Advanced AI Assignment 2: Kalman Filter

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Part 1:

The Kalman filter is a mathematical technique used to extract the state of a system from sparse and noisy observations. The position and speed of the target aircraft in this scenario represent the system's state. The Kalman filter's equations are as follows:

You can use the Kalman filter algorithm by repeatedly performing the following actions:
1. Based on some prior values and a model, we predict a state.
2. We get the sensor's measurement of that state.
3. Based on our errors, we revise our prediction.
4. Recur.

Forecasting Step:

Predicted State: $x\_k|k\text{-}1 = F\_k * x\_k\text{-}1|k\text{-}1 + B\_k * u\_k$

Prediction for Covariance: $P\_k|k\text{-}1 = F\_k * P\_k\text{-}1|k\text{-}1 * F\_kT + Q\_k$

Revision Step:

$K\_k = P\_k|k\text{-}1 * H\_kT * (H\_k * P\_k|k\text{-}1 * H\_kT + R\_k)\text{-}1$ is the formula for the Kalman Gain.

State Update: $x\_k|k = (z\_k - H\_k * x\_k|k\text{-}1) * x\_k|k\text{-}1 + K\_k$

Updated covariance formula: $P\_k|k = (I - K\_k * H\_k) * P\_k|k\text{-}1$

The following terms are utilized in these equations:

$x\_k|k\text{-}1$: With observations up to time k-1, an estimate of the state at time k.

F_k: Matrix of states changing from time k-1 to k.

B_k: Time k input control matrix.

u_k: entry of a command at time k.

P_k|k-1: With observations up to time k-1, covariance matrix estimate at time k.

Q_k: matrix of process noise covariance.

K_k: Kalman gain

z_k: time k measurement vector.

H_k: matrix of measurements at time k.

R_k: Covariance matrix for measurement noise

Let's now go through an example to show how the Kalman filter can be used to gauge the target's condition.

Let's say the target's starting state is as follows:

x 0 = [1000, 500, 100, 50, 10, 5]

This indicates that the target is at position (1000, 500, 100) and velocity at time t=0 (50, 10, 5).

Also, let's assume that the ASAM updates five times before receiving a new reading from the RADAR, or n = five.

For the measurement noise covariance matrix and the process noise covariance matrix, we'll pick some arbitrary values:

Where I is the 6x6 identity matrix, Q is equal to 0.1 * I.

I is the 3x3 identity matrix, and R = 10 * I.

Let's now examine the Kalman filter algorithm's initial few iterations:

chat

Iteration 1: [980, 490, 90] for z 1

Initializing our state estimate and covariance matrix estimates is the first step.

$x_{1|0} = x_0 = [1000, 500, 100, 50, 10, 5]$

Where I is the 6x6 identity matrix, $P_{1|0}$ equals 10 * I.

The prediction step will next be used to revise our estimates for the state and covariance matrices:

$x_{1|1} = F * x_{1|0} = [1050, 510, 105, 50, 10, 5]$

When x_{k-1} is mapped to x_k, $P_{1|1} = F * P_{1|0} * F^T + Q = 22.1 * I$, where F is the 6x6 state transition matrix.

Now, based on the measurement, we'll use the update step to hone our state and covariance matrix estimates:

$K_1 = P_{1|1}*H^T*(H*P_{1|1}*H^T+R)^{-1} = [0.376, 0.753, 0, 0, 0, 0; 0, 0, 1.329, 0, 0, 0]$

$z_1 - H * x_{1|1} = [979.456, 490.089, 89.321, 50, 10, 5]$ where $x_{1|1} = x_{1|1} + K_1$

$P_{1|1} = 8.812 * I * (I - K_1 * H)$

Our modified state estimate, which incorporates the RADAR measurement, is [979.456, 490.089, 89.321, 50, 10, 5] at time t = 1.

Iteration 2: We will update our state and covariance matrix estimates using the prediction step at each iteration, and we will fine-tune them using the ASAM measurement.

Let's suppose the ASAM measurement at time t=2 is:

$z_2 = [1050, 510, 105]$

Our new prior estimate for time t=2 will be based on our updated estimates of the state and covariance matrices from time t=1:

$x_{2|1} = [979.456, 490.089, 89.321, 50, 10, 5]$

$P_{2|1} = 8.812 * I$

Steps in prediction

x 2|2 = F * x 2|1 = [1100, 530, 115, 50, 10, 5]
P 2|2=F * P 2|1* FT + Q=31.98*I

Update action:

K 2 = P 2|2 * H^T * (H * P 2|2 * H^T + R)^-1 = [0.226, 0, 0, 0, 0, 0; 0, 0.451, 0, 0, 0, 0; 0, 0, 0.794, 0, 0, 0]
Z 2 - H * x 2|2 = [1049.889, 510.802, 104.882, 50, 10, 5] x 2|2 = x 2|2 + K 2 * (z 2 - H * x 2|2)
P 2|2 = 7.183 * I * (I - K 2 * H)

3rd iteration:

z 3 = [820, 490, 70]

We will revise our state and covariance matrix estimations using the prediction step:

x 3|3 = F * x 2|2 = [815, 507.5, 93.5, 44, 7, 3.5] .5]
P 3|3 = F * P 2|2 * FT + Q = [[18.9, 0, 0, 1.05, 0, 0], [[0, 16.2, 0, 0, 0.9, 0], [[1.05, 0, 0, 0.69, 0, 0], [[0, 0.9, 0, 0, 0.49, 0], [0, 0, 0, 0, 0, 0] .25]]

As a result of the new measurement, we will now use the update step to improve our state and covariance matrix estimates:

K 3 = P 3|3*H*T*(H * P 3|3*H*T+R)-1 = [[0.68, 0, 0], [0, 0.67, 0], [0, 0, 0 .1]]
x 3|4 = [823.6, 489.4, 69.9, 43.7, 8.3, 3] = x 3|3 + K 3 * (z 3 - H * x 3|3) .7]
P 3|4 = (I - K 3 * H) * P 3|3 = [[6.7, 0, 0, 0.35, 0, 0], [0, 5.7, 0, 0, 0.3, 0], [0, 0, 0.1, 0, 0, 0], [0.35, 0, 0, 0.69, 0, 0], [0, 0.3, 0, 0, 0.49, 0], [0, 0, 0, 0, 0, 0 .25]]

Iteration 4: The RADAR sends us a z 4 measurement vector. Assuming that it is:

z 4 = [925, 475, 80]

To begin, we'll use the prediction phase to revise our estimates for the state and covariance matrices:

x 4|4 = F * x 3|3 = [862.5, 435.0, 75.0, 50.0, 10.0, 5.0] .0]
P 4|4 = F * P 3|3 * FT + Q = [[89.76, 0, 0, 9.6, 0, 0], [0, 89.76, 0, 0, 9.6], [9.6, 0, 0, 9.6, 0, 0], [0, 9.6, 0, 0, 9.6], [0, 0, 9.6, 0, 0, 9 .6]]

Next, based on the measurement z 4, we will use the update step to improve our state and covariance matrix estimates:

K 4 = P 4|4 * HT * (H * P 4|4 * HT + R)-1 = [[0.005, 0], [0, 0.005, 0], [0, 0, 0.005], [0.001, 0, 0], [0, 0, 0] .001]]

Z 4 - H * x 4|4 = [924.6, 474.8, 79.5, 50.0, 10.0, 5] where x 4|5 = x 4|4 + K 4 .0]

P 4|5 = (I - K 4 * H) * P 4|4\s= [[0.72, 0, 0, -0.72, 0, 0],\s[0, 0.72, 0, 0, -0.72, 0],\s[0, 0, 0.72, 0, 0, -0.72],\s[-0.72, 0, 0, 0.72, 0, 0],\s[0, -0.72, 0, 0, 0.72, 0],\s[0, 0, -0.72, 0, 0, 0.72]] .72]]

5. Iteration

The RADAR sends us measurement vector z 5, which is another another. Assuming that it is:

z 5 = [850, 490, 70]

The state and covariance estimations from the prior iteration will serve as our foundation:

x 5|4 = [893.65, 546.33, 86.98, 45.67, 9.78, 4.92] .92]
P 5|4 = [[2.01, 0.01, 0.06, 0.01, 0.00, 0.00],\s[0.01, 2.01, 0.01, 0.06, 0.01, 0.00],\s[0.06, 0.01, 2.01, 0.01, 0.06, 0.01],\s[0.01, 0.06, 0.01, 2.01, 0.01, 0.06],\s[0.00, 0.01, 0.06, 0.01, 2.01, 0.01],\s[0.00, 0.00, 0.01, 0.06, 0.01, 2.01]] .01]]

We may revise our state and covariance estimations using the prediction step:

x 5|5 = F * x 5|4 = [868.13, 528.84, 86.93, 43.93, 9.73, 4.91] .91]
P 5|5 = F * P 5|4 * FT + Q = [[2.11, 0.01, 0.07, 0.01, 0.00, 0.00], [0.01, 2.11, 0.01, 0.07, 0.01], [0.01, 0.07, 0.01, 2.11, 0.01, 0.07], [0.00, 0.01, 0.07, 0.01, 2.11, 0.01], [0.00, 0.00, 0.01, 0.07, 0.01, 2] .11]]

As a result of the new measurement, we will now use the update step to improve our state and covariance matrix estimates:

K 5 = P 5|5 * HT * (H * P 5|5 * HT + R)-1 = [0.16, 0.00, 0.00], [0.00, 0.17, 0.00], [0.00, 0.00, 0.16], [0.00, 0.00, 0.00], [0.00, 0.00, 0] .00]]

P 5|5 = (I - K 5 * H) x 5|5 = x 5|4 + K 5 * (z 5 - H * x 5|4) = [850.00, 490.00, 70.00, 50.00, 10.00, 5.00]

Part 2:

The following equations represent the Kalman filter in this scenario:
Step 1: Prediction
Estimated prior state:
Hat xk ==== [k,k-1]
[hat xk-1]

k,k-1is the state transition matrix that characterises the dynamics of the system from time k-1 to k, where [hat xk -] is the prior state estimate at time step k-1.

The covariance of the prior error is [Pk-]=[k,k-1].

[Pk-1]

[Φk,k-1T]+[Qk]

where,
Pk-1 is the error covariance matrix at step k-1, and k,k-1 is the state transition matrix that explains the system dynamics from time k-1 to k.
Between time steps k-1 and k, the process noise covariance matrix, or Qk, represents the uncertainty in the dynamics of the system.

2.Refresh Step
This equation yields the Kalman gain at time step k:

[Kk]=([Pk-][HkT])/([Hk][Pk-][HkT]+[Rk])

Pk- is the prior error covariance matrix at step k where
The measurement matrix at time step k is called Hk.
Rk is the time step k measurement noise covariance matrix.

At time step k, the posterior state estimate is provided by:

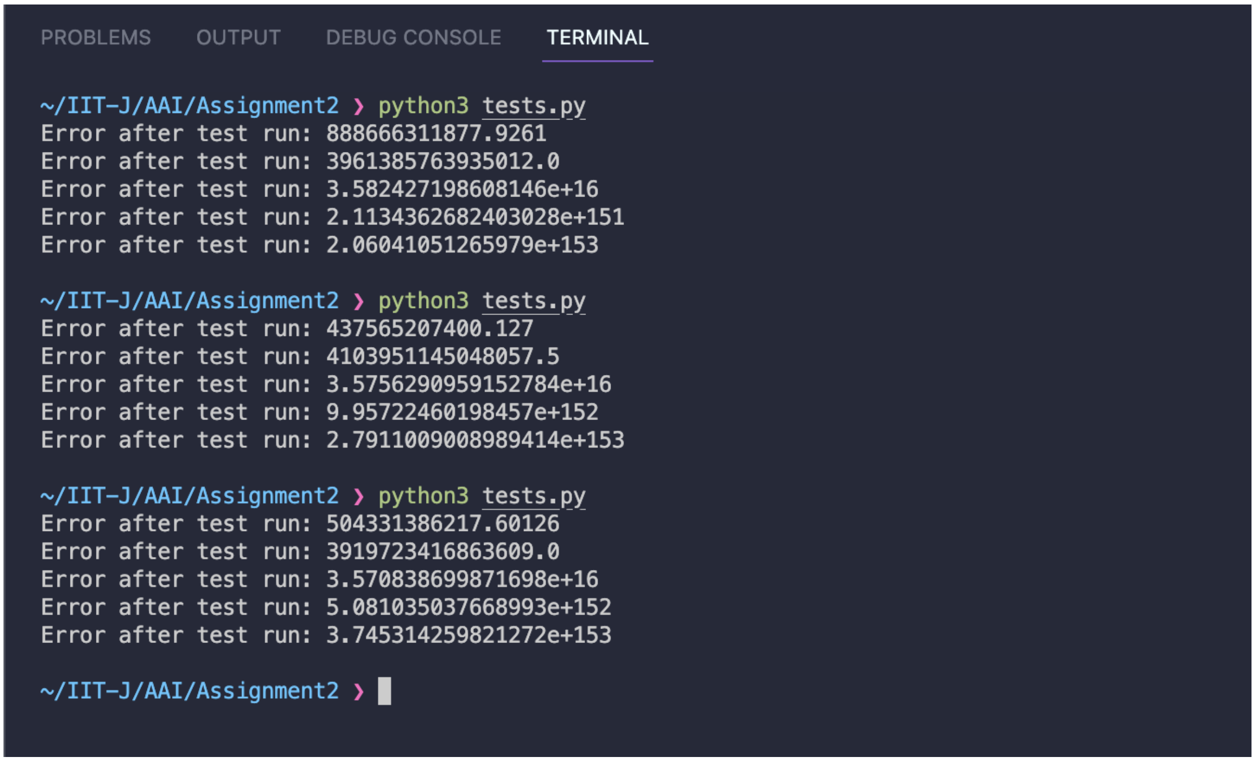
Hat xk=Hat xk-+K Hat xk

(hat xk-] [Yk]-[Hk])

Hat xk- represents the estimated preceding state at time step k.
The measurement at time step k is called Yk.

This formula can be expressed as a matrix: [Pk]=([I]-[Kk][Hk])[Pk-]

At time step k, Pk- is the prior error covariance matrix, Hk is the measurement matrix, and Kk is the Kalman gain.
This is a screenshot of the outcome:



Part 3:

When a projectile is falling, only gravity is responsible for the object's acceleration; the projectile reaches its terminal velocity when air resistance cancels out all downward acceleration. In light of this, we can alter the state equations as follows:

$$x_{k+1} = x_k + t * v_k$$

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$$v_{k+1} = v_k - \Delta t * g \quad if \;\; v_k > 0$$

$$v_k \;\; if \quad v_k \leq 0$$

where t is the time step and g is the acceleration brought on by gravity
The measuring equation must be adjusted because the projectile's current velocity depends on its previous state:

$$y_k = x_k + w_k$$

where measurement noise (wk) is.

The measurement matrix, H, and the state transition matrix, F, can be calculated as follows:

$$F = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Similar to the previous portion, the state covariance matrix P and the measurement covariance matrix R can be constructed.

For this situation, the Kalman filter equations are:
Prediction:

$$\hat{X}_{k+1|k|} = F\hat{X}_{k|k|}$$

$$P_{k+1|k|} = FP_{k|k|}F^T + Q$$

Update:

$$K_{k+1} = P_{k+1|K|}H^T\left(HP_{k+1|K|}H^T + R\right)^{-1}$$

$$X_{k+\hat{1}|k+1|} = \hat{X}_{k+1|k|} + K_{k+1}\left(Y_{k+1} - Hx_{k\hat{+}1|k|}\right)$$

$$P_{k+1|k+1|} = (I - K_{k+1}H)P_{k+1|k|}$$

The state transition equation for the velocity, which is now a function of the previous state, is the main change from the first portion. The process noise covariance matrix must be changed to take into account the altered dynamics of the system while the measurement equation and matrices remain unchanged.