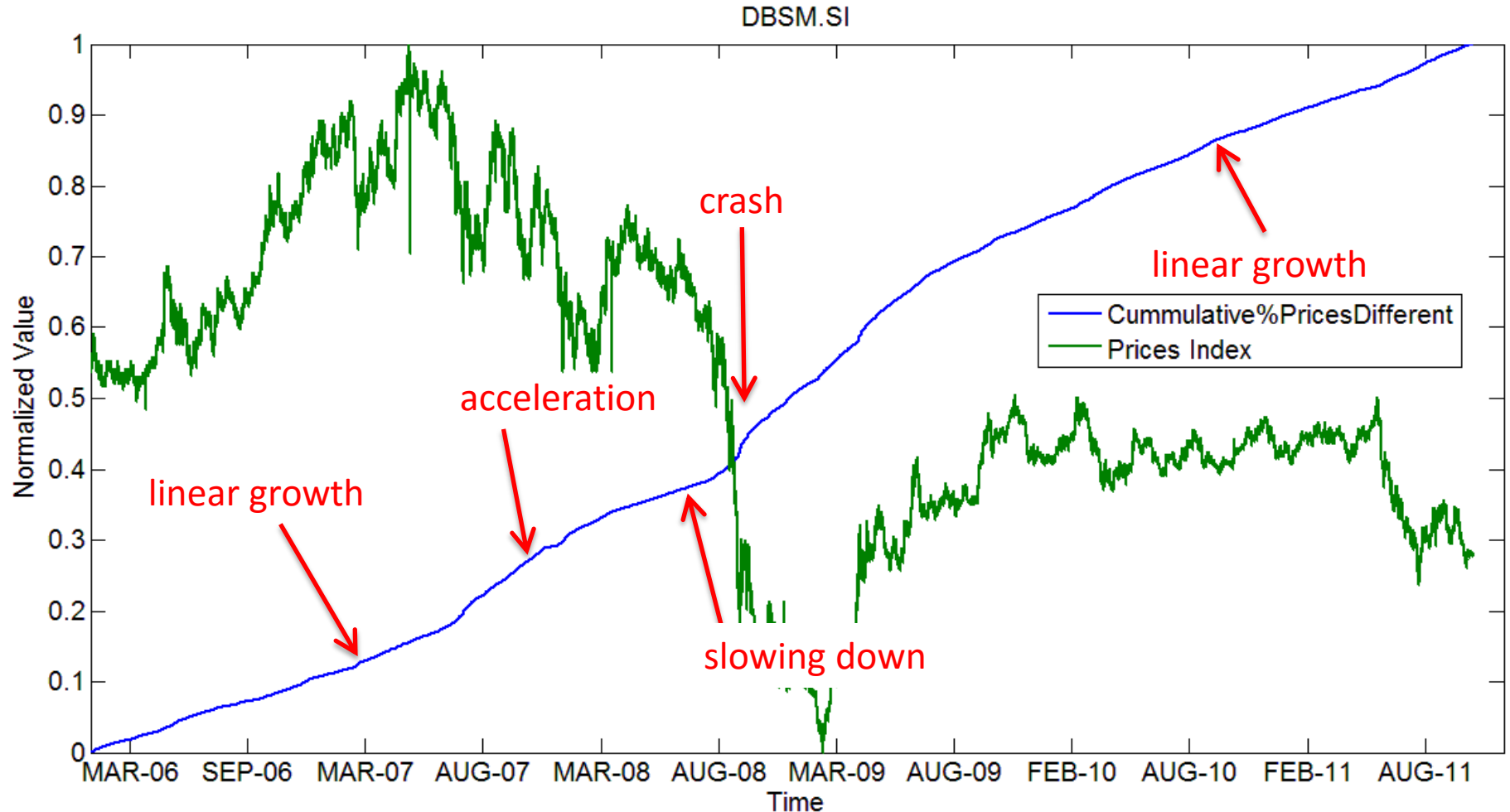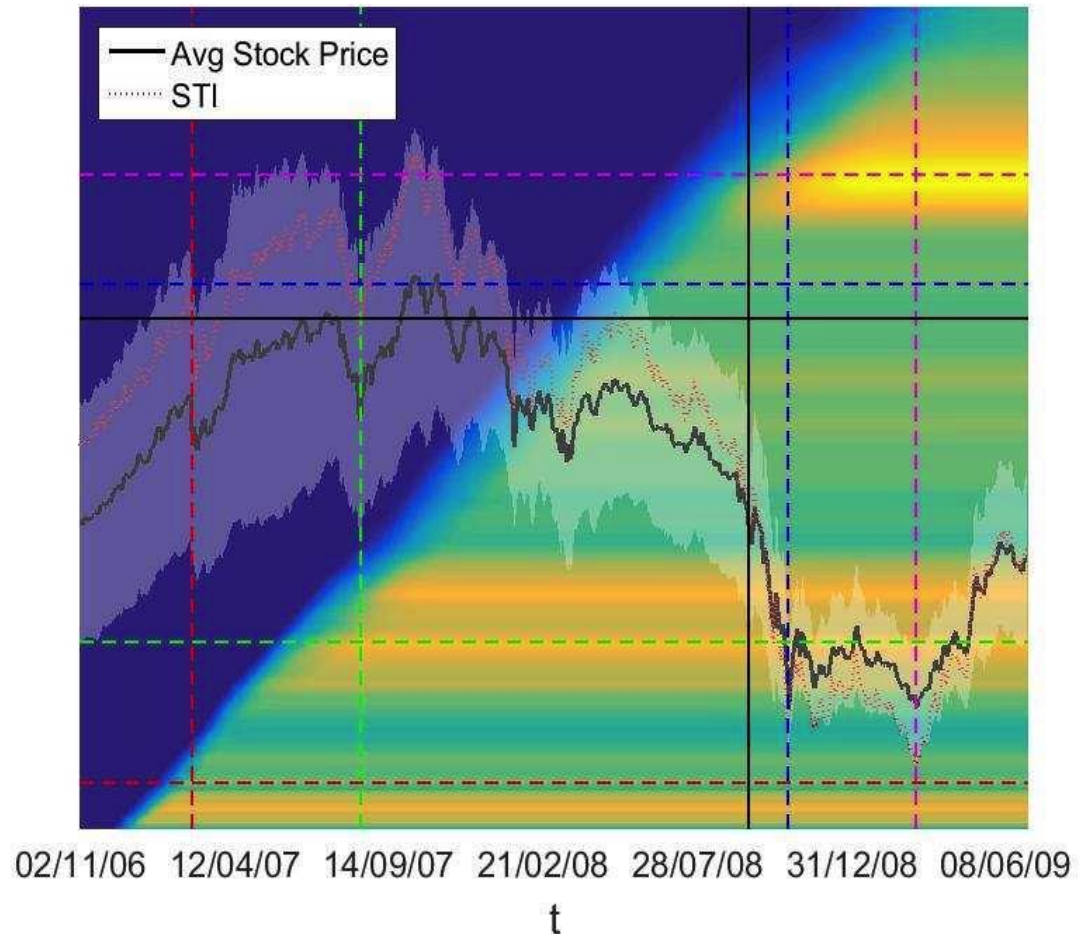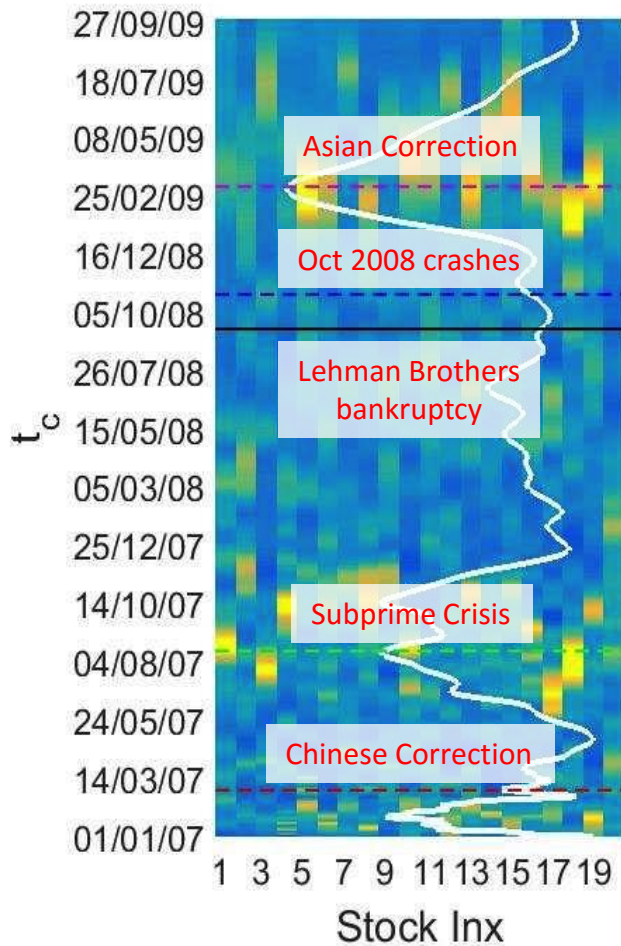
suppression of smaller earthquakes

r_0
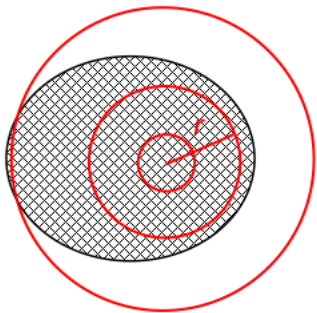
Sep 1999 Chi-Chi Earthquake

Cheong et al., *Sci. Rep.* **4**, 3624 (2014)

Oct 2008 SGX Crash: Precursors

Oct 2008 SGX Crash: Prediction

Teh & Cheong, in preparation

Summary

- Regimes & regime shifts
  - Universal phenomenology
  - Landau Theory
- Early warning
  - Critical fluctuations
  - Critical slowing down
- Quantitative forecasting
  - Log-periodic power law
  - SOG forecasting

References