Contents

Query answering

A bad algorithm:

input query q;
for every document d in database
    check if d matches q;
    if so, add its docid to list L;
output list L (perhaps sorted in some way);

Time should be largely independent of database size.
(Unavoidably) proportional to answer size.

Central Data Structure: Inverted file

A vocabulary or lexicon or dictionary, usually kept in main memory, maintains all the indexed terms (*set*, *map*...)

- Collection: document $\rightarrow$ words contained in the document

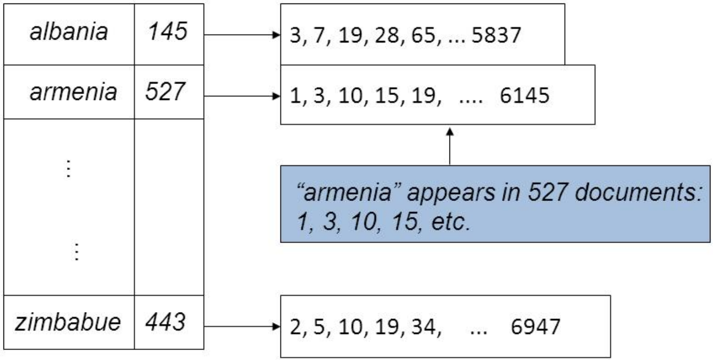
Central Data Structure: Inverted file

A vocabulary or lexicon or dictionary, usually kept in main memory, maintains all the indexed terms (*set*, *map*. . .)

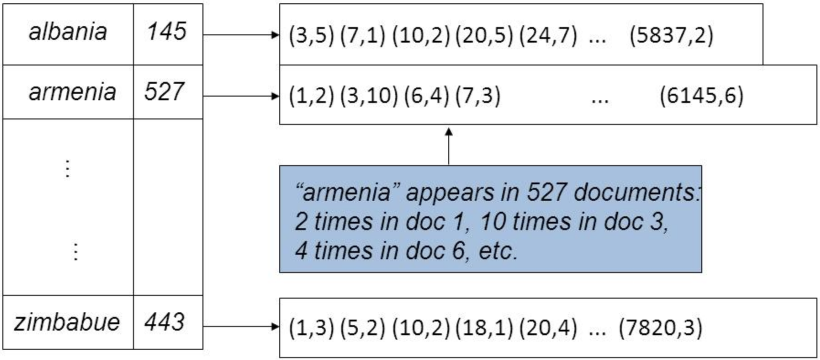
- Collection: document $\rightarrow$ words contained in the document
- Inverted file: word $\rightarrow$ documents that contain the word

Built at preprocessing time, not at query time: can afford to spend some time in its construction.

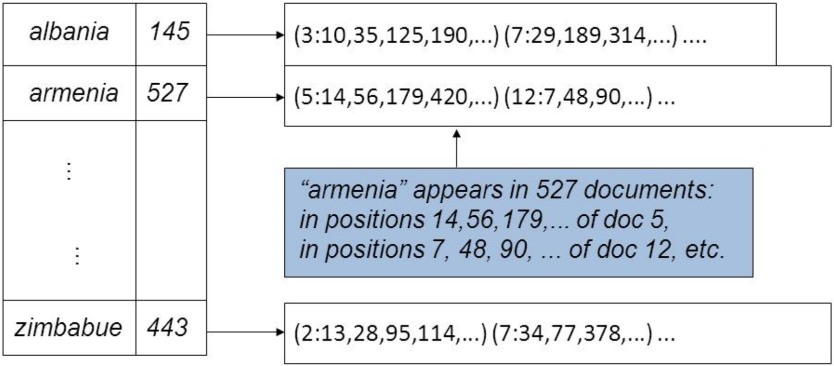
The inverted file: Variant 1



The inverted file: Variant 2



The inverted file: Variant 3



Postings

The inverted file is made of incidence/posting lists

We assign a *document identifier*, docid to each document.
The dictionary may fit in RAM for medium-size applications.

- For each indexed term, a posting list: list of docid's (plus maybe other info) where the term appears.
- Posting lists stored in disk for largish collections.
- Almost always sorted by *docid*.
- often compressed: minimize info to bring from disk!

Implementation of the Boolean Model

Simplest: Traverse posting lists

Conjunctive query: a AND b

- get the posting lists of a and b from inverted file
- ... and intersect them
- if sorted: can do a merge-like intersection;
- time: order of the sum of the lengths of posting lists.

Exercise. Similar algorithms for OR and BUTNOT.

Implementation of the Boolean Model

```
def intersect(L1,L2):
    i = j = 0
    Lres = []
    while i < len(L1) and j < len(L2):
        if L1[i] < L2[j]:
            ++i
        else if L1[i] > L2[j]
            ++j
        else  # L1[i] == L2[j]
            Lres.append(L1[i])
            ++i
            ++j
    return Lres
```

Query Optimization

Query Optimizer $\rightarrow$ evaluation plan for each query:

- Rewriting the query using laws of Boolean algebra
- Choosing other algorithms for intersection and union
- Using more data structures (computed offline)

Query Rewriting

What is the most efficient way to compute a AND b AND c?

- (a AND b) AND c?
- (b AND c) AND a?
- (a AND c) AND b?

The following are equivalent. Which is cheapest?

- (a AND b) OR (a AND c)?
- a AND (b OR c)?

The cost of an execution plan depends on the sizes of the lists and the sizes of intermediate lists.

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Worst cases:

- $|L1 \cap L2| \leq \min(|L1|, |L2|)$
- $|L1 \cup L2| \leq |L1| + |L2| - |L1 \cap L2| \leq |L1| + |L2|$

Query Rewriting

a AND b AND $c \rightarrow$ (a AND b) AND c

Assume: $|L_a| = 1.000$, $|L_b| = 2.000$, $|L_c| = 300$.

Minimum comparisons if using sequential scanning = $1.000 + 2.000 + 300 = 3.300$.

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Instruction	Comparisons	Result $\leq$
1. $L_{a \cap b} =$ intersect(L_a, L_b)	1.000 + 2.000 = 3.000	1.000
2. $L_{res} =$ intersect$(L_{a \cap b}, L_c)$	1.000 + 300 = 1.300	300
Total comparisons	3.000 + 1.300 = **4.300**	–

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Instruction	Comparisons	Result $\leq$
1. $L_{a \cap c} =$ intersect(L_a, L_c)	1.000 + 300 = 1.300	300
2. $L_{res} =$ intersect$(L_{a \cap c}, L_b)$	300 + 2.000 = 2.300	300
Total comparisons	1.300 + 2.300 = **3.600** < 4.300	–

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Total comparisons	1.300 + 2.300 = **3.600** < 4.300	–

Heuristic for AND-only queries: Intersect from shortest to longest.

Query Rewriting

a AND (b OR c)

Assume: $|L_a| = 300$, $|L_b| = 4.000$, $|L_c| = 5.000$.

Minimum comparisons if using sequential scanning =
$300 + 4.000 + 5.000 = 9.300$.

Query Rewriting

a AND (b OR c)

Assume: $|L_a| = 300$, $|L_b| = 4.000$, $|L_c| = 5.000$.

Minimum comparisons if using sequential scanning = $300 + 4.000 + 5.000 = 9.300$.

Instruction	Comparisons	Result $\leq$
1. $L_{b \cup c} =$ union(L_b, L_c)	4.000+5.000 = 9.000	9.000
2. $L_{res} =$ intersect$(L_a, L_{b \cup c})$	9.000 + 300 = 9.300	300
Total comparisons	9.000 + 9.300 = **18.300**	–

Query Rewriting

a AND (b OR c) $\rightarrow$ (a AND b) OR (a AND c)

Assume: $|L_a| = 300$, $|L_b| = 4.000$, $|L_c| = 5.000$.

Minimum comparisons if using sequential scanning = $300 + 4.000 + 5.000 = 9.300$.

Query Rewriting

a AND (b OR c) $\rightarrow$ (a AND b) OR (a AND c)

Assume: $|L_a| = 300$, $|L_b| = 4.000$, $|L_c| = 5.000$.

Minimum comparisons if using sequential scanning = $300 + 4.000 + 5.000 = 9.300$.

Instruction	Comparisons	Result $\leq$
1. $L_{a \cap b} =$ intersect(L_a, L_b)	300+4.000 = 4.300	300
2. $L_{a \cap c} =$ intersect(L_a, L_c)	300+5.000 = 5.300	300
3. $L_{res} =$ union$(L_{a \cap b}, L_{a \cap c})$	300 + 300 = 600	600
Total comparisons	4.300 + 5.300 + 600 = **9.900** < 18.300	–

Query Rewriting

The combinatorics may get complicated . . .

$$(a \text{ AND } b \text{ AND } d) \text{ OR } (a \text{ AND } (c \text{ OR } d) \text{ AND } e)$$
$$\equiv$$
$$((a \text{ AND } d) \text{ AND } b) \text{ OR } (a \text{ AND } c \text{ AND } e) \text{ OR } ((a \text{ AND } d) \text{ AND } e)$$

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Consider distributing so that we can compute intersect(L_a, L_d) once and store for reuse.

Exercise: Write the new plan as a sequence of instructions.
Exercise: Find cases where the new plan is more efficient.

Sublinear time intersection: Binary Search

Alternative: traverse one list and look up every docid in the other via binary search.

- Time: length of shortest list times log of length of longest.

Sublinear time intersection: Binary Search

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- Time: length of shortest list times log of length of longest.

If $|L1| \ll |L2|$

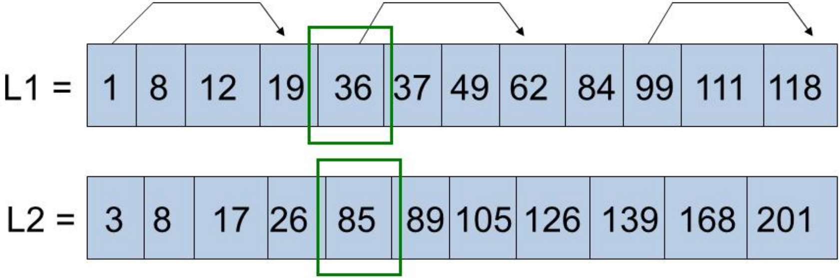
$$|L1| \cdot \log(|L2|) < |L1| + |L2|$$

Query Optimization

Example:

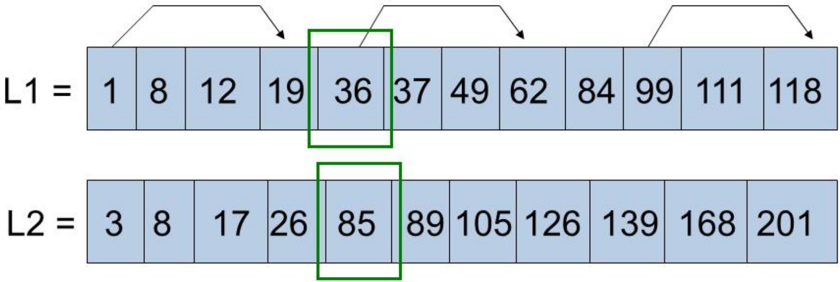
- $|L1| = 1.000$, $|L2| = 1.000$:
  - sequential scan: 2.000 comparisons,
  - binary search: $1.000 * 10 = 10.000$ comparisons.
- $|L1| = 100$, $|L2| = 10.000$:
  - sequential scan: 10.100 comparisons,
  - binary search: $100 * \log(10.000) = 1.400$ comparisons.

Sublinear time intersection: Skip pointers



- We've merged 1 . . . 19 and 3 . . . 26.
- We are looking at 36 and 85.
- Since pointer(36)=62 < 85, we can jump to 84 in L1.

Sublinear time intersection: Skip pointers



- Forward pointer from some elements.
- Either jump to next segment, or search within next segment (once).
- Optimal: in RAM, $\sqrt{|L|}$ pointers of length $\sqrt{|L|}$.
- Needs random access - not so easy if in disk.

Implementation of the Vector Model, I

Problem statement

Fixed similarity measure $sim(d, q)$:

Retrieve

documents d_i which have a similarity to the query q

- either
  - above a threshold sim_{min}, or
  - the top r according to that similarity, or
  - all documents,
- sorted by decreasing similarity to the query q.

Must react very fast (thus, careful to the interplay with disk!), and with a reasonable memory expense.

Implementation of the Vector Model

Obvious non-solution

```
for each d in D:
    sim(d,q) = 0
    get vector representing d
    for each w in q:
        sim(d,q) += tf(d,w) * idf(w)
    normalize sim(d,q) by |d|*|q|
sort results by similarity
```

idf_w and $|d|$ can be precomputed and stored in the index.
$|q|$ computed now.

…too inefficient for large D

Implementation of the Vector Model

Towards a faster algorithm

Most documents include a small proportion of the available terms.

Queries usually include a humanly small number of terms.

Only a very small proportion of the documents will be relevant.

Inverted file available!

Implementation of the Vector Model

Idea: Invert the loops, use inverted file

```
for each w in q:
    L = posting list for w, from inverted file
    for each d in L:
        if d seen for first time:
            sim(d,q) = 0
        sim(d,q) += tf(d,w) * idf(w)
for each d seen:
    normalize sim(d,q) by |d|*|q|
sort results by similarity
```

Implementation of the Vector Model

Idea: Invert the loops, use inverted file

After a few outer loops:

- Instead of having all of $sim(d, q)$ for some d's
- We have partially computed $sim(d, q)$ for all d's

= scan the document-term matrix by columns, not by rows

Index compression, I

Why?

A large part of the query-answering time is spent

bringing posting lists from disks to RAM.

Need to minimize amount of bits to transfer.

Index compression schemes use:

- Docid's sorted in increasing order.
- Frequencies usually very small numbers.
- Can do better than e.g. 32 bits for each.

Index compression, II

Topic for self-study. At least:

- Unary self-delimiting code.
- Gap compression + Elias Gamma code.
- Continuation bit.
- Typical compression ratios.

E.g. books listed in the Presentation part of these notes.

Getting fast the top r results

The last line of the algorithm was

```
sort the documents in answer by value of sim(d,q)
```

Time $O(R \log R)$, where R = #docs with $sim(d, q) > sim_{min}$.
Noticeable if R is large (milions).

User usually wants really fast the top-r, where $r \ll R$.
E.g., $r = 10$.

Example with $R = 10$ and $r = 3$

$$L = [5, 8, 3, 6, 4, 1, 10, 2, 7, 9]$$

docid	sim(docid, q)
1	0.53
2	0.26
3	0.72
4	0.32
5	0.25
6	0.36
7	0.52
8	0.88
9	0.50
10	0.19

Getting fast the top r results

Let $L = [d_1 ... d_R]$ be the answer (random order)

```
put [d_1, ..., d_r] in a minheap
for i = r+1..R:
    min_val = sim(d,q) for d = top of the heap
    if sim(d_i,q) > min_val:
        replace smallest element in heap with d_i
        reorganize heap
```

Claim: After any iteration, the heap contains the top-r documents among the first i.

Claim: If the similarities in L are randomly ordered, the expected running time of this algorithm is $O(R + r \cdot \ln(r) \cdot \ln(R/r))$.

Getting fast the top r results

Let $L = [d_1, \ldots, d_R]$ be the answer

- Time to put r elements in heap: $O(r)$
  - (recall why it is better than the obvious $O(r \log r)$)
- $Pr(d_i$ enters the heap$) =$

  $= Pr[d_i$ among r largest in $d_1, \ldots, d_i] = r/i$
- $E[$time to process $d_i] = \frac{r}{i} O(\log r) + \frac{i-r}{i} O(1)$
- E[Running time] = $O(r) + \sum_{i=r+1}^{R} \left(\frac{r}{i} O(\log r) + \frac{i-r}{i} O(1) \right)$

  $= \ldots$ (use $H(n) \simeq \ln(n)$, H harmonic function)

  $= O(R) + O(r \ln(r) \ln(R/r))$

For $r \ll R$, we go from $O(R \log R)$ to $O(R)$.