Lògica en la Informàtica
Deducció en Lògica de Primer Ordre (LPO)

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Crèdits

El material utilitzat en aquesta presentació ha estat extret del elaborat pel professor Robert Nieuwenhuis (Dept. CS, UPC) per l'assignatura *Lògica en la Informàtica* de la FIB.

En particular, del llibre *Lógica para informáticos* - Farré, R. [et al.], Marcombo, 2011. ISBN: 9788426716941.

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

A: All people that have electric cars are ecologists.
B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.
C: A person is an ecologist if his/her mother is an ecologist.
D: Mary is John's grandmother.
E: Mary has an electric car.
F: John is an ecologist.

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F: John is an ecologist.

Resposta (predicats):

$$
\begin{array}{lcl}
\mathit{hasEcar}(x) & \equiv & \text{“}x \text{ has an electric car”} \\
\mathit{isEcologist}(x) & \equiv & \text{“}x \text{ is an ecologist”} \\
\mathit{mother}(x, y) & \equiv & \text{“}y \text{ is the mother of x”} \\
\mathit{grandma}(x, y) & \equiv & \text{“}y \text{ is the grandmother of x”}
\end{array}
$$

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\mathit{mother}(x, y) & \equiv & \text{“}y \text{ is the mother of x”} \\
\mathit{grandma}(x, y) & \equiv & \text{“}y \text{ is the grandmother of x”}
\end{array}
$$

A: $\forall x \, (\mathit{hasEcar}(x) \rightarrow \mathit{isEcologist}(x) \,)$

A $\neg \mathit{hasEcar}(x) \vee \mathit{isEcologist}(x)$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.

$hasEcar(x)$ $\equiv$ "x has an electric car"
$isEcologist(x)$ $\equiv$ "x is an ecologist"
$mother(x, y)$ $\equiv$ "y is the mother of x"
$grandma(x, y)$ $\equiv$ "y is the grandmother of x"

NO és correcte:

$$\forall x \ (\exists y \, (\, grandma(x, y) \rightarrow \exists z \, (mother(x, z) \wedge mother(z, y) \,) \ ) \ )$$
$$\forall x \ (\exists y \, (\, \neg grandma(x, y) \vee \exists z \, (mother(x, z) \wedge mother(z, y) \,) \ ) \ )$$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.

NO és correcte:

$\forall x \ (\exists y \, (\neg grandma(x, y) \vee \exists z \, (mother(x, z) \wedge mother(z, y)) \ ) \ )$

Aquesta formalització del llenguatge natural no és adecuada:
si tenim una situació I amb persones $D_I = \{p1, p2, avia\}$ i on l'àvia de $p1$ es $avia$: $grandma_I(p1, avia) = 1$
i on tota la resta és fals (ningú és mare de ningú, etc.) llavors I satisfà la fórmula, perquè $\forall x \exists y \neg grandma(x, y)$. De fet, amb aquesta formalització no és possible obtenir la clàusula buida.

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.

El que sí és correcte és:

$\forall x \forall y \; (\; grandma(x, y) \; \rightarrow \; \exists z \, (\, mother(x, z) \wedge mother(z, y) \,) \,)$

- Elim. $\rightarrow$

$\forall x \forall y \; (\; \neg grandma(x, y) \; \vee \; \exists z \, (mother(x, z) \wedge mother(z, y) \,) \,)$

- Skolem:

$\forall x \forall y \; (\; \neg grandma(x, y) \; \vee \; (\, mother(x, f_z(x, y)) \wedge mother(f_z(x, y), y) \,) \,)$

- Distrib:

B1 $\neg grandma(x, y) \vee mother(x, f_z(x, y))$

B2 $\neg grandma(x, y) \vee mother(f_z(x, y), y))$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

C: A person is an ecologist if his/her mother is an ecologist.

$$
\begin{array}{lcl}
\mathit{hasEcar}(x) & \equiv & \text{“}x \text{ has an electric car”} \\
\mathit{isEcologist}(x) & \equiv & \text{“}x \text{ is an ecologist”} \\
\mathit{mother}(x,y) & \equiv & \text{“}y \text{ is the mother of x”} \\
\mathit{grandma}(x,y) & \equiv & \text{“}y \text{ is the grandmother of x”}
\end{array}
$$

C: $\forall x \, (\; \exists y \, (\, \mathit{mother}(x,y) \wedge \mathit{isEcologist}(y) \,) \; \rightarrow \; \mathit{isEcologist}(x) \,)$

$\forall x \, (\neg \exists y \, (\, \mathit{mother}(x,y) \wedge \mathit{isEcologist}(y) \,) \; \vee \; \mathit{isEcologist}(x) \,)$

$\forall x \, (\quad \forall y \, \neg \, (\, \mathit{mother}(x,y) \wedge \mathit{isEcologist}(y) \,) \; \vee \; \mathit{isEcologist}(x) \,)$

$\forall x \, (\quad \forall y \, (\neg \, \mathit{mother}(x,y) \vee \neg \, \mathit{isEcologist}(y) \,) \vee \mathit{isEcologist}(x) \,)$

C $\quad \neg \, \mathit{mother}(x,y) \vee \neg \, \mathit{isEcologist}(y) \vee \mathit{isEcologist}(x)$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

$hasEcar(x)$	$\equiv$	"x has an electric car"
$isEcologist(x)$	$\equiv$	"x is an ecologist"
$mother(x, y)$	$\equiv$	"y is the mother of x"
$grandma(x, y)$	$\equiv$	"y is the grandmother of x"

D: Mary is John's grandmother.

D $grandma(john, mary)$

E: Mary has an electric car.

E $hasEcar(mary)$

F: John is an ecologist.

$\neg$ F: John is not an ecologist.

$\neg$ F $\neg\, isEcologist(john)$

Examen final de 2020 tardor. Exercici 6

Volem demostrar que $A \wedge B \wedge C \wedge D \wedge E \models F$.

I això passa ssi $A \wedge B \wedge C \wedge D \wedge E \wedge \neg F$ és insatisfactible.

A	$\neg hasEcar(x) \vee isEcologist(x)$
B1	$\neg grandma(x, y) \vee mother(x, f_z(x, y))$
B2	$\neg grandma(x, y) \vee mother(f_z(x, y), y))$
C	$\neg mother(x, y) \vee \neg isEcologist(y) \vee isEcologist(x)$
D	$grandma(john, mary)$
E	$hasEcar(mary)$
$\neg$F	$\neg isEcologist(john)$

He d'obtenir la $\square$ mitjançant resolució a partir d'aquestes 7 clàusules.

Examen final de 2020 tardor. Exercici 6

A $\quad \neg hasEcar(x) \vee isEcologist(x)$
B1 $\quad \neg grandma(x,y) \vee mother(x, f_z(x,y))$
B2 $\quad \neg grandma(x,y) \vee mother(f_z(x,y), y))$
C $\quad \neg mother(x,y) \vee \neg isEcologist(y) \vee isEcologist(x)$
D $\quad grandma(john, mary)$
E $\quad hasEcar(mary)$
$\neg$F $\quad \neg isEcologist(john)$

res entre **mgu**

E+A $\quad \{x = mary\}$ obtenim:
1. $isEcologist(mary)$

D+B1 $\quad \{x = john, y = mary\}$ obtenim:
2. $mother(john, f_z(john, mary))$

D+B2 $\quad \{x = john, y = mary\}$ obtenim:
3. $mother(f_z(john, mary), mary)$

2+C $\quad \{x = john, y = f_z(john, mary)\}$ obtenim:
4. $\neg isEcologist(f_z(john, mary)) \vee isEcologist(john)$

4+$\neg$F $\quad \{\}$ obtenim:
5. $\neg isEcologist(f_z(john, mary))$

Examen final de 2020 tardor. Exercici 6

A $\quad \neg hasEcar(x) \vee isEcologist(x)$

B1 $\quad \neg grandma(x,y) \vee mother(x, f_z(x,y))$

B2 $\quad \neg grandma(x,y) \vee mother(f_z(x,y), y))$

C $\quad \neg mother(x,y) \vee \neg isEcologist(y) \vee isEcologist(x)$

D $\quad grandma(john, mary)$

E $\quad hasEcar(mary)$

$\neg$F $\quad \neg isEcologist(john)$

res entre mgu

1. $isEcologist(mary)$
2. $mother(john, f_z(john, mary))$
3. $mother(f_z(john, mary), mary)$
4. $\neg isEcologist(f_z(john, mary)) \vee isEcologist(john)$
5. $\neg isEcologist(f_z(john, mary))$

   3+C $\quad \{x = f_z(john, mary), y = mary\}$ obtenim:
6. $\neg isEcologist(mary) \vee isEcologist(f_z(john, mary))$

   6+5 $\quad \{\ \ \}$ obtenim:
7. $\neg isEcologist(mary)$

   1+7 $\quad \{\ \}$ obtenim:
8. $\square$

Per a estudiar teoria de LI:

- repassa els materials que hem estudiat.
- FÉS ELS EXÀMENS PENJATS, començant pels últims, cap als anteriors, treballant sempre primer l'enunciat SENSE resoldre, i després l'examen resolt.
- continua fent els exercicis del tema 5.