Math Cheatsheet

Trigonometry

Remember: *SOH CAH TOA*

Sine

$$\frac{a}{sin(\alpha)} = \frac{b}{sin(\beta)} = \frac{c}{sin(\gamma)}$$

Right Angles

$$sin(\theta) = \frac{o}{h} = \frac{opposite}{hypotenuse}$$

Cosine

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cdot cos(\alpha) \\ b^2 &= a^2 + c^2 - 2ac \cdot cos(\beta) \\ c^2 &= a^2 + b^2 - 2ab \cdot cos(\gamma) \end{aligned}$$

Right Angles

$$cos(\theta) = \frac{a}{h} = \frac{adjacent}{hypotenuse}$$

Tangent

$$tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

Right Angles

$$tan(\theta) = \frac{o}{a} = \frac{opposite}{adjacent}$$

Geometry

Similarity

$$\begin{aligned} s &= k \cdot s_0 \\ A &= k^2 \cdot A_0 \\ V &= k^3 \cdot V_0 \end{aligned}$$

Triangle

Equilateral

$$\begin{aligned} h &= \frac{\sqrt{3}}{2}a \\ A &= \frac{\sqrt{3}}{4}a^2 \end{aligned}$$

Right Angle

$$A = \frac{a \cdot b}{2}$$

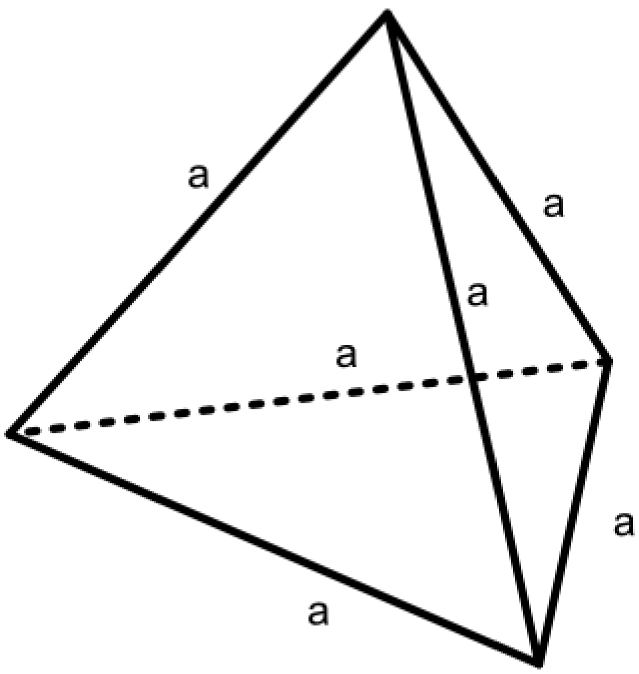
Circle

$$\begin{aligned} A &= \pi \cdot r^2 \\ U &= \pi \cdot 2r = \pi \cdot d \end{aligned}$$

Pyramid

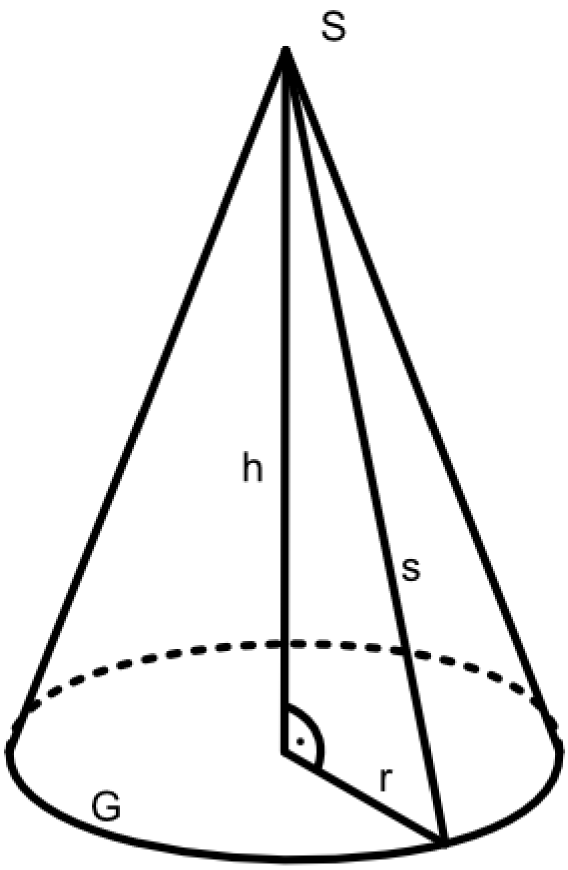
$$V = \frac{G \cdot h}{3}$$

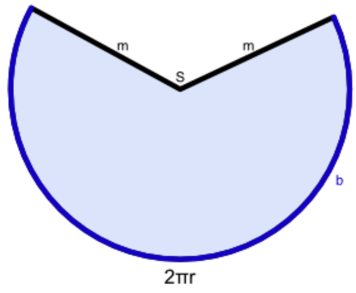
Tetrahedron



$$V = \frac{\sqrt{2}}{12}a^3$$

Kegel





$$V = \frac{G \cdot h}{3} = \frac{r^2\pi \cdot h}{3}$$
$$b = 2\pi \cdot r$$
$$M = \frac{b \cdot m}{2} = r\pi \cdot m$$
$$m = G + M$$

$$m^2 = h^2 + r^2$$
$$\alpha = 360^\circ \cdot \frac{r}{m}$$

Kugel

$$V = \frac{4}{3} r^3 \pi$$
$$A = 4\pi r^2$$

Vector Geometry

Definition

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$$
$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$$

Getting a Vector from Two Points

$$B = (11; -2) \quad C = (8; 6)$$

$$\overrightarrow{BC} = \begin{pmatrix} C_x \\ C_y \end{pmatrix} - \begin{pmatrix} B_x \\ B_y \end{pmatrix}$$

Stretching

$$k \cdot \vec{a}$$
$$k \cdot \begin{pmatrix} a_x \\ a_y \end{pmatrix}$$
$$\begin{pmatrix} k \cdot a_x \\ k \cdot a_y \end{pmatrix}$$
$$\begin{pmatrix} a'_x \\ a'_y \end{pmatrix}$$

Linear Combination

AKA aligning the tip of each vector to the tail of another vector, resulting in a chain, which then gives you the starting point of the combination of vectors.

$$\vec{x} = k_1\vec{a} + k_2\vec{b} + k_3\vec{c}$$

Linear Dependency

2 Vectors are linearly dependent if you can achieve one of the vectors by multiplying the other vector with any value of k, where k can't be 0.

$$\vec{a} = k \cdot \vec{b}$$

If there is a solution where k_1, k_2, k_3, ... are all != 0 then the three vectors are linearly dependent.

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = k_1 \cdot \vec{a} + k_2 \cdot \vec{b} + k_3 \cdot \vec{c}$$

Scalar Absolute Value (Length)

$$|\vec{a}| = \sqrt{{a_x}^2 + {a_y}^2}$$
$$|\vec{b}| = \sqrt{{b_x}^2 + {b_y}^2 + {b_z}^2}$$
$$\vdots$$

Unit vector

A unit vector is a vector that equals to 1 in its absolute value. The following formula can be used to determine the unit vector pointing in the same direction as the vector a.

$$\vec{e}_a = \frac{1}{|\vec{a}|} * \vec{a} = \frac{\vec{a}}{|\vec{a}|}$$

You can use the following formula to figure out what the new vector should be if it's to point in the same direction as the given one a but have a specific length l.

$$\vec{a}_l = \frac{l}{|\vec{a}|} * \vec{a}$$

Dot product

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$
$$\vdots$$

OR

$$\vec{a} \cdot \vec{b} = cos(\alpha) \cdot |\vec{a}| \cdot |\vec{b}|$$

The Scalar Absolute value can be calculated with the use of the `dotp` function using the TI-Nspire CX CAS as follows.

```
function dotp(veca, vecb):
    return veca.x * vecb.x + veca.y * vecb.x


function dotp(veca, vecb):
    return veca.x * vecb.x + veca.y * vecb.y + veca.z * vecb.z
```

$$|\overrightarrow{AB}| = \sqrt{dotp(\overrightarrow{AB}, \overrightarrow{AB})}$$

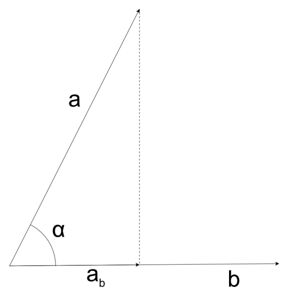
Angles

Vectors need to point ***AWAY*** from angle!

$$\begin{aligned}
\overrightarrow{AC} \cdot \overrightarrow{AB} &= \cos(\alpha) \cdot |\overrightarrow{AC}| \cdot |\overrightarrow{AB}| \\
\cos(\alpha) &= \frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{|\overrightarrow{AC}| \cdot |\overrightarrow{AB}|} \\
\alpha &= \cos^{-1}(\frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{|\overrightarrow{AC}| \cdot |\overrightarrow{AB}|})
\end{aligned}$$

Vector Shadow

$$\begin{aligned}
|\vec{a_b}| &= |\vec{a}| \cdot \cos(\alpha) \\
\vec{a_b} &= |\vec{a_b}| \cdot \vec{e_b} \\
\vec{a_b} &= |\vec{a}| \cdot \cos(\alpha) \cdot \vec{e_b}
\end{aligned}$$



Straights

Parameter Definition

$$g : \vec{r} = \vec{a} + t \cdot \vec{u}$$

Positional Relationships

Two straights can be in one of four relations: *identical*, *intersecting in one point*, *parallel and non-intersecting*, *not parallel*. By defining both of the straights generally

$$\begin{aligned}
g &: \vec{r} = \vec{a} + t \cdot \vec{u} \\
h &: \vec{r} = \vec{b} + s \cdot \vec{v}
\end{aligned}$$

we can make some determinations about when which of these cases are true.

If the equation $\vec{a} + t \cdot \vec{u} = \vec{b} + s \cdot \vec{v}$ has

1. one solution then *g* and *h* intersect in one point.
2. no solutions
    1. and $\vec{u} = k\vec{v}$ is valid the straights *g* and *h* are parallel.
    2. and $\vec{u} = k\vec{v}$ is invalid the straights *g* and *h* are not parallel.
3. infinite solutions then *g* and *h* are the same.

For straights in a 2D space the case 2.2 is not possible.

Exponential functions

$$E = S \cdot q^t$$

You get the resulting value E by multiplying the starting value s with the result of the growth factor q to the power of time t that has passed. This can be more generally described as:

$$f(x) = b \cdot a^x$$

Logistical functions

$$f(t) = \frac{a \cdot S}{a + (S - a)e^{-kt}}$$

Here a is defined as the y-axis intercept of the function, or the starting value if you will. S is defined as the value of $f(t)$ when t is infinitely big or otherwise referred to as carrying capacity. This can be more generally described as (might be inaccurate as I can't find a concise formula from different sources):

$$f(x) = \frac{a \cdot S}{a + (S - a)e^{-kx}}$$

Acknowledgements

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