Parallelism (PAR)

Unit 2: Understanding parallelism

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Course 2024/25 (Fall semester)

Outline

Expressing and understanding parallelism

- Can the computation be divided in parts?[1]
  - Task decomposition: based on the processing to do (e.g. functions, loop iterations)
  - Data decomposition: based on the data to be processed (e.g. elements of a vector, rows of a matrix) (implies task decomposition)
  - There may be (data or control) dependencies between tasks
- Metrics to understand how our task/data decomposition can potentially behave
- Factors: granularity and overheads

[1] Different strategies covered during this course

Learning material for this lesson

- Atenea: Unit 2.1 Understanding parallelism I
  - Video lesson 2: expressing tasks
  - Questions after video lesson 2
  - Going further: Excel to explore the effect of task decomposition overheads
- Atenea: Unit 2.2 Understanding parallelism II
  - Video lesson 3: speed-up and efficiency
  - Questions after video lesson 3
  - Going further: Excel to explore the effect of data sharing overheads
- These slides to dive deeper into the concepts in Unit 2
- Collection of Exercises: problems in Chapter 2

Outline

Outline

Concepts in video lesson 2

- TDG: directed acyclic graph to represent tasks and dependencies between them
- Metrics:
  - $T_1 = \sum_{i=1}^{nodes}(work\_node_i)$
  - $T_\infty = \sum_{i \in critical\_path}(work\_node_i)$, assuming sufficient (infinite) resources
  - $Parallelism = T_1/T_\infty$
  - P_{min} is the minimum number of processors necessary to achieve $Parallelism$
- Task granularity vs. number of tasks

Outline

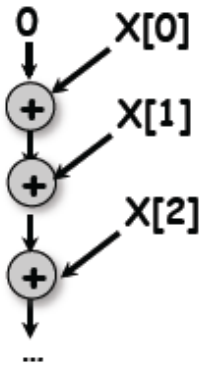
Example 1: vector sum

Compute the sum of elements `X[0] ... X[n-1]` of a vector `X`

```
sum = 0; for ( i=0 ; i< n ; i++ ) sum += X[i];
```

Task definition: each iteration of the `i` loop is a task.

- **TDG** (with input data):



- **Metrics:**

$T_1 \propto n$

$T_\infty \propto n$

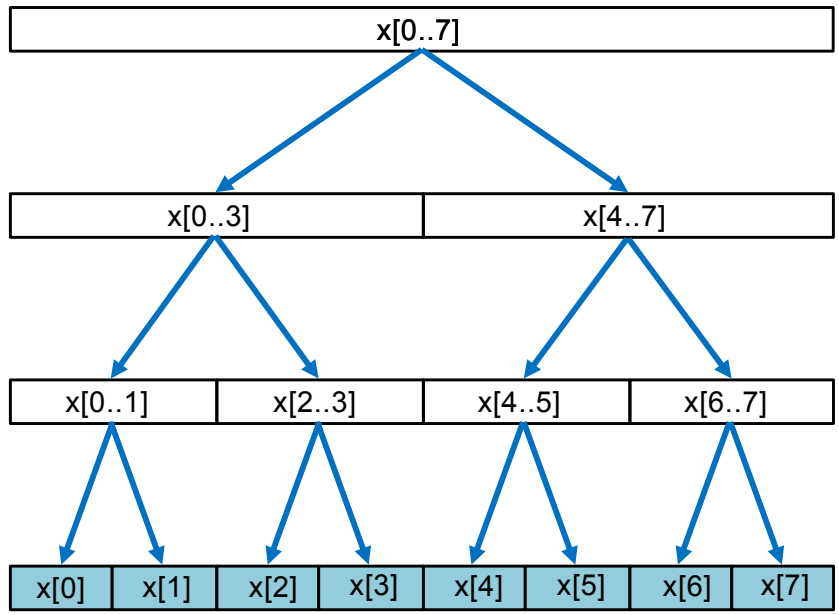
$Parallelism = 1$

How can we design an algorithm which leads to a TDG with more parallelism?

Example 1: vector sum

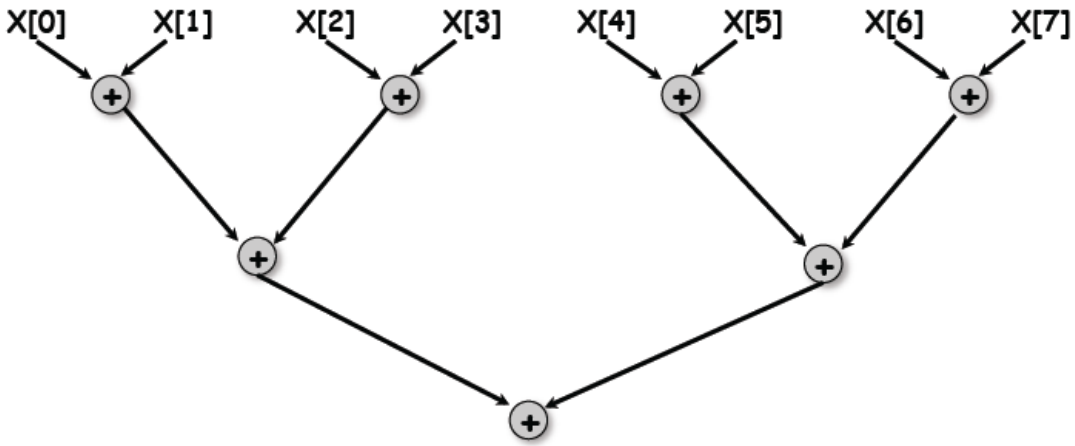
Writing a **recursive version** of the sequential program to compute the sum of elements `X[0]` ... `X[n-1]` of a vector `X` , following a *divide–and–conquer* strategy:

```
int recursive_sum(int *X, int n) {
   int ndiv2 = n/2;
   int sum=0;

   if (n==1) return X[0];

   sum1 = recursive_sum(X, ndiv2);
   sum2 = recursive_sum(X+ndiv2, n-ndiv2);
   return sum1+sum2;
}

void main() {
   int sum, X[N];
   ...
   sum = recursive_sum(X,N);
   ...
}
```



Example 1: vector sum

- **Task definition:** each invocation to `recursive_sum`
- **TDG** (with input data):



- **Metrics**:
  $T_1 \propto n$; $T_\infty \propto \log_2(n)$; $Parallelism \propto (n \div \log_2(n))$
- Same problem can be expressed with different algorithms/implementations leading to different metrics

Outline

Granularity and parallelism

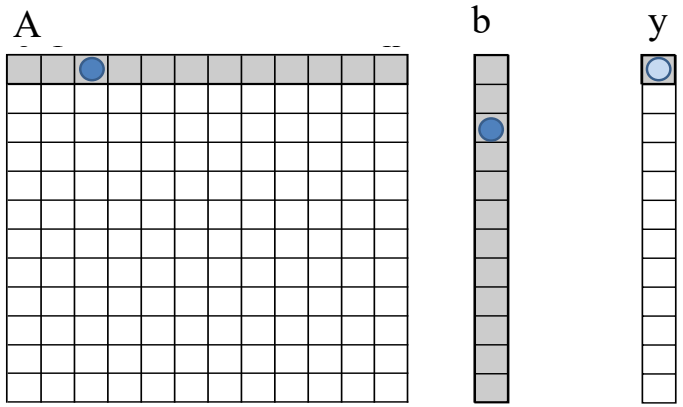
Given a sequential program, the number of tasks that one can generate and the size of the tasks (what is called **granularity**) are related one to the other.

- Fine-grained tasks vs. coarse-grained tasks
- The parallelism increases as the decomposition becomes finer in granularity (small tasks) and vice versa

Granularity and parallelism: fine-grained decomposition

Example: matrix-vector product (n by n matrix):

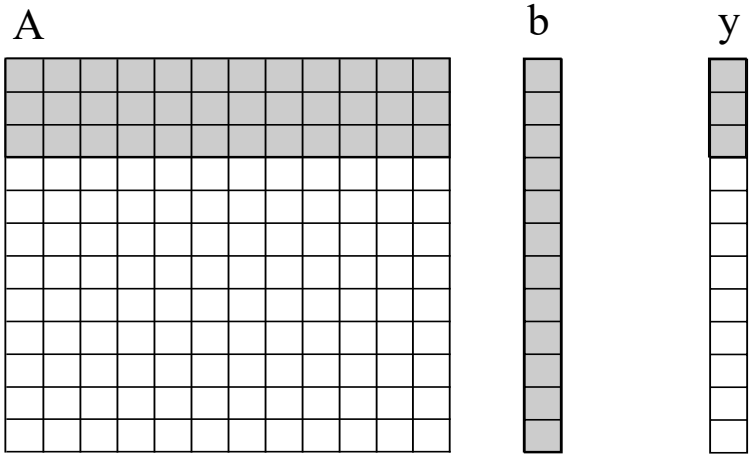
- A task could be each individual $\times$ and $+$ in the dot product that contributes to the computation of an element of y ($y[i] = y[i] + A[i][j] * b[j]$)



- A task could also be each complete dot product to compute an element of y ($y[i] = y[i] + \sum_{j=1}^{j=n}(A[i][j] * b[j])$)

Granularity and parallelism: coarse-grained decomposition

- A task could be in charge of computing a number of consecutive elements of y (e.g. three elements)



- A task could be in charge of computing the whole vector y

Granularity and parallelism: fine vs. coarse-grained

- It would appear that the parallel time can be made arbitrarily small by making the decomposition finer in granularity but...
  - Inherent bound on how fine the granularity of a computation can be
    - *e.g. matrix-vector multiply:* (n^2) *concurrent tasks.*
  - Tradeoff between the granularity of a decomposition and associated overheads (sources of overhead: creation of tasks, task synchronization, exchange of data between tasks, ...)
  - The granularity may determine performance bounds

Example 2: stencil computation using Jacobi solver

Stencil algorithm that computes each element of matrix `utmp` using 4 neighbor elements of matrix `u`, both matrices with $n \times n$ elements

```
void compute(int n, double *u, double *utmp) {
    int i, j;
    double tmp;

    for (i = 1; i < n-1; i++) {
        for (j = 1; j < n-1; j++) {
            tmp = u[n*(i+1) + j] + u[n*(i-1) + j] + // elements u[i+1][j] and u[i-1][j]
                  u[n*i + (j+1)] + u[n*i + (j-1)] - // elements u[i][j+1] and u[i][j-1]
                  4 * u[n*i + j];                   // element u[i][j]
            utmp[n*i + j] = tmp / 4;                // element utmp[i][j]
        }
    }
}
```

Example 2: stencil computation using Jacobi solver

What tasks can be? Assume: 1) the innermost loop body takes t_{body} time units; and 2) n is very large, so that $n - 2 \simeq n$

Task is ... (granularity)	**Num. tasks**	**Task cost**	**T_1**	**T_∞**	**Parallelism**
All iterations of i and j loops	1	$n^2 \cdot t_{body}$	$n^2 \cdot t_{body}$	$n^2 \cdot t_{body}$	1
Each iteration of i loop	n	$n \cdot t_{body}$	$n^2 \cdot t_{body}$	$n \cdot t_{body}$	n
Each iteration of j loop	n^2	t_{body}	$n^2 \cdot t_{body}$	t_{body}	n^2
r consecutive iterations of i loop	$n \div r$	$n \cdot r \cdot t_{body}$	$n^2 \cdot t_{body}$	$n \cdot r \cdot t_{body}$	$n \div r$
c consecutive iterations of j loop	$n^2 \div c$	$c \cdot t_{body}$	$n^2 \cdot t_{body}$	$c \cdot t_{body}$	$n^2 \div c$
A block of r x c iterations of i and j, respectively	$n^2 \div (r \cdot c)$	$r \cdot c \cdot t_{body}$	$n^2 \cdot t_{body}$	$r \cdot c \cdot t_{body}$	$n^2 \div (r \cdot c)$

Finer grain task decomposition $\rightarrow$ higher parallellism, but ...

Example 2: stencil computation using Jacobi solver

... what if each task creation takes t_{create}?

Task is ... (granularity)	**Num. tasks**	**Task cost**	**Task creation ovh**
All iterations of i and j loops	1	$n^2 \cdot t_{body}$	t_{create}
Each iteration of i loop	n	$n \cdot t_{body}$	$n \cdot t_{create}$
Each iteration of j loop	n^2	t_{body}	$n^2 \cdot t_{create}$
r consecutive iterations of i loop	$n \div r$	$n \cdot r \cdot t_{body}$	$(n \div r) \cdot t_{create}$
c consecutive iterations of j loop	$n^2 \div c$	$c \cdot t_{body}$	$(n^2 \div c) \cdot t_{create}$
A block of r x c iterations of i and j, respectively	$n^2 \div (r \cdot c)$	$r \cdot c \cdot t_{body}$	$(n^2 \div (r \cdot c)) \cdot t_{create}$

Trade-off between task granularity and task creation overhead

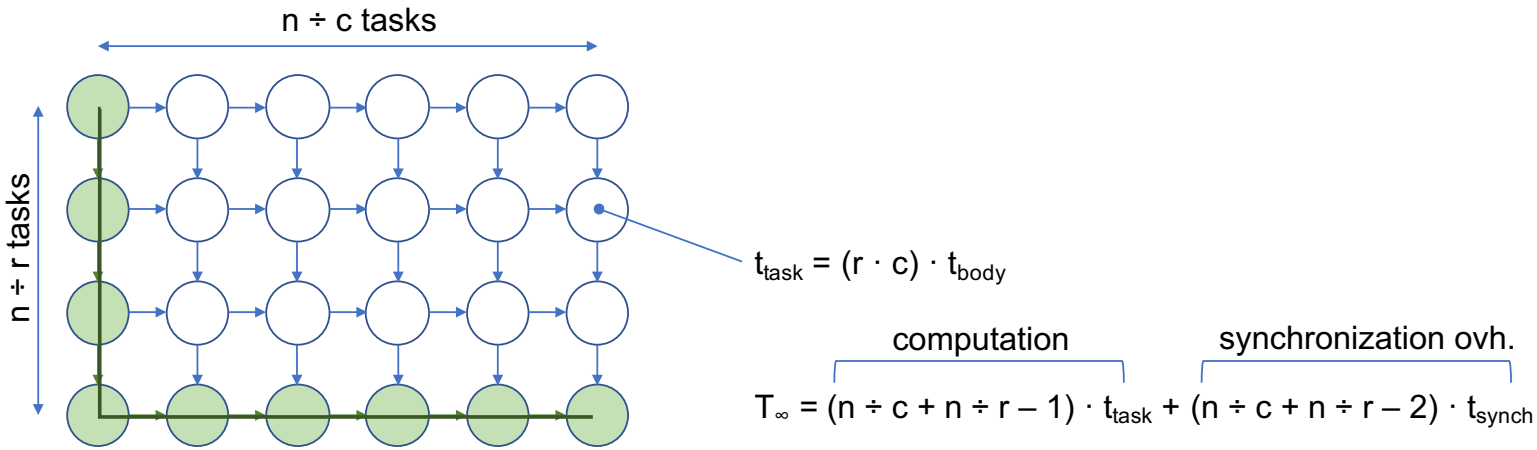
Example 3: stencil computation using Gauss-Seidel solver

Stencil algorithm that **updates** (in place) each element of matrix u using its 4 neighbors, matrix size $n \times n$ elements.

```
void compute(int n, double *u, double *utmp) {
    int i, j;
    double tmp;

    for (i = 1; i < n-1; i++) {
        for (j = 1; j < n-1; j++) {
            tmp = u[n*(i+1) + j] + u[n*(i-1) + j] + // elements u[i+1][j] and u[i-1][j]
                  u[n*i + (j+1)] + u[n*i + (j-1)] - // elements u[i][j+1] and u[i][j-1]
                  4 * u[n*i + j];                   // element u[i][j]
            u[n*i + j] = tmp / 4;                   // element u[i][j]
        }
    }
}
```

Example 3: stencil computation using Gauss-Seidel solver

Assuming: 1) each task computes a block of $r \times c$ iterations of the `i` and `j` loops, respectively; and 2) each task synchronization takes t_{synch}



Again, trade-off between task granularity and task synchronisation overhead

Outline

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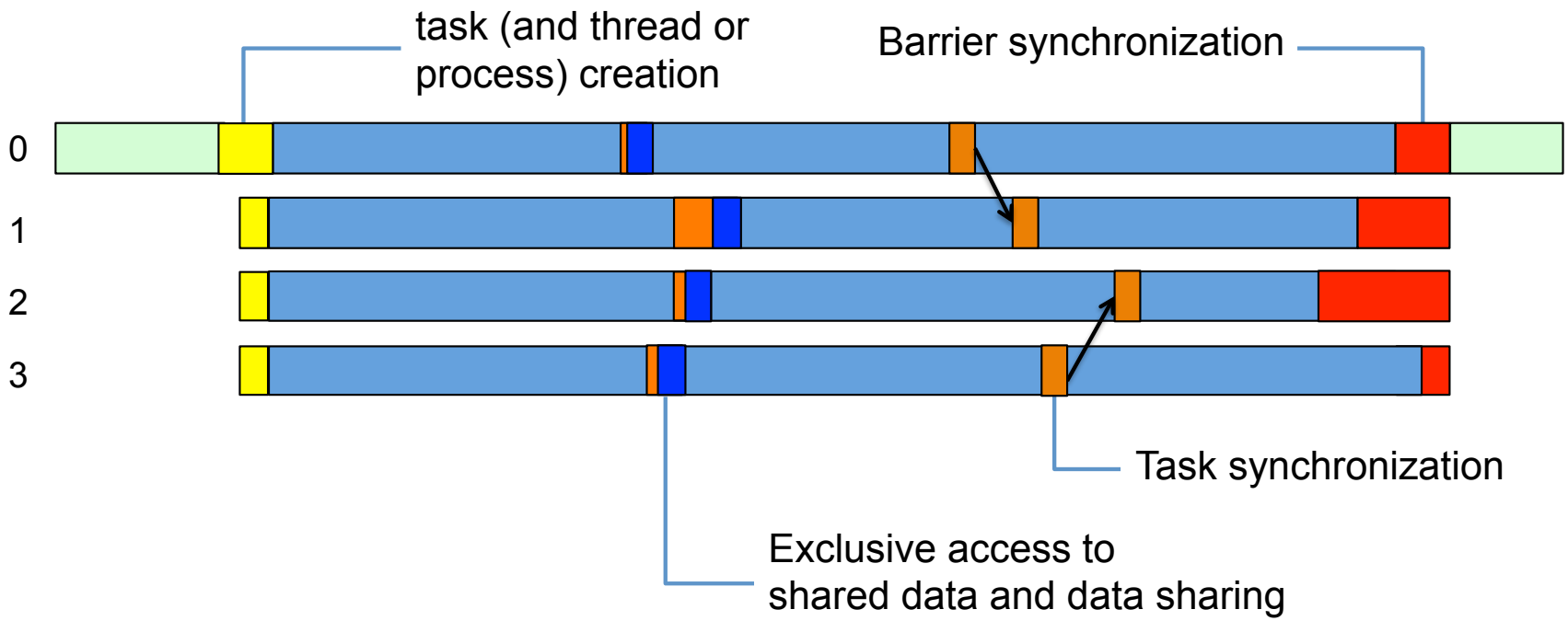
Concepts in video lesson 3

- Task scheduling (mapping) on P processors
- Metrics:
  - T_p: execution time on P processors
  - Speed–up $S_p = \frac{T_1}{T_p}$
  - Efficiency $E_p = \frac{S_p}{P}$
- Strong vs. weak scaling
- Amdahl's law, or the negative effect of the serial parts in your parallel application
  - parallel fraction ϕ
  - $S_p = \frac{1}{(1-\phi)}$ when $p \to \infty$ (ideally, assuming all parallel regions scale to ∞ processors)

Sources of overhead until now ...

Parallel computing is not for free, we should account overheads (i.e. any cost that gets added to a sequential computation so as to enable it to run in parallel)



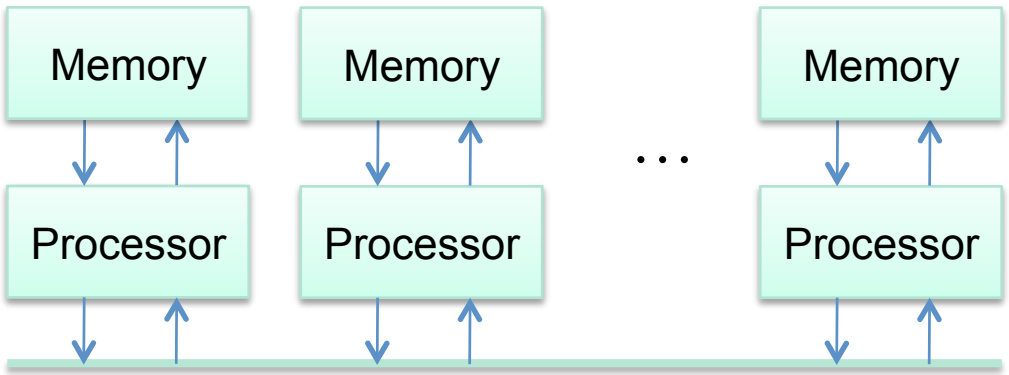
Outline

Other sources of overhead

- **Data sharing:** can be explicit via messages, or implicit via a memory hierarchy (caches)
- **Idleness:** thread cannot find any useful work to execute (e.g. dependences, load imbalance, poor communication and computation overlap or hiding of memory latencies, ...)
- **Computation:** extra work added to obtain a parallel algorithm (e.g. replication)
- **Memory:** extra memory used to obtain a parallel algorithm (e.g. impact on memory hierarchy, ...)
- **Contention:** competition for the access to shared resources (e.g. memory, network)

How to model data sharing overhead?

We start with a simple architectural model in which each processor P_i has its own memory, interconnected with the other processors through an interconnection network.



- Processors access to local data (in its own memory) using regular load/store instructions
- We will assume that local accesses take zero overhead.

How to model data sharing overhead?

- Processors can access remote data (in other processors) using a message-passing model (remote load instruction[2])
- To model the time needed to access remote data we will use two components:
  - Start up: time spent in preparing the remote access (t_s)
  - Transfer: time spent in transferring the message (number of bytes m, time per byte t_w) from/to the remote location

$$t_{access} = t_s + m \times t_w$$

- Synchronization between the two processors involved may be necessary to guarantee that the data is available

[2] Remote store is also possible, not used in our model.

How to model data sharing overhead?

Assumptions (to make simpler the model)

- At a given moment, a processor P_i can only perform one remote memory access to another processor P_j
- At a given moment, a processor P_i can only serve one remote memory access from another processor P_k
- Both actions can be performed simultaneously: $P_i \rightarrow P_j$ and $P_i \leftarrow P_k$

Back to example 2: Jacobi solver

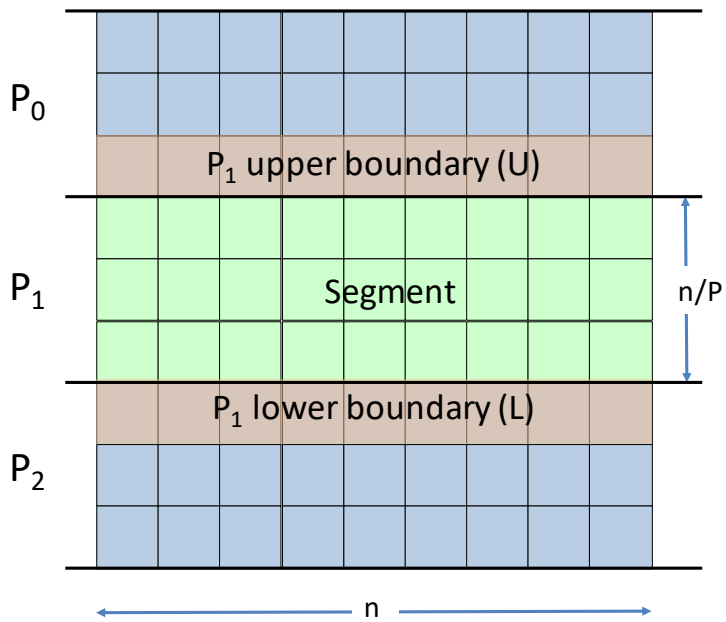
Stencil algorithm: each element of matrix `utmp` computed using 4 neighbor elements of matrix `u`, both matrices with $n \times n$ elements

```
void compute(int n, double *u, double *utmp) {
    int i, j;
    double tmp;

    for (i = 1; i < n-1; i++) {
        for (j = 1; j < n-1; j++) {
            tmp = u[n*(i+1) + j] + u[n*(i-1) + j] + // elements u[i+1][j] and u[i-1][j]
                  u[n*i + (j+1)] + u[n*i + (j-1)] - // elements u[i][j+1] and u[i][j-1]
                  4 * u[n*i + j];                   // element u[i][j]
            utmp[n*i + j] = tmp / 4;                // element utmp[i][j]
        }
    }
}
```

Task definition: $n \div P$ consecutive iterations of i loop, being P the number of processors

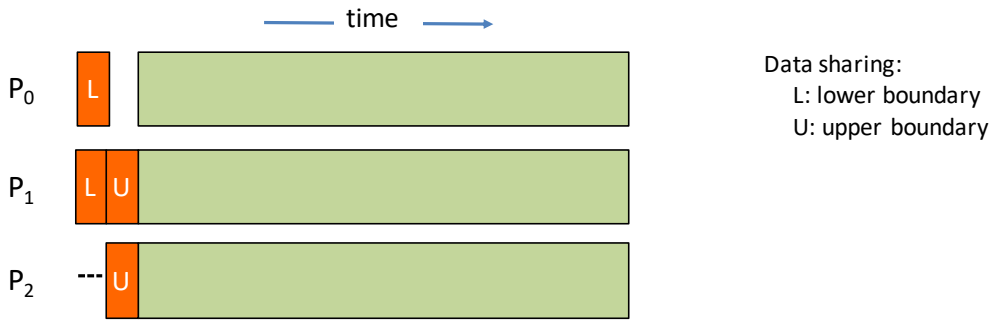
Example 2: data decomposition and movement



- Row decomposition, each processor with n^2/P elements of `u` matrix and n^2/P elements of `utmp` matrix (segment)
- Upper and lower boundaries, each with n elements
- No need to gather `utmp` in one of the processors at the end of the computation

Example 2: parallel execution timeline

- Parallelization strategy:
  1. Exchange boundaries with the two adjacent processors
  2. Each processor computes the elements of its `utmp` segment



- Questions:
  1. What is the data sharing time per segment assuming each boundary is accessed using a single message?
  2. What is the total time (computation and data sharing)?
  3. Obtain the expression for the speed–up S_P

Example 2: parallel execution time and speedup

With the same assumptions as before:

- The sequential execution time is $T_1 = n^2 \times t_{body}$
- The parallel execution time, considering both computation and data movement, is:

$$T_P = \frac{n^2}{P} \times t_{body} + 2 \times (t_s + n \times t_w)$$

- The corresponding expression for the speedup is

$$S_P = \frac{T_1}{T_P} = \frac{n^2 \times t_{body}}{\frac{n^2}{P} \times t_{body} + 2 \times (t_s + n \times t_w)}$$

Back to example 3: Gauss-Seidel solver

Stencil algorithm that **updates** (in place) each element of matrix u using its 4 neighbours, matrix size $n \times n$ elements.

```
void compute(int n, double *u, double *utmp) {
    int i, j;
    double tmp;

    for (i = 1; i < n-1; i++) {
        for (j = 1; j < n-1; j++) {
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                  4 * u[n*i + j];                   // element u[i][j]
            u[n*i + j] = tmp / 4;                   // element u[i][j]
        }
    }
}
```
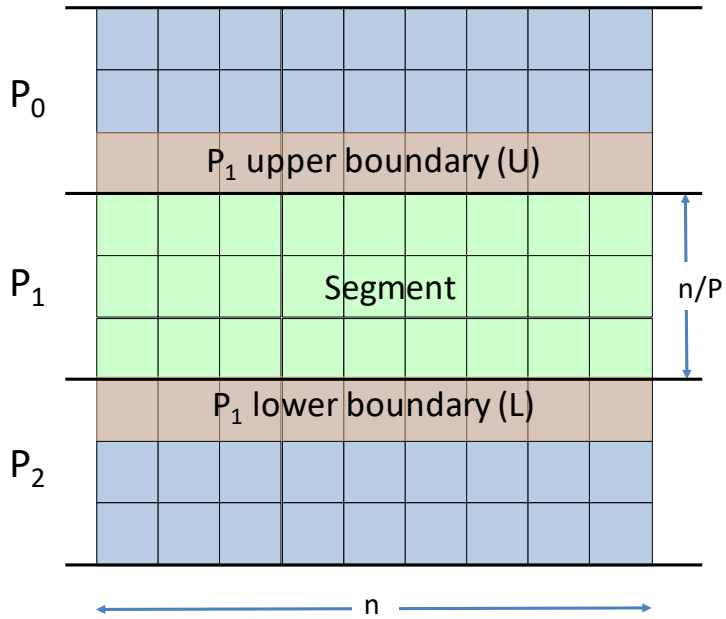

Task definition 1: $n \div P$ consecutive iterations of i loop, being P the number of processors

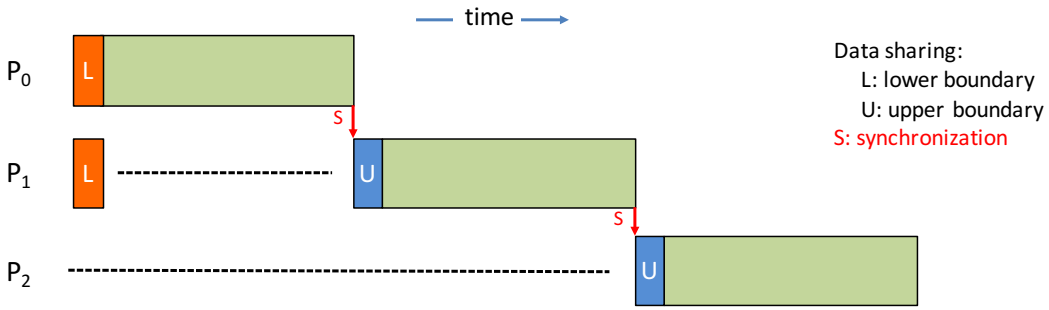
Example 3: data decomposition and movement



- Row distribution, each processor with n^2/P elements (segment)
- Upper and lower boundaries needed to compute segment, each with n elements (data sharing)
- No need to gather final `u` in one of the processors

Example 3: parallel execution timeline

- Parallelization strategy:
  1. Access to lower boundary L in next processor (if any)
  2. Wait for termination of task in previous processor, if any (dependence)
  3. Access to upper boundary U computed by that task (if any)
  4. Apply stencil algorithm to segment



- Questions:
  1. What is the data sharing time per segment?
  2. What is the total time (computation and data sharing)?

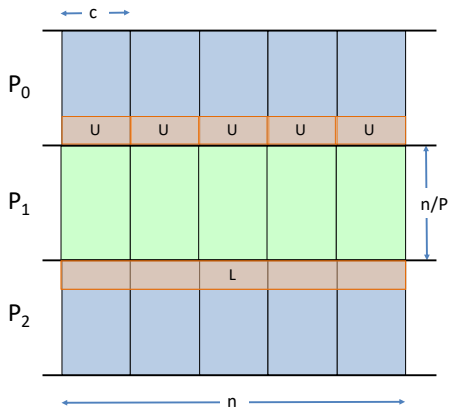
Example 3: Gauss-Seidel solver

Stencil algorithm that **updates** (in place) each element of matrix u using its 4 neighbours, matrix size $n \times n$ elements.

```
void compute(int n, double *u, double *utmp) {
    int i, j;
    double tmp;

    for (i = 1; i < n-1; i++) {
        for (j = 1; j < n-1; j++) {
            tmp = u[n*(i+1) + j] + u[n*(i-1) + j] + // elements u[i+1][j] and u[i-1][j]
                  u[n*i + (j+1)] + u[n*i + (j-1)] - // elements u[i][j+1] and u[i][j-1]
                  4 * u[n*i + j];                   // element u[i][j]
            u[n*i + j] = tmp / 4;                   // element u[i][j]
        }
    }
}
```

Task definition 2: block of $n \div P$ by c consecutive iterations of i and j loops, respectively; P is the number of processors

Example 3: blocking parallelization

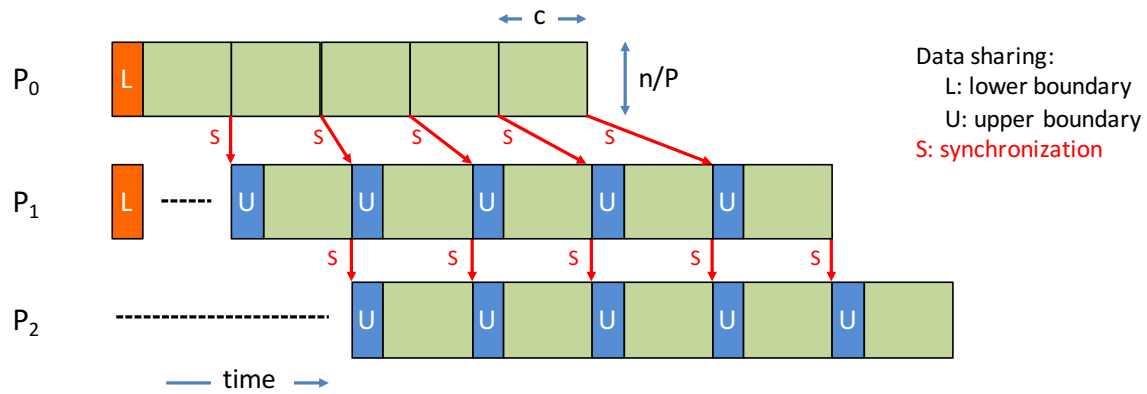


- Data decomposition:
  - Row distribution, each processor with n^2/P elements
  - Tasks compute segments of $n \div P$ rows by c columns

- Parallelization strategy:

  Once: Access to n elements for lower boundary L (if any)
  For each block (task):
  - Wait for termination of task computing the same block in previous processor, if any (dependence)
  - Access to the c elements in that block that belong to the upper boundary U (if any)
  - Apply stencil algorithm to the block

Example 3: parallel execution timeline with blocking



Questions:

1. What is the overhead of data sharing (before and during the parallel computation)?
2. What is the total time (computation and data sharing)?
3. Is there an optimum value for c?

Example 3: parallel execution time and speedup

Assuming that t_{body} is the computation time for the innermost loop body and n is very large, so that $n - 2 \simeq n$

$$T_P = (\frac{n}{c} + P - 1) \times (\frac{n}{P} \times c) \times t_{body} \quad +$$

$$(t_s + n \times t_w) + ((\frac{n}{c} + P - 2) \times (t_s + c \times t_w))$$

The corresponding expression for the speedup would be

$$S_P = \frac{T_1}{T_P} = \frac{n^2 \times t_{body}}{T_P}$$

Example 3: optimum blocking factor

Assuming $P >> 2$:

$$T_P \simeq (\frac{n}{c} + P)(\frac{n}{P} \times c)t_{body} + (t_s + n \times t_w) + (\frac{n}{c} + P)(t_s + c \times t_w)$$

The optimum block size c_{opt} is obtained applying the derivative to T_P and equal it to zero

$$\frac{\partial T_P}{\partial c} = n \times t_{body} - t_s \frac{n}{c^2} + P \times t_w = 0$$

$$c_{opt} = \sqrt{\frac{n \times t_s}{n \times t_{body} + P \times t_w}} = \sqrt{\frac{t_s}{t_{body} + t_w \frac{P}{n}}}$$

If $n >> P$ then

$$c_{opt} \simeq \sqrt{\frac{t_s}{t_{body}}}$$

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