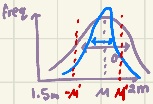
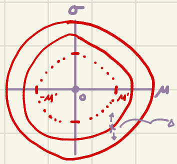
Week 1

#1. vectors.



unit: height (cm)



$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left\{\frac{-(x-\mu)^2}{2\sigma^2}\right\}$

$\begin{bmatrix}\mu \\ \sigma\end{bmatrix} \begin{bmatrix}\mu' \\ \sigma'\end{bmatrix}$

• vector : list of numbers.

Q) $3\cdot 4 - y = 2$

$12 - 2 = y$, $y = 10$

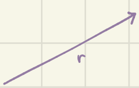
$3x - 2y = 7$
$2x - 2y = 2$ −
$x + 0 = 5$, $x = 5$
$\therefore 10 - 2y = 2$, $8 = 2y$, $y = 4$

$6x - 4y = 8$
$6x + 3y = 15$ −
$0 - 7y = -7$, $y = 1$
$\therefore 6x + 3\cdot 1 = 15$
$6x = 12$, $x = 2$

$-6x + \cancel{6y} = 60$
~~$3(-2x + 2y) = 3\cdot 20$~~
~~$2(5x + 3y) = 6\cdot 2$~~
$10x + \cancel{6y} = 12$ −
$-16x + 0 = 48$
$x = -3$,
$-\cancel{2(-3)} + 2y = 20$ (6)
$2y = 14$, $y = 7$

$\cancel{3x} - \cancel{2y} + z = 7$
$\cancel{3x} - \cancel{2y} - z = 3$ −
$2z = 4$, $z = 2$

$x + y + 2 = 2$
$3x - 2y + 2 = 7$ ⇒
$2x + \cancel{2y} = 0$
$3x - \cancel{2y} = 5$ +
$5x = 5$
$x = 1$, $y = -1$, $z = 2$

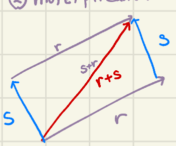


120 sqm²
2 bed Rs
1 bath R
$ 150,000

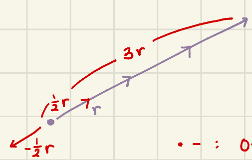
→ $\begin{bmatrix}120 \\ 2 \\ 1 \\ 150\end{bmatrix}$

① addition

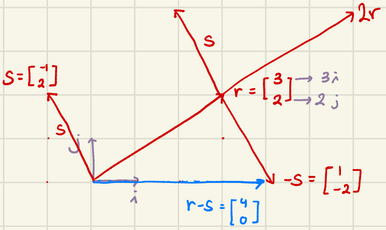
② multiplication by a scalar numbers.



s+r = r+s



• - : other side.



$$r - s = \begin{bmatrix} 3 - -1 \\ 2 \ - 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Associative

- $r + s = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = s + r$
- $(r+s) + t = r + (s + t)$

$$2r = \begin{bmatrix} 2 \cdot 3 \\ 2 \cdot 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$$-r = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$r + -r = \begin{bmatrix} 3 + -3 \\ 2 + -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

120 sqm^2
2 bed Rs
1 bath R
\$ 150,000

$$\rightarrow \begin{array}{cc} \begin{bmatrix} 120 \\ 2 \\ 1 \\ 150 \end{bmatrix} & \begin{array}{l} sqm^2 \\ bed\,R \\ bath R \\ \$,000 \end{array} \end{array}$$

+  =

$$2\begin{bmatrix} 120 \\ 2 \\ 1 \\ 150 \end{bmatrix} = \begin{bmatrix} 240 \\ 4 \\ 2 \\ 300 \end{bmatrix}$$

Q) $a = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$d = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$b = \begin{bmatrix} +1 \\ -2 \end{bmatrix}$

$-b = \begin{bmatrix} -1 \\ +2 \end{bmatrix}$

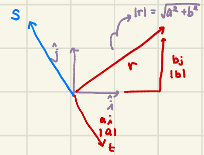
$u = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$u + v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$b = \begin{bmatrix} 1 \\ -2 \end{bmatrix} + e = \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$d - b = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

Week 2



$r = a\cdot\hat{i} + b\cdot\hat{j} = \begin{bmatrix} a \\ b \end{bmatrix}$

$r = \left|\begin{matrix} a \\ b \end{matrix}\right|$

$|r| = \sqrt{a^2+b^2}$

$r = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} r_i \\ r_j \end{bmatrix}$

$s = \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} s_i \\ s_j \end{bmatrix}$

- dot

$r\cdot s = r_i\cdot s_i + r_j\cdot s_j = 3\cdot -1 + 2\cdot 2 = 4-3 = 1$

- commutative

$r.s = s.r$

- associative over scalar multiplication

$r.(\overset{\text{scalar num}}{a}s) = a(r.s)$

$r_1(as_1) + r_2(as_2) = a(r_1s_1 + r_2s_2)$
$= a(r.s)$

- distributive over addition.

$r.(s+t) = r.s + r.t$

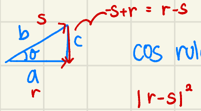
$r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$ $s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{bmatrix}$ $t = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix}$

$r.(s+t) = r_1(s_1+t_1) + r_2(s_2+t_2) \cdots + r_n(s_n+t_n)$

$= r_1s_1 + r_1t_1 + r_2s_2 + r_2t_2 + \cdots + r_ns_n + r_nt_n$

$= r.s + r.t$

cos, dot product



cos rule : $c^2 = a^2 + b^2 - 2ab\cos\theta$

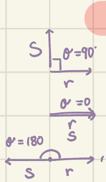
$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$ — ①

$(r-s).(r-s) = r.r - s.r + \underbrace{r.-s}_{=-s.r} + -s.-s$

$= |r|^2 - 2s.r + |s|^2$ — ②

①=② ⇒ $\cancel{|r|^2} - \cancel{|r|^2} + \cancel{|s|^2} - \cancel{|s|^2} - 2|r||s|\cos\theta + 2s.r = 0$, $\cancel{2}s.r = \cancel{2}|r||s|\cos\theta$

$r.s = |r||s|\cos\theta$

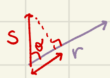


, $\cos 90° = 0$, $r.s = |r||s|\cdot 0 = 0$

, $\cos 0 = 1$, $r.s = |r||s|$

, $\cos 180 = -1$, $r.s = -|r||s|$

Projection



$\cos\theta = \frac{adj}{hyp} = \frac{adj}{|s|}$

$r.s = |r|\underbrace{|s|\cos\theta}_{adj}$

$|r| \times$ projection

scalar projection : $\frac{r.s}{|r|} = |s|\cos\theta$, vector projection : $r\cdot\frac{r.s}{|r||r|} = \frac{r.s}{r.r}r$

Q) $S = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 2 \end{bmatrix}$ $|S| = \sqrt{1^2 + 3^2 + 4^2 + 2^2} = \sqrt{\underbrace{1+9}_{10} + \underbrace{16+4}_{20}}$
$= \sqrt{30}$

$a \cdot b = a_1 b_1 + a_2 b_2 + \cdots$

$r \cdot S = -5 \cdot 1 + 3 \cdot 2 + 2 \cdot (-1) + 8 \cdot 0$
$= -5 + 6 - 2 + 0 = -1$

$S \text{ onto } r = \frac{r \cdot S}{|r|} = |S| \cos\theta$

$= \frac{30 + -20 + 0}{\sqrt{3^2 + (-4)^2 + 0^2}} = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$
$9 + 16$

vector Projection : $r \cdot \frac{r \cdot s}{|r||r|} = \frac{r \cdot s}{r \cdot r} r$

$S \text{ onto } r = \frac{r \cdot S}{r \cdot r} \times r$

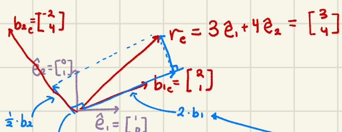
$= \frac{3 \cdot 10 - 20 + 0}{9 + 16 + 0} \times r$

$= \frac{2\cancel{10}}{5\cancel{25}} \times \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 6/5 \\ -8/5 \\ 0 \end{bmatrix}$

$|a+b| = \sqrt{3^2 + 5^2 + 16^2} = \sqrt{290} = 17.029$
$\wedge$ $\wedge$
$|a| + |b| = \sqrt{3^2 + 0 + 4^2} + \sqrt{0 + 5^2 + 12^2}$
$= \sqrt{9+16} + \sqrt{25+144} = \sqrt{25} + \sqrt{169} = 5 + 13 = 18$

Changing basis



how do we know 90°?

$\cos\theta = \frac{b_1 \cdot b_2}{|b_1||b_2|} = 0$, $\cos 90° = 0$

$b_1 \cdot b_2 = 2 \cdot -2 + 1 \cdot 4 = 0$

ratio from e to b

then, $r_b = \begin{bmatrix} ? \\ ? \end{bmatrix} = \begin{bmatrix} 2 \\ 1/2 \end{bmatrix}$

$\frac{r_e \cdot b_1}{|b_1|^2} = \frac{3 \times 2 + 4 \times 1}{2^2 + 1^2} = \frac{10}{5} = 2$, $\frac{r_e \cdot b_1}{|b_1|^2} \cdot b_1 = 2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$\frac{r_e \cdot b_2}{|b_2|^2} = \frac{3 \times -2 + 4 \times 4}{(-2)^2 + 4^2} = \frac{-6+16}{4+16} = \frac{10}{20} = \frac{1}{2}$, $\frac{r_e \cdot b_2}{|b_2|^2} \cdot b_2 = \frac{1}{2} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

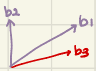
$\therefore \begin{bmatrix} 4 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Basis

1) are not linear combinations of each other (linearly independent)

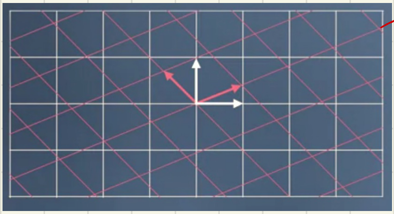
2) span the space

- The space is then n-dimentional



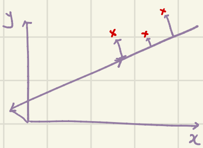
$b_3 \neq a_1 b_1 + a_2 b_2$

↑ linearly indipendent.



→ different dimension.

Changing basis ex



⇒ Linear regression 할때, changing basis를 통해 최적의 직선을 찾을 수 있다.

Week 3

Vector → matrix

ex) a: apple , b: banana

$2a + 3b = 8$
$10a + 1b = 13$

Matrix ⇒ $\begin{pmatrix} \boxed{\begin{matrix}2\\10\end{matrix}} & \boxed{\begin{matrix}3\\1\end{matrix}} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix}$

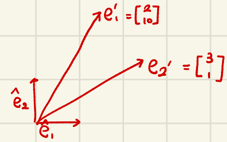
2x2 2x1 2x1 columns.

$$\begin{pmatrix} 2a + 3b \\ 10a + 1b \end{pmatrix} = \begin{pmatrix} 8 \\ 13 \end{pmatrix}$$

$$\begin{pmatrix} \boxed{\begin{matrix}2\\10\end{matrix}} & \boxed{\begin{matrix}3\\1\end{matrix}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$$

$2\times1 + 3\times0$
$10\times1 + 1\times0$

$$\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$



Matrices transform space

$$\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$$

$A \quad r = r'$

$A(nr) = nr'$

$A(r+s) = Ar + As$

$A(n\hat{e}_1 + m\hat{e}_2) = nA\hat{e}_1 + mA\hat{e}_2$
$= ne_1' + me_2'$

ex) matrix multiply

$$\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 32 \end{bmatrix}$$

vector multiply

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix} \left(3\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = 3\left(\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) + 2\left(\begin{bmatrix} 2 & 3 \\ 10 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$= 3\left(\begin{bmatrix} 2 \\ 10 \end{bmatrix}\right) + 2\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 12 \\ 32 \end{bmatrix}$$

ex2) $\begin{bmatrix} 7 & -6 \\ 12 & 8 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 35 - 36 \\ 60 + 48 \end{bmatrix} = \begin{bmatrix} -1 \\ 108 \end{bmatrix}$

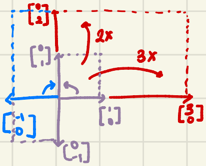
Types of matrix transformation.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

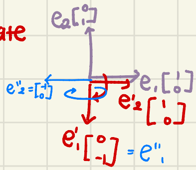
$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$



$A_2\,(A_1 \cdot r)$

90° clockwise rotate



$A_1=\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

$A_2A_1=\begin{bmatrix}0&-1\\-1&0\end{bmatrix}$

$A_2A_1=\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

$=\begin{bmatrix}0&-1\\-1&0\end{bmatrix}$

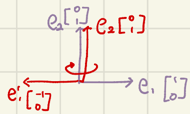
$-1\cdot0+0\cdot-1=0$

$-1\cdot1+0\cdot0=-1$

$0\cdot1+1\cdot0=0$

$0\cdot0+1\cdot-1=-1$

Mirror rotate



$A_2=\begin{bmatrix}-1&0\\0&1\end{bmatrix}$

- not commutative
  $A_2A_1 \neq A_1A_2$
- Assostative
  $A_3(A_2\,A_1)$
  $(A_3\,A_2)\,A_1$

ex)

$A_1\cdot A_2=\begin{bmatrix}0&1\\-1&0\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}$

$=\begin{bmatrix}0\cdot1+1\cdot0 & 0\cdot1+1\cdot1\\-1\cdot1+0\cdot0 & -1\cdot1+0\cdot1\end{bmatrix}$

$=\begin{bmatrix}0&1\\-1&-1\end{bmatrix}$

Q) $Ar=\begin{bmatrix}\frac{1}{2}&-1\\0&\frac{3}{4}\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}\frac{3}{2}-\frac{4}{2}\\0+\frac{3}{2}\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}\\\frac{3}{2}\end{bmatrix}$

$As=\begin{bmatrix}\frac{1}{2}&-1\\0&\frac{3}{4}\end{bmatrix}\begin{bmatrix}-2\\4\end{bmatrix}=\begin{bmatrix}-1&-4\\0&+3\end{bmatrix}=\begin{bmatrix}-5\\3\end{bmatrix}$

$M=\begin{bmatrix}1&0\\0&8\end{bmatrix}\begin{bmatrix}1&0\\-\frac{1}{2}&1\end{bmatrix}=\begin{bmatrix}1\cdot1+\cancel{0\cdot-\frac{1}{2}}^{\,0} & \cancel{1\cdot0}^{\,0}+0\cdot0\\\cancel{0\cdot1}_{\,0}+\cancel{8\cdot-\frac{1}{2}}_{\,-4} & \cancel{0\cdot1}_{\,0}+\cancel{8\cdot1}_{\,8}\end{bmatrix}$

$=\begin{bmatrix}1&0\\-4&8\end{bmatrix}$

Gaussian elimination

$$\underbrace{\begin{pmatrix} 2 & 3 \\ 10 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{r} = \begin{pmatrix} 8 \\ 13 \end{pmatrix} = S$$

- $A^{-1} A = I$
  Inverse
- $\underbrace{\cancel{A^{-1}A}}_{I} \boxed{r = A^{-1} S} \rightarrow$ know $A^{-1} \rightarrow$ solve r (a? b?)

$A^{-1}A = I$

$$\begin{array}{l} 1. \\ 2. \\ 3. \end{array} \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 21 \\ 13 \end{bmatrix}$$

row 1 off of row 2, 3

기준

$$1. \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ -2 \end{bmatrix}$$

multiply row 3 by -1

'elimination'

기준

$$\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & +1 \end{pmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ +2 \end{bmatrix}$$

triangular matrix : everything below the body diagonal is 0

'back-substitution'

put answer of c back into the first two rows.

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \\ 2 \end{pmatrix} \begin{array}{l} \rightarrow 15-4 \\ \rightarrow 6-2 \\ \; \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} \Rightarrow \begin{array}{l} a = 5 \\ b = 4 \\ c = 2 \end{array}$$

$A^{-1}A = I$

$A \; B = I$

$B = A^{-1}$

$$\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

row column

$$\begin{pmatrix} A \end{pmatrix} \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

elimination, back-substitution

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -2 & 0 & 3 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix}$$

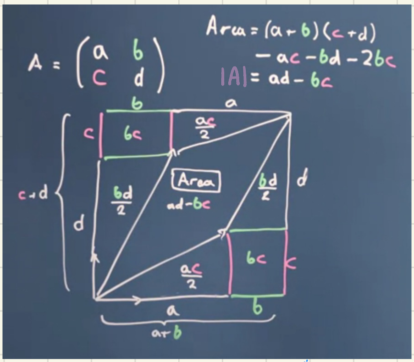
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{pmatrix} \leftarrow \begin{array}{l} A^{-1} \\ B \end{array}$$

Q $\begin{pmatrix}1&1&1\\3&2&1\\2&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{bmatrix}15\\28\\23\end{bmatrix}$

$\begin{pmatrix}1&1&1\\0&1&1\\0&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{bmatrix}15\\12\\5\end{bmatrix}$

$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{bmatrix}3\\7\\5\end{bmatrix}$

Determinants and Inverse



$\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}d&-b\\-c&a\end{pmatrix}=\begin{pmatrix}ad-bc & \overset{0}{\cancel{-ab+ba}}\\ \underset{0}{\cancel{cd-cd}} & -cb+ad\end{pmatrix}$

$\Rightarrow \dfrac{1}{ad-bc}\underbrace{\begin{pmatrix}a&b\\c&d\end{pmatrix}}_{A}\underbrace{\begin{pmatrix}d&-b\\-c&a\end{pmatrix}}_{A^{-1}} = \begin{pmatrix}\cancel{\frac{ad-bc}{}}^{1} & 0\\ 0 & \cancel{\frac{ad-cb}{}}_{1}\end{pmatrix}$

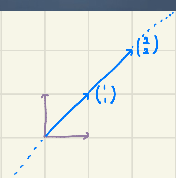
$= \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$= I$

$A = \begin{pmatrix}1&2\\1&2\end{pmatrix}$

$|A| = 0$

determinant $= 0$



$\begin{pmatrix}1&1&3\\1&2&4\\2&3&7\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}12\\17\\29\end{pmatrix}$

row ③ = row ① + row ②

column ③ = 2 × col ① + col ②

$\begin{pmatrix}1&1&3\\0&1&1\\0&0&0\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}12\\5\\0\end{pmatrix}$

$0c = 0$

Week 4

Intro: Einstein summation convention & the summary of the dot product.

$$\underset{A}{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & \\ \vdots & \vdots & & \\ a_{n1} & & & a_{nn} \end{pmatrix}} \underset{B}{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & \\ \vdots & \vdots & & \\ b_{n1} & & & b_{nn} \end{pmatrix}} = \underset{AB}{\begin{pmatrix} & & \\ & ab_{23} & \\ & & \end{pmatrix}}$$

AB : row A × column B

row ↗ ↖ column

$$(ab)_{23} = a_{21} \cdot b_{13} + a_{22} \cdot b_{23} + \cdots + a_{2n} \cdot b_{n3}$$

$$ab_{ik} = \sum_j a_{ij} b_{jk} = a_{ij} b_{jk} \quad (i, k = 1 \cdots n)$$

$AB = C$

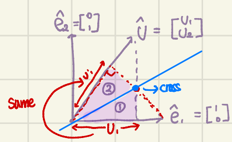
$C_{ik} = a_{ij} b_{jk}$

$$\begin{array}{c} \text{2 rows} \end{array} \overset{\text{3 col}}{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}} \times \overset{\text{4 col}}{\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}} \text{3 rows} = \text{2 rows} \overset{\text{4 col}}{\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}}$$

• revisit the dot product

$$\overset{U}{\begin{pmatrix} U_i \\ \\ \end{pmatrix}} \cdot \overset{V}{\begin{pmatrix} V_i \\ \\ \end{pmatrix}} \Rightarrow [U_1 \, U_2 \cdots U_n] \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

$U_i \cdot V_i$



① = ②

$U_1 = U'_1$

Q) $C_{21} = A_{2j} B_{j1}$

$C_{mn} = A_{mj} B_{jn}$

$C_{21} = A_{21} B_{11} + A_{22} B_{21}$

$= 4 \cdot 1 + 0 \cdot 0 = 4$

Q2) $C_{mn} = A_{mj} B_{jn}$ (→2 row, →3 col)

$$2 \overset{3}{\begin{pmatrix} 4 & 3 & 5 \\ 5 & 4 & 1 \end{pmatrix}}$$

$C_{11} = A_{11} B_{11} + A_{12} B_{21} + A_{13} B_{31}$

$= 1 + \cancel{2 \cdot 0} + 3 \cdot 1 = 4$

$C_{12} = A_{11} B_{12} + A_{12} \cdot B_{22} + A_{13} \cdot B_{32}$

$= 1 \cdot 1 + 2 \cdot 1 + \cancel{3 \cdot 0}$

$= 1 + 2 + 0 = 3$

$C_{13} = A_{11} \cdot B_{13} + A_{12} B_{23} + A_{13} B_{33}$

$= \cancel{1 \cdot 0} + 2 \cdot 1 + 3 \cdot 1 = 5$

$C_{21} = A_{21} B_{11} + A_{22} B_{21} + A_{23} B_{31}$

$= 4 \cdot 1 + \cancel{0} + 1 \cdot 1 = 5$

$c_{22} = A_{21}B_{12} + A_{22}B_{22} + A_{23}B_{32}$

$= 4\cdot 1 + \cancel{0} + \cancel{1\cdot 0} = 4$

$c_{23} = A_{21}B_{13} + A_{22}B_{23} + A_{23}B_{33}$

$= \cancel{4\cdot 0} + \cancel{0} + 1\cdot 1 = 1$

Q3) $2\cdot 1 + 4\cdot 3 + 5\cdot 2 + 6\cdot 1 = 2 + 12 + 10 + 6 = 14 + 16 = 30.$

Q4) $1\cdot 2 + 3\cdot 4 + 2\cdot 5 + 1\cdot 6 = 30$

Q5)

$$\begin{array}{cc} & 4 \\ 3 & \begin{pmatrix} 2 & 2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \end{array}$$

$C_{11} = A_{11}B_{11} + A_{12}B_{21} = \cancel{2\cdot 0} + -1\cdot -2 = 2$

$C_{12} = A_{11}B_{12} + A_{12}B_{22} = 2\cdot 1 + \cancel{-1\,0} = 2$

$C_{13} = A_{11}B_{13} + A_{12}B_{23}$