Домашнее задание

№ 237

$$\det\begin{pmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{pmatrix} = -a\begin{vmatrix} 4 & 4 & -3 \\ 2 & 3 & -2 \\ 4 & 5 & -4 \end{vmatrix} + b\begin{vmatrix} 5 & 2 & -1 \\ 2 & 3 & -2 \\ 4 & 5 & -4 \end{vmatrix}$$

$$-c\begin{vmatrix} 5 & 2 & -1 \\ 4 & 4 & -3 \\ 4 & 5 & -4 \end{vmatrix} + d\begin{vmatrix} 5 & 2 & -1 \\ 4 & 4 & -3 \\ 2 & 3 & -2 \end{vmatrix} =$$

$$= -a(-48-32-30+36+32+40)$$
$$+b(-60-16-10+12+16+50)$$
$$-c(-80-20-24+16+82+75)$$
$$+d(-40-12-12+8+16+45) =$$

$$= 2a - 8b + c + 5d$$

№280

$$\begin{vmatrix} -3 & 9 & 3 & 6 \\ -5 & 8 & 2 & 7 \\ 4 & -5 & -3 & -2 \\ 7 & -8 & -4 & -5 \end{vmatrix} = -9\begin{vmatrix} -5 & 2 & 7 \\ 4 & -3 & -2 \\ 7 & -4 & -5 \end{vmatrix} + 8\begin{vmatrix} -3 & 3 & 6 \\ 4 & -3 & -2 \\ 7 & -4 & -5 \end{vmatrix}$$

$$+5\begin{vmatrix} -3 & 3 & 6 \\ -5 & 2 & 7 \\ 7 & -4 & -5 \end{vmatrix} - 8\begin{vmatrix} -3 & 3 & 6 \\ -5 & 2 & 7 \\ 4 & -3 & -2 \end{vmatrix} =$$

$$= -9(-75 + 28 - 112 + 147 + 40 + 40)$$
$$+8(-45 - 96 - 42 + 126 + 60 + 24)$$
$$+5(30 + 147 - 120 - 84 + 75 - 84)$$
$$-8(12 + 84 + 90 - 48 - 30 - 63) =$$

$$= -9 \cdot 12 + 8 \cdot 27 + 5 \cdot 54 - 8 \cdot 45 =$$

$$= 18.$$

№290

$$\begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2-x & 1 & \dots & 1 \\ 1 & 1 & 3-x & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & n+1-x \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1-x & 0 & \dots & 0 \\ 0 & 0 & 2-x & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & n-x \end{vmatrix} =$$

$$= (1-x)(2-x)\cdot\ldots\cdot(n-x)$$

№293

$$\begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2-x^2 & 2 & 3 \\ 2 & 3 & 1 & 5 \\ 2 & 3 & 1 & 9-x^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 1-x^2 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & 3-x^2 \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 1-x^2 & 0 & 0 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 4-x^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 1-x^2 & 0 & 0 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & 4-x^2 \end{vmatrix} =$$

$$= -3(1-x)(1+x)(2-x)(2+x)$$

~~1327~~ 1297

$$\begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 0 & a_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & a_n \end{vmatrix} = (-1)^{n+1} \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ a_1 & 0 & 0 & \dots & 0 \\ 0 & a_2 & 0 & \dots & 0 \\ \vdots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & a_{n-1} & 0 \end{vmatrix} +$$

$$+ \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 0 & a_2 & \dots & 0 \\ \vdots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & a_{n-1} \end{vmatrix} \cdot a_n = -a_1 a_2 \cdot \dots \cdot a_n + a_n \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 0 & a_2 & \dots & 0 \\ \vdots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & a_{n-1} \end{vmatrix} =$$

$$= -a_1 a_2 \cdot \dots \cdot a_n + a_n D_{n-1} = -a_1 a_2 \cdot \dots \cdot a_n \left(\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right)$$

№30 1

$$\begin{vmatrix} 7 & 5 & 0 & \dots & 0 \\ 2 & 7 & 5 & \dots & 0 \\ 0 & 2 & 7 & \dots & 0 \\ - & - & - & - & - \\ 0 & 0 & 0 & \dots & 7 \end{vmatrix}$$

$$D_n = 7D_{n-1} - 10D_{n-2}$$

$$x^2 - 7x + 10 = 0$$

$$x_1 = 2$$

$$x_2 = 5$$

$$\begin{cases} 7 = C_1 \cdot 2 + C_2 \cdot 5 \\ \begin{vmatrix} 7 & 5 \\ 2 & 7 \end{vmatrix} = C_1 \cdot 4 + C_2 \cdot 25 \end{cases}$$

$$\begin{cases} 14 = C_1 \cdot 4 + C_2 \cdot 10 \\ 39 = C_1 \cdot 4 + C_2 \cdot 25 \end{cases}$$

$$15C_2 = 25 \ ; \ C_2 = \frac{5}{3}$$

$$C_1 = -\frac{2}{3}$$

$$D_n = \frac{5^{n+1} - 2^{n+1}}{3}$$

№303

$$\begin{vmatrix} 1 & 2 & 0 & 0 & 0 & \dots & 0 & 0 \\ 3 & 4 & 3 & 0 & 0 & \dots & 0 & 0 \\ 0 & 2 & 5 & 3 & 0 & \dots & 0 & 0 \\ 0 & 0 & 2 & 5 & 3 & \dots & 0 & 0 \\ 0 & 0 & 0 & 2 & 5 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & \dots & 2 & 5 \end{vmatrix} =$$

$D_n = 5D_{n-1} - 6D_{n-2}$

$\alpha^2 - 5\alpha + 6 = 0$

$\alpha_1 = 3$

$\alpha_2 = 2$

$$\begin{cases} 1 = 3C_1 + 2C_2 \\ -2 = 9C_1 + 4C_2 \end{cases}$$

$3C_1 = 4$, $C_1 = -\frac{4}{3}$

$C_2 = \frac{5}{2}$

$D_n = 5 \cdot 2^{n-1} - 4 \cdot 3^{n-1}$

№ 304

$$\begin{vmatrix} \alpha+\beta & \alpha\beta & 0 & 0 & \dots & 0 \\ 1 & \alpha+\beta & \alpha\beta & 0 & \dots & 0 \\ 0 & 1 & \alpha+\beta & \alpha\beta & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & \alpha+\beta \end{vmatrix}$$

$$D_n = (\alpha+\beta)D_{n-1} - \alpha\beta D_{n-2}$$

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

$$x_1 = \alpha$$

$$x_2 = \beta$$

$$\begin{cases} \alpha+\beta = \alpha C_1 + \beta C_2 \\ \alpha^2 + \cancel{2\beta} + \beta^2 = \alpha^2 C_1 + \beta^2 C_2 \end{cases}$$

$$C_1 = \varphi$$

$$C_2 = \varphi$$

$$D_n = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta}$$

№365.

Фибоначчи: $a_n = a_{n-1} + a_{n-2}$; $a_1 = 1$; $a_2 = 2$

$$D_n = \begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & & -1 & 1 \end{vmatrix} =$$

Доказать, что $D_n = a_n$

$D_n = D_{n-1} + D_{n-2}$, т.к. м-да рекурренты

$\cancel{x^2 - x - 1 = 0}$.

$D_1 = 1$

$$D_2 = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2.$$

ч.т.д.

1329

$$\begin{vmatrix} a^n & (a-1)^n & \dots & (a-n)^n \\ a^{n-1} & (a-1)^{n-1} & \dots & (a-n)^{n-1} \\ \dots & \dots & \dots & \dots \\ a & a-1 & \dots & a-n \\ 1 & 1 & \dots & 1 \end{vmatrix} \overset{A^T}{=} \begin{matrix} (1)\leftrightarrow(n) \\ (2)\leftrightarrow(n-1) \\ (3)\leftrightarrow(n-2) \\ \dots \end{matrix} \begin{vmatrix} 1 & a-n & \dots & (a-n)^{n-1} & (a-n)^n \\ 1 & a-n+1 & \dots & (a-n+1)^{n-1} & (a-n+1)^n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & a-1 & \dots & (a-1)^{n-1} & (a-1)^n \\ 1 & a & \dots & a^{n-1} & a^n \end{vmatrix} =$$

определитель вандермонда

$$= n(n-1)(n-2)\cdot\ldots\cdot 1 \cdot (n-1)(n-2)\cdot\ldots\cdot 1 \cdot \ldots \cdot 2\cdot 1 \cdot 1 =$$

$$= n!\cdot(n-1)!\cdot(n-2)!\cdot\ldots\cdot 2!\cdot 1! = \prod_{k=1}^{n} k!$$