GME: A New Equilibrium?

Dennis Leet
Texas A&M university

February 10, 2022

1 Introduction

The last three years have been tumultuous times for the stock market. Cryptocurrencies dominant financial news and several unexpected securities both made fortunes and broke hedge funds. Game Stop, a brick and mortar video game retailer is one such stock. There is much excitement surrounding the fortunes which stand to be made from GME's fluctuations.

Our goal here is multifaceted: explain the daily returns transformation, model the distribution of the daily returns and finally to prescribe ARMA and ARMA + GARCH models for predicting future price fluctuations. This work is also intended for a wide audience, hence the explanations of short selling and short squeezes. Lastly it is my intention to provide traders with models that are useful. Most of these models are easily implemented within current trading software.

2 Short Selling

Understanding short selling is integral to our understanding of GME's meteoric rise in price *(figure 1)*. To those familiar with market mechanics this idea is standard. However, it is the goal of this report to appeal to a wide audience. Thus I have chosen to explain short selling with the following simplified example:

Consider a retail level US based investor Peter and a publicly traded company **Short Corp** (ticker: **SC**). Peter fully believes that Short Corp is massively over valued and will fall in value in the near future. Peter's reasoning for the price fall could exist for a variety of reasons: unflattering news about the company, a poor earnings report, insider information, or even just a hunch. Regardless of reasoning, Peter is convinced that the price will fall in the future and he wants to profit from the price drop. How does he intend to make money when the stock's value decreases? He will **Short** the stock.

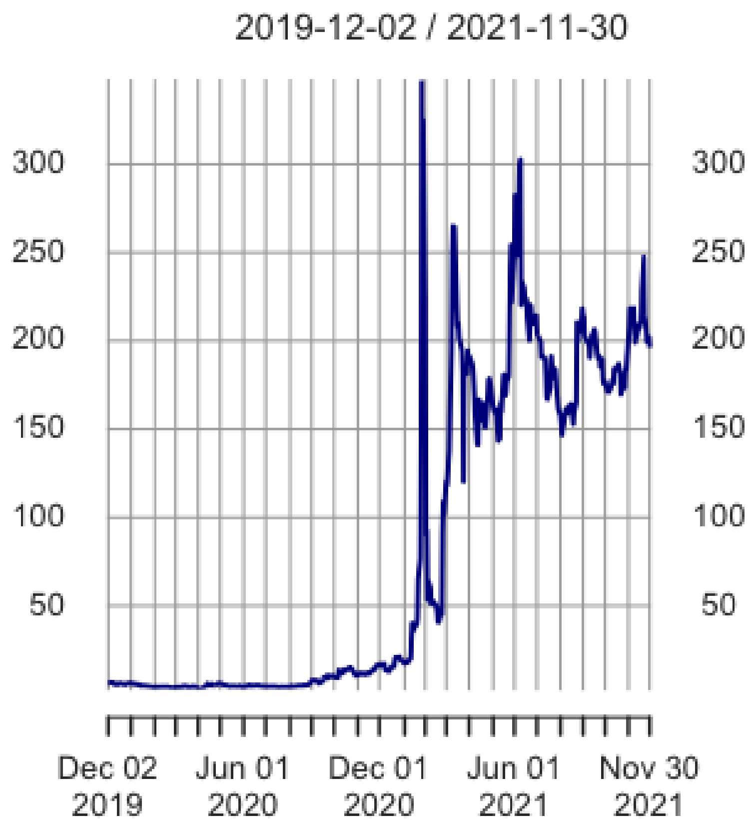

Figure 1:
GME Adj. Closing Price: Dec. 02, 2019 - Nov. 11 2021

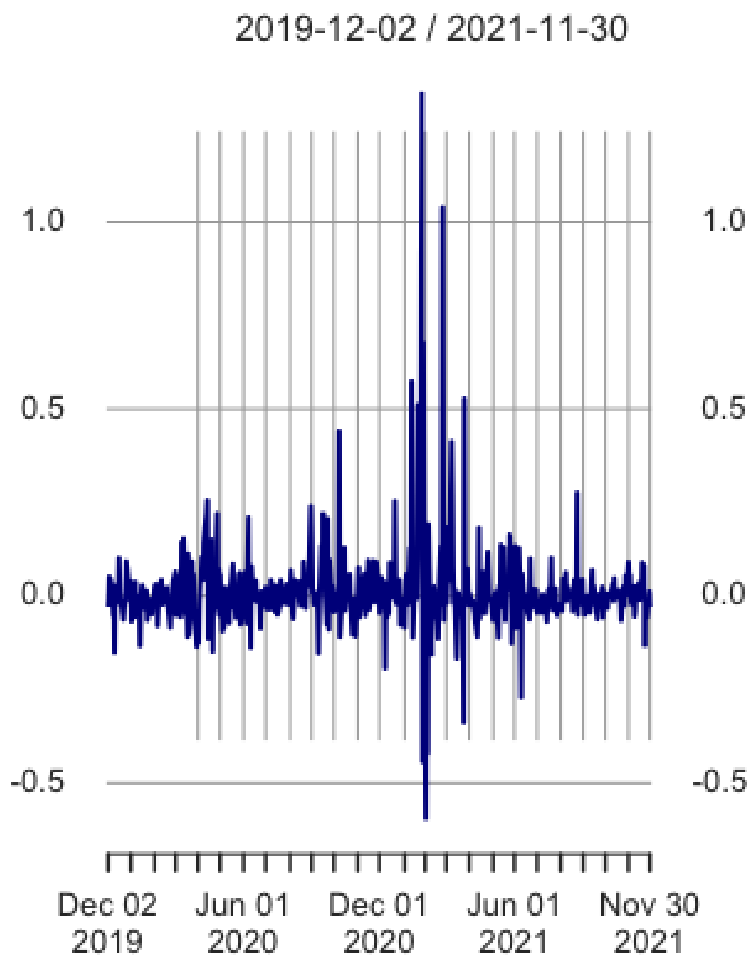

Figure 2:
GME Daily Returns: Dec. 02, 2019 - Nov. 11 2021

To enter into a short position, Peter borrows K many shares from his investment institution and sells those shares on the open market at $price = p_0$ and $time = t_0$. This agreement with his institution stipulates that he must return the shares to them at a specified time t_n. However, he may return them earlier if we wishes. If all goes as planned Peter will buy back the shares from the open market at a lower price p_k and realize a profit, discounting brokerage fees, of $p_0 - p_k$. If $p_0 > p_k$ he profits and if $p_0 < p_k$ he loses money. Less formally, if Peter borrows and sells 100 shares of **SC** on the open market at \$10 each and then repurchases them later for \$3 each, he realizes a profit of \$7 per share for a total profit of $(100 \text{ shares}) \times \$7 = \$700$.

Coming out on top isn't without stress. While the price of a stock can not fall below zero, there exists no price ceiling. Normally an investor purchasing stock at price P and intending to hold can lose a maximum of P dollars per share. However, the investor in a short position might have to repurchase the stock at a much higher price with no ceiling on their losses.

3 The Anatomy of a Short Squeeze

A short squeeze is best understood through example. Consider again the Short Corp (**SC**) stock from earlier. Lets say that a large groups of investors foresee that the price of **SC** will decline in the future. These investors take the appropriate short positions, waiting for the price to drop.

Alternatively, say another group of investors, possibly through coordination, decides to take aggressive long positions. As this group buys into **SC** the price is pushed upwards. It could be that for a time the price of **SC** stays within some tolerable level of loss for those in short positions - if they only knew that the squeeze is just around the corner!

The price of **SC** continues to rise and the short investors get nervous. In an attempt to stop bleeding money, they either hedge their short with a long position or decide to close out their position and purchase the shares of **SC** at the market price - taking the loss on the nose. Demand from short sellers drives up the price of **SC**, which in turn energizes investors to either jump in the market or increase their current long positions. As this cycle continues, the price goes higher and higher.

The Game Stop **GME** short squeeze unfolded in this usual fashion. The squeeze is generally credited as beginning in January 2021. The company's stock steadily slid in value for years and generally traded with little volatility. The mean adjusted closing prices from 01/01/2020 - 01/01/21 and 01/02/2021 \$7.14 and \$166.55 respectively.The respective standard deviations for these periods are

\$4.32 and \$62.19. The question is now if we can predict intense movements in GME's price.

4 Modeling GME Daily Returns

When modeling stock behavior the general practice is to model the returns of the investment. Before proceeding into the thrust of our ARMA + GARCH analysis we want to take a detour and make some obervations about GME's returns. A stock's returns at time 't' denoted as r_t are defined as follows:

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$$

where p_t equals the price at time t. Going forward our goal will be to model the returns on a daily time scale of GME *(figure 2)*. When working with time series it is important to transform the data to stationary. If you are unfamiliar with stationarity, observe the plot of the returns below compared to a chart of the price of GME above.

The returns of securities are generally found to follow a bell shape. This could mean either the normal distribution or several other candidates. Usually this distribution is centered somewhere close zero. As far as the spread of the daily returns, the most extreme behaviors tend to cluster as poor (or exceptional) performance and often begat more of the same. While these sorts of distributions might not perfectly fit our returns, it is a good place to start.

We begin with four candidate models: T, Normal, Skewed T and Skewed Normal. Compared to the normal distribution the T-distribution simply has fatter tails while remaining symmetric. The two skewed options are similar to their symmetric counterparts but lean slightly to one side or the other. We proceed to model fitting through maximum likelihood estimation (MLE) and diagnostics through both visual inspection and the Bayesian Information Criterion (BIC).

Using MLE we obtained one candidate from each of the four distributions by running through a sequence of parameter choices and picking the parameter which is most likely to generate the data. We then used BIC to pick the best model which was the skewed t-distribution. The BIC values for each model are in the table below:

Distribution:	T	Skewed T	Normal	Skewed Normal
BIC:	-995.40	-1005.08	-637.82	-637.82

For visual comparison we plot the kernel density estimate (KDE) of the asset's returns over top of our skewed t-distribution. While it is standard to plot the fitted distribution over a histogram in other contexts, assigning bin width proved problematic over such a small domain. Using the KDE also gives us the opportunity to view both distributions as continuous.

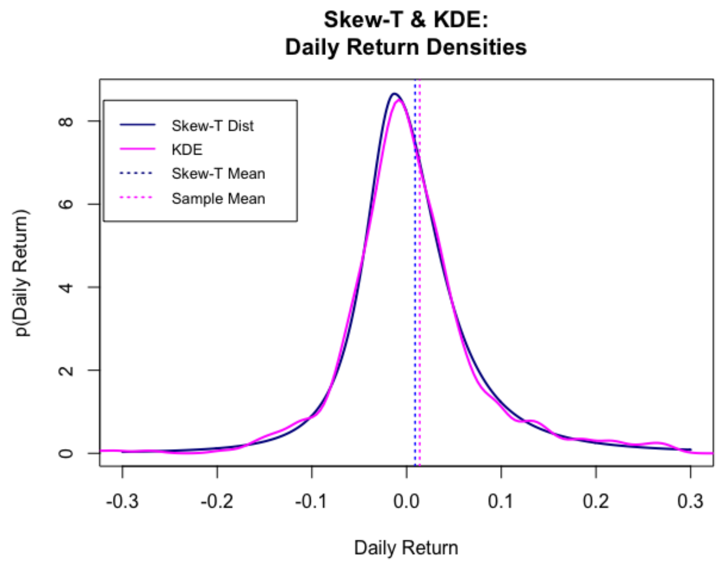

Figure 3:
The KDE and Skewed-T Distributions

The KDE and skewed t-distribution fit very well over each other. If you look closely it is apparent that both distributions seem to lean slightly to the left with means only slightly larger than zero. We observe that most of the negative daily returns tend to congregate on the interval (-0.1,0) while the positive returns seem to have a wider dispersion. I will include an exhaustive list of quantiles in the results section of the report.

5 Methods

Now motivated with a better understanding of GME's return structure, we move to the bulk of our analysis. From here forward we will be working with both Autoregressive Moving Average (ARMA) and Generalized Autoregessive Moving Average (GARCH) models. We begin this section of the report further motivating and discussing both of these models.

We made extensive use of Maximum Likelihood Estimation (MLE) throughout this report by use of the **arima()** and **sarima()** function in R. As one of the workhorses of frequentist statistics, MLE provides easy to recreate results. In addendum we also pursued Yule Walker estimation for the ARMA portion of model building. While the results for Yule Walker and MLE estimation are often similar, we simply consider another form of estimation as a way to bolster the case for our model. We also perform model selection using BIC with its tendency to parsimony.

Consider our conversation earlier on clustering. We established that bullish days and bearish days on the market tend to happen on consecutive days. This should come as no surprise to anyone who has ever watched the market and attempted to exploit a trend. For our analysis and model creation we intend to do this very same thing. If we know that the change in price today is strongly correlated to the change in price over the last five days, how would this affect our trading decisions? Even if it was weakly correlated, we would still want to take it under consideration. ARMA models provide us with a concise framework for carrying out this plan.

ARMA models are composed of both autoregressive and moving average components. So that we can continue directly with our analysis, lets take a quick look at an ARMA(2,1) model:

$$x_t = c + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \theta_1 \epsilon_{t-1} + \epsilon_t$$

What does this mean? Allow x_t to be the price level of a stock at the current time 't' in days. What we are saying with the above equation is that the price at time 't' is a combination of the price from the last two days, x_{t-2} and x_{t-1} accompanied by a combination of "white noise" denoted as ϵ_{t-1} and ϵ_t.

Analogous to model the mean return with ARMA is also using GARCH to model the variance or risk. Instead of assuming a constant amount of variance over time, GARCH enables us to model the variance as changing conditionally on the data. Once again this seems very logical. If an investor experiences a few turbulent prior days on the market, they would also suspect that the next day also had a high chance of risk. Without becoming too technical, we consider that the risk today is a combination of risk on prior days and some white noise or randomness.

Since it is my goal for the reader to walk away with a useful model, lets go ahead and fit a model to the GME data. Afterwards, we will further discuss interpretation. I want to provide my methods but also give clear and ample evidence for my choices.

6 ARMA-GARCH Model Estimation & Selection

To begin fitting the AMRA portion of our model we first look at the autocorrelation function (ACF) and its partial (PACF) counter part. In doing so we hope to gain understanding into the internal relationship structure of GME's returns.

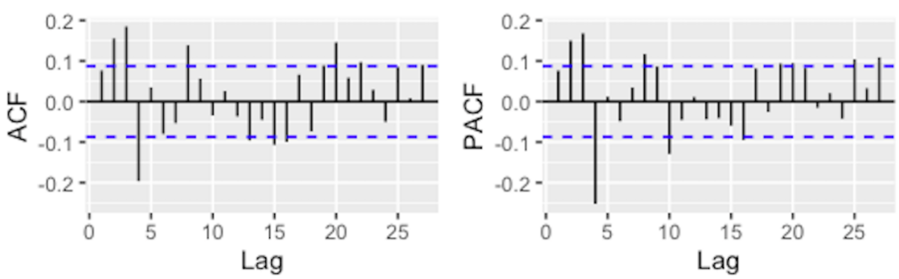

Figure 4:
ACF & PACF of GME Daily Returns

Observing *figure 1*, we see a clear correlation structure in the ACF and PACF. We look at the x axis labeled "Lag". Any bar which is higher than the blue dotted line indicates that there is possible significance for a lagged variable at the level. Since both the ACF and PACF exhibit this pattern, precedent in time series modeling establishes that we should being with a model of mixed AR and MA portions. The significance in lags 1-4 dictates that we want to begin with an ARMA(4,4). However, we want to observe the virtue of parsimony in our model building. While we might start with an ARMA(4,4) model, it is our intention to "whittle down" our lagged variables to something more manageable.

We utilize the sarima() function in r to generate candidate models. This is a balance between minimizing the BIC and making sure that the 95% confidence intervals on our coefficient estimates do not include zero. Inclusion of zero means that the AR or MA term may not be significant. I also want to make it known to the reader that I wish to bias our selection towards AR models. This is due to the fact that AR models tend to be more tangible for users than MA models with their reliance on moving averages of white noise. Thus we want the AR component of our model to be lagged greater than 2 but less than 5. We will then explore which MA component works best with the two different AR levels. The following table showcases the BIC value for different choices of p and q. Remember that p is the autoregressive lag and q corresponds to the lag on the moving average component.

BIC —— ARMA(p,q).		
	p=3	p=4
q=1	-1.2567	-1.2649
q=2	-1.2562	-1.2540
q=3	-1.2530	-1.2536
q=4	-1.2504	-1.2467

The answer appear to be simple - ARMA(4,1)! Not so fast. Upon closer examination the first lag of the AR component does not appear to be significant. Thus we instead turn to the ARMA(3,1). The model is written out in mathematical long form as follows. Note that the subscripts below the coefficients represent their respective standard errors. For $w_t \sim Normal(0, \sigma^2)$, $t \in \mathbf{Z}$:

$$x_t = 0.014_{(.008)} - 0.566_{(.09)} x_{t-1} + 0.175_{(.05)} x_{t-2} + 0.298_{(.04)} x_{t-3} + 0.646_{(.10)} w_{t-1} + w_t$$

We want to take a minute and look at this model. Notice how the coefficients are much larger than their respective standard errors - this is a good sign that they are very likely significant. We see that x_t is a combination of three prior x_t entries and two white noise distributed variables. We now proceed to analyze the residuals. As with any model there are both wins and loses within the plots.

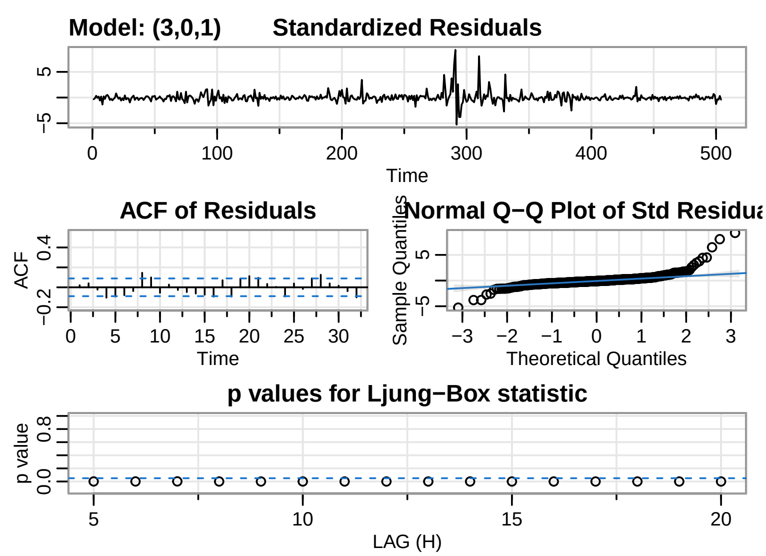

Figure 5:
Plots and ACF of ARMA(3,1) Residuals.

With the exception of the subset corresponding to the short squeeze, (around

300 on the top plot) the residuals seem to be distributed normally with the same variance across time. I anticipated this and while it isn't ideal, it doesn't mean that we need to throw away our model. The ACF of the residuals is promising but not perfect. It appears that we have taken most of the auto-correlation from the residuals through our model however some still remains.

The Normal QQ-plot caries with it the standard problems of modeling financial data - specifically returns. Ideally we want all of the hollow points fairly close to the blue line. While returns are often distributed Gaussian-like, there is a tendency towards fat tails. For us these fat tail entries are tied directly to the short squeeze where the returns changed 100 fold from one day to the next. Once again clustering also comes into play. Remember how extremely bullish or bearish activity tends to clump together? This causes our model to have more error as groupings of extreme values throw off our estimate both individually and as a group. Essentially after a group of extreme entries, our model has to reacclimate to more typical behavior which also causes more error. This is a downside of predicted entries as a combination of prior realizations. Lastly, the Ljung-Box results show that there is still some autocorrelation left in the residuals as a group. This is not necessarily surprising after looking at the residual ACF plot earlier.

What are we to make of these diagnostics? They could be better. Out of the eight models whose BIC we checked, this is the best option taking into consideration our data set and specifically the transformation from price to returns made earlier. At it's base level the idea of this project is to give investors a useful tool. There are undoubtedly many transformations which stationarize the time series more efficiently than turning the price into daily returns. However, what is called for is a model with easy interpretation. Thus we sacrifice model fit for ease of implementation and interpretation within the framework of swing-trading. Hence we move forward to GARCH fit and prediction.

We now want to model our variance as inconsistent across time. This sensible approach is accomplished through GARCH as mentioned in the methods section. It should be acknowledge that this second approach to modeling our data will give us a similar model structure with different coefficients in the ARMA portion.

Observing the squared residuals from our ARCH fit, we see a clear pattern that GARCH(1,1) would be an appropriate fit for the variance across time. Utilizing the **ugarchfit()** in R we construct and examine the BIC values for 4 different ARMA models with an attached GARCH(1,1) component. Their BIC values are as follows:

	ARMA(2,1)	ARMA(2,2)	ARMA(3,1)	ARMA(3,2)
BIC:	-1.6345	-1.6765	-1.6561	-1.6985

Despite our desire to simplify, the slightly more complex ARMA(3,2) component is selected by BIC to accompany our GARCH(1,1) variance model. This model is specified as follows:

$$x_t = -1.313_{(.003)}x_{t-1} + -0.243_{(.06)}x_{t-2} + 0.128_{(.07)}x_{(t-3)} + 1.322_{(.005)}w_{t-1} + 0.366_{(.003)}w_{t-2} + w_t$$

$$\sigma_t^2 = 0.135_{(.01)}\epsilon_{t-1}^2 + 0.865_{(.02)}\sigma_{t-1}^2$$

Where ϵ_t is defined as the error of our model at time t and $w_t \sim Normal(0, \sigma^2)$. We can see the ARMA model on top which only differs slightly from our last AMRA model. What is new is how σ_t^2. We see that it is created by a combination of the error from the t-1 term and the variance of the last time period. This make our variance nimble and able to react to quick changes in market volatility.

The residual analysis isn't much better than than that of our ARMA model. We see a clear QQ-plot of the residuals below.

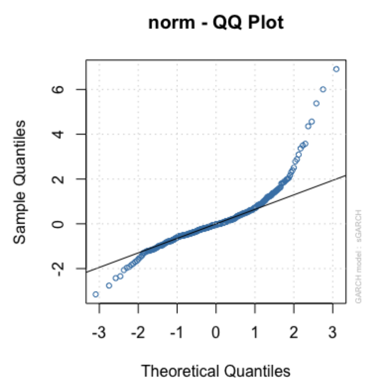

Figure 6:
QQ-plot of ARMA(3,2)+GARCH(1,1).

7 Results and Forecasting

Towards the beginning of this work we fitted a skewed t-distribution over the returns of GME. A more exhaustive list of the quantiles follows:

Percentile	Return Value
0.01	-0.216
0.05	-0.097
0.10	-0.065
0.25	-0.032
0.50 (median)	-0.013
0.75	0.036
0.90	0.097
0.95	0.144
0.99	0.357

Once again we see how the distribution is essentially centered at zero and is skewed right (notice how much further away from 0 the 99th percentile is than the 1st). Often times it is difficult for traders to understand if a particular day's return is out of the ordinary. It is my intention that this table will assist traders in knowing whether or not they are in a volatile or calm market. For instance a daily return of -0.04 might not cause any alarm bells to go off but a return of -0.08 might be worth examining for downward momentum.

We are pushing towards prediction and would like to know how often we are able to predict and interval in which the returns occur. Regardless of its fit, the ARMA+GARCH model remains useful. View the plot below (*figure 7*). The orange line is two times the predicted standard deviation, $2 \times \sigma_t$ across the return series. Notice how often we "capture" fluctuations in price. Over the course of two years the price only exceeded our predicted interval 10 times. This is because the conditional variance is able to open up and show us when the market is more volatile.

The ARMA only model, while useful for motivating our understanding of ARMA+GARCH, proved not to be as useful for prediction compared to the ARMA+GARCH model. This is specifically due to the utility gained through GARCH's conditionally expanding and contracting variance. The five day forecast *(figure 8)* from our ARMA + GARCH model predicted three out of five daily returns within one standard deviation and all returns five days out within two standard deviations.

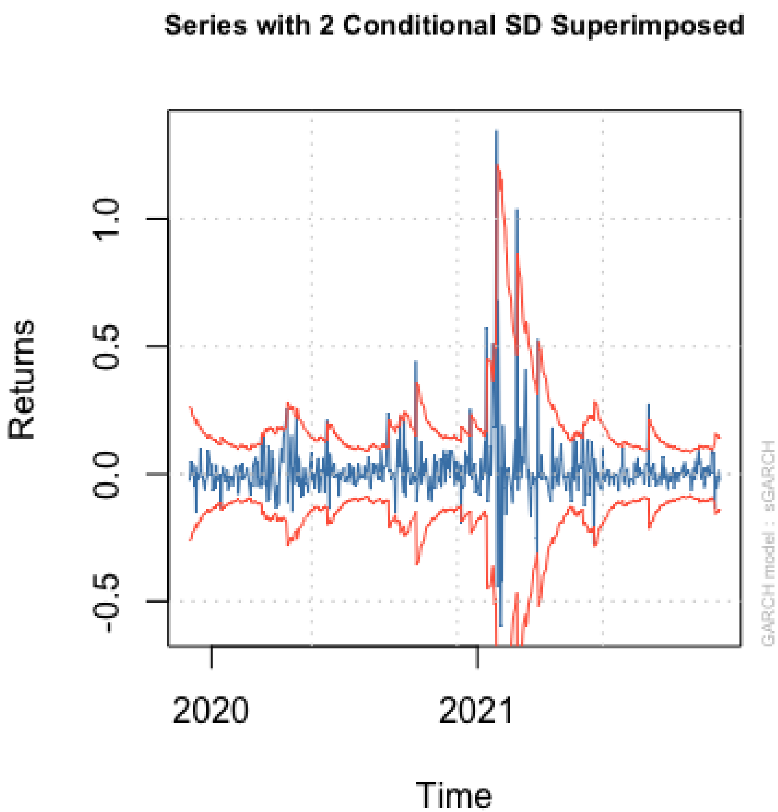
Series with 2 Conditional SD Superimposed
Returns
1.0
0.5
0.0
-0.5
2020
2021
Time
GARCH model : sGARCH

Figure 7:

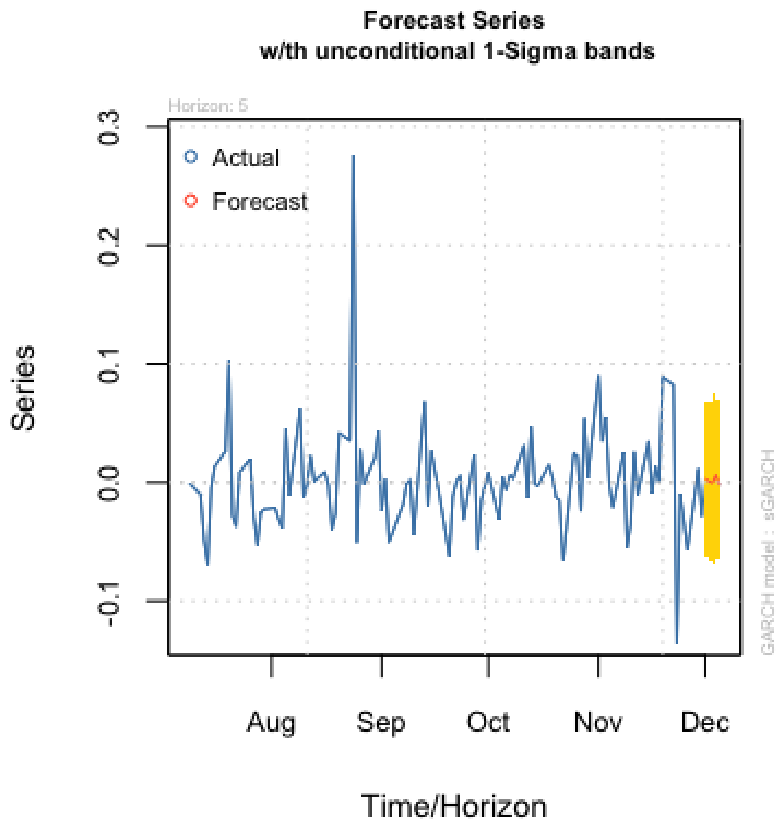
Forecast Series
w/th unconditional 1-Sigma bands
Horizon: 5
Actual
Forecast
Series
0.3
0.2
0.1
0.0
-0.1
Aug
Sep
Oct
Nov
Dec
Time/Horizon
GARCH model : sGARCH

Figure 8:

8 Conclusion

While I do not claim to have a crystal ball (wouldn't that be nice!) I hope that these models are useful to swing traders making decision concerning GME. I believe that the ARMA(3,2)+GARCH(1,1) model is especially ripe for forecasting intervals to catch the daily return. These models should also be easy to implement within various software applications. A word of caution. These models are meant to be used as tools instead of sole decision criterion. Ideally they should be inside of a bouquet of different tools to help traders make smart decisions. Over time the behavior of GME's returns will change. When that will happen is difficult to predict but eventually it will be necessary to create new models which better approximate the current behavior. Until then, I hope that these models assist you in your endeavors as a trader.

9 Sources

Gamestop SEC Filing / General Corporate Information
https://www.sec.gov/ix?doc=/Archives/edgar/data/1326380/000132638021000032/gme-20210130.htm

Short Selling / Short Squeeze (p. 1,4 - 5):
https://www.sec.gov/comments/s7-08-08/s70808-318.pdf

For further information on Financial Time Series consult:

Chatfield, C., amp; Xing, H. (2019). The analysis of Time Series: An introduction with R. CRC Press,Taylor amp; Francis Group.

Ruppert, D., amp; Matteson, D. S. (2015). Statistics and data analysis for Financial Engineering: With R examples. Springer.

All data downloaded in .csv form from:
www.yahoo.finance.com