INDETERMINACIONS $\frac{\infty}{\infty}$, $\frac{0}{0}$

- **infinits** en: un punt / l'infinit

$\lim_{x \to \text{punt}} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$

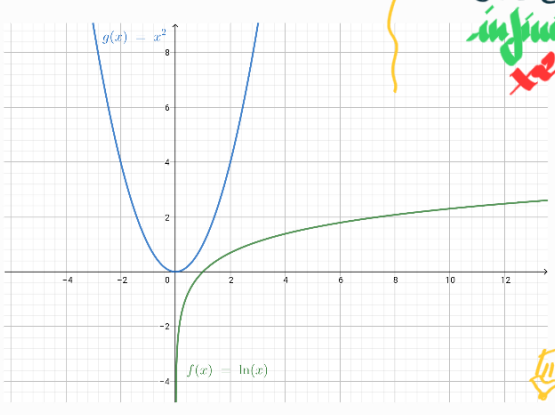
$\lim_{x \to 1} \frac{1}{x-1} = \infty$

$\lim_{x \to \infty} x = \infty$

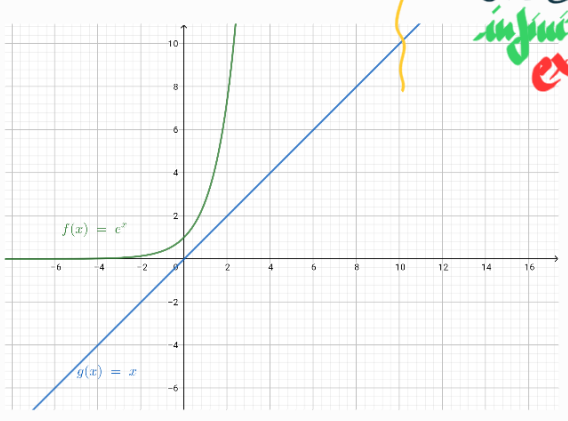
→ amb **diferent ordre** en $x \to \infty$

ordre infinit	ordre infinit	ordre infinit	ordre infinit
x^x	a^x	x^n	$\log_a x$

$\lim_{x \to +\infty} \frac{\ln x}{x^2} = \frac{\infty}{\infty} =$ Comparant ordres d'infinit: ordre infinit x^2 > ordre infinit $\ln x$ $= 0$



$\lim_{x \to +\infty} \frac{e^x}{x} = \frac{\infty}{\infty} =$ Comparant ordres d'infinit: ordre infinit e^x > ordre infinit x $= \infty$



$\lim_{x \to +\infty} \frac{\log_2 x}{\sqrt{x}} = 0$

↑ Comprovar

$\lim_{x \to +\infty} \frac{15^x}{\sqrt{x}} = +\infty$

→ amb mateix ordre: "infinits EQUIVALENTS" $\sim$

Si $\begin{cases} \lim\limits_{x \to \text{punt o } \infty} f = \infty \\ \lim\limits_{x \to \text{punt o } \infty} g = \infty \end{cases}$ i $\boxed{\lim\limits_{x \to \text{punt o } \infty} \dfrac{f}{g} = 1} \iff f \sim g$

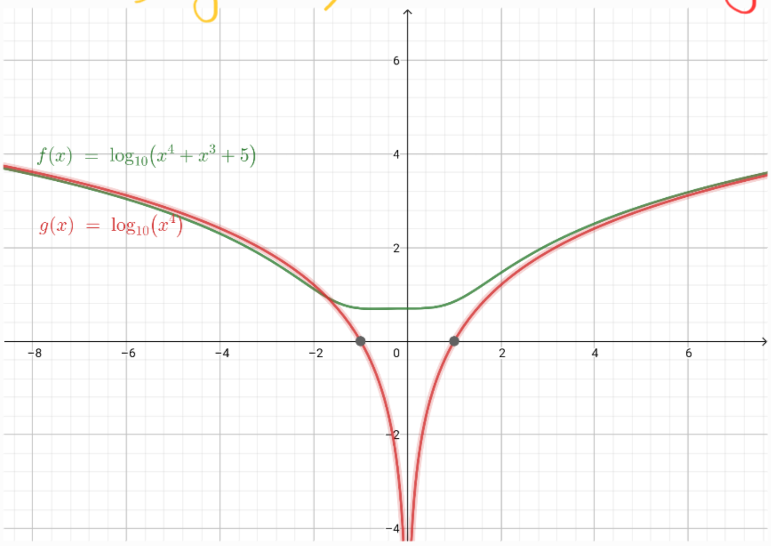
més usuals en $x \to \infty$

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_0 \sim a_n x^n$$

$$\sqrt[m]{a_n x^n + a_{n-1} x^{n-1} + \dots + a_0} \sim \sqrt[m]{a_n x^n}$$

$$\log_a (a_n x^n + a_{n-1} x^{n-1} + \dots + a) \sim \log_a (a_n x^n)$$

Ex: $\lim\limits_{x \to +\infty} \dfrac{\log(x^4+x^3+5)}{\log(x^6-3)} = \dfrac{\infty}{\infty} \overset{IE}{=} \lim\limits_{x \to +\infty} \dfrac{\log x^4}{\log x^6} = \lim\limits_{x \to +\infty} \dfrac{4 \log x}{6 \log x} = \dfrac{4}{6} = \dfrac{2}{3}$



infinitèsims en un punt / l'infinit

$\lim_{x \to \text{punt}} f(x) = 0$

$\lim_{x \to 1} x - 1 = 0$

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

amb mateix ordre: infinitèsims EQUIVALENTS $\sim$

$$\text{si} \begin{cases} \lim_{x \to \text{punt}_{0\ \infty}} f = 0 \\ \lim_{x \to \text{punt}_{0\ \infty}} g = 0 \end{cases} \quad \text{i} \quad \boxed{\lim_{x \to \text{punt}_{0\ \infty}} \frac{f}{g} = 1 \iff f \sim g}$$

més usuals en $x \to 0$

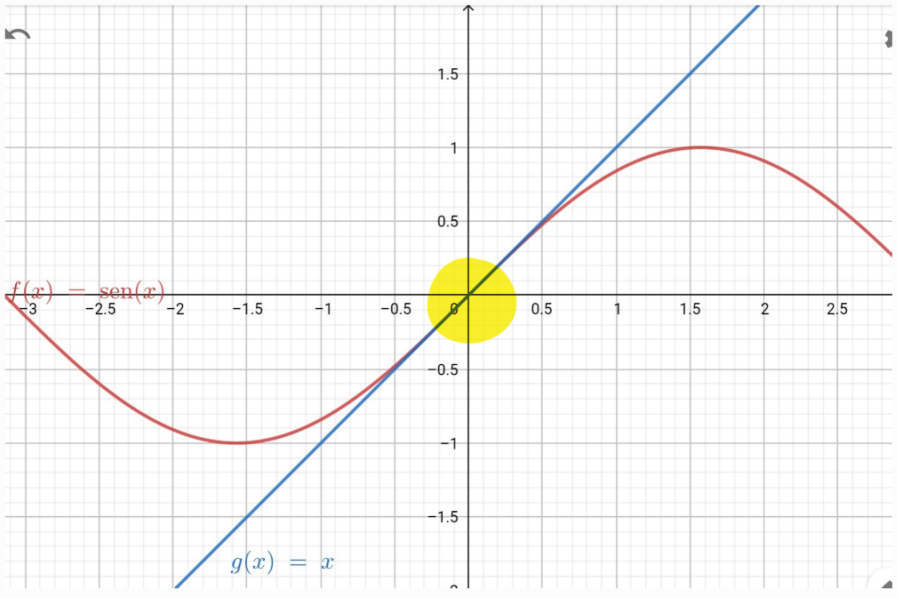
$\sin x \sim x$

$1 - \cos x \sim \frac{x^2}{2}$

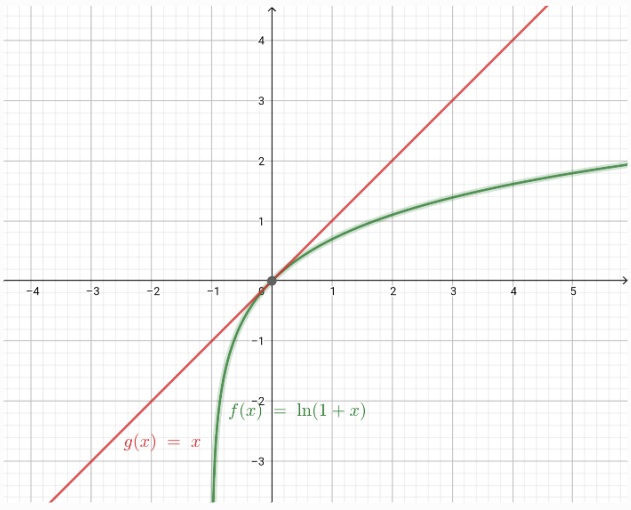
$\ln(x+1) \sim x$

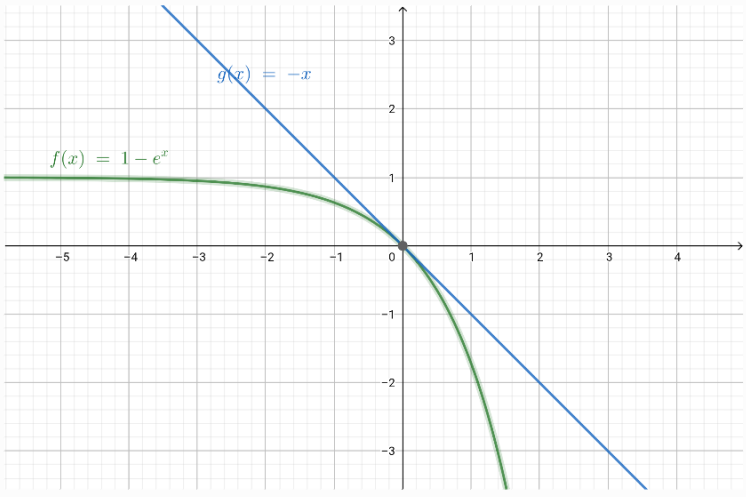
$e^x - 1 \sim x$

$\rightarrow \lim_{x \to 0} \frac{\sin x}{x} = 1$ o $\lim_{x \to 0} \frac{x}{\sin x} = 1$



és a dir, $\sin x$ s'apropa a 0 quan $x \to 0$ amb un comportament semblant a com s'apropa x a 0 quan $x \to 0$


$g(x) = x$
$f(x) = \ln(1+x)$


$g(x) = -x$
$f(x) = 1 - e^x$