① Least Squares. / POPULATION

$$\beta := \arg\min_{b \in \mathbb{R}^p} E\,(Y - X'b)^2$$

* $\underline{\text{FOC}}$ $\dfrac{\partial \arg\min}{\partial b} = 0 \Rightarrow \beta$

$$E\,(Y - X'\beta)X = 0$$

$$\boxed{\beta = E(X'X)^{-1} E(XY)}$$

$$\boxed{\epsilon = Y - X'\beta}$$

$$2 \rightarrow \boxed{Y = X'\beta + \epsilon}$$

$$E\,\epsilon X = 0$$

$$\epsilon \perp X$$

$$\rightarrow \beta = E(X'X)^{-1} E(XY)$$

* $(2\ 4\ 2\ 4\ 5\ 8\ 2\ 3\ 4\ 5)$

$$\frac{3}{10} \rightarrow \frac{1}{36}$$

$\hookrightarrow$ 1000000

$\hookrightarrow \hat{\beta} \longrightarrow \beta$ ✓

② Least Squares / Sample

$$\hat{\beta} := \arg\min_{b \in \mathbb{R}^p} \mathbb{E}(Y_i - X_i' b)^2$$

FOC

$$\mathbb{E}_n \big(\overbrace{Y_i - X_i' \hat{\beta}}^{\hat{\epsilon}_i} \big) X = 0$$

$$\hat{\beta} = \mathbb{E}_n (X_i' X)^{-1} (X_i' Y_i)$$

$$\hat{\epsilon}_i = \underline{Y_i - X_i' \hat{\beta}}$$

$$\boxed{y_i = x_i'\hat{\beta} + \hat{\epsilon}_i}$$

$$y_i = \hat{y}_i + \hat{\epsilon}_i$$

$$i \longrightarrow \infty$$

$$\hat{\beta} \longrightarrow \beta$$

* FRISCH - WAUGH - Lovell theorem

* POPULATION

$$X = (D'; W')' \quad \ldots \quad (1)$$

OBJETIVO

$$Y = D'\beta_1 + W'\beta_2 + \varepsilon \quad \ldots \quad (2)$$

a) $\tilde{V} = V - W'\gamma_{VW}$, $\hat{V}$

$$\gamma_{VW} = \arg\min_{b \in \mathbb{R}^p} E(V - W'b)^2$$

$$\gamma_{VW} = E(W'W)^{-1} E(W'V)$$

b) $Y = V + U \rightarrow \tilde{Y} = \tilde{V} + \tilde{U}$

$$\tilde{Y} = Y - W'\gamma_{YW}$$

$$\tilde{U} = U - U'\gamma_{VW}$$

$a \cap b \longrightarrow 2$

$$\tilde{Y} = \tilde{D}'\beta_1 + \tilde{W}'\beta_2 + \tilde{\epsilon}$$

$$\tilde{Y} = \tilde{D}'\beta_1 + \epsilon$$

$$\tilde{W} = W - W'\underbrace{\gamma_{WW}}_{\downarrow} = 0$$

$$X'\hat{\beta} = \hat{Y}$$

$$\tilde{Y} = \hat{\epsilon} \longrightarrow \hat{\epsilon} = Y - \hat{Y}$$

$$\tilde{Y} = \hat{\epsilon} = Y - X'\hat{\beta}$$

$$\boxed{\tilde{Y} = Y - X'\gamma_{YX}}$$

$$* \quad \tilde{Y} = \tilde{D}'\beta_1 + \tilde{W}'\beta_2 + \tilde{\epsilon}$$

$$0$$

tarea

$$\tilde{Y} = \tilde{D}'\beta_1 + \epsilon_*$$

$$\boxed{E\,\epsilon\,\tilde{D} = 0}$$

$$\beta_1 = E[\tilde{D}'\tilde{D}]^{-1} E[\tilde{D}'\tilde{Y}]$$

$$\hat{\epsilon} = \tilde{D} = D - W'\gamma_{DW}$$

$$\boxed{\beta_1 = E[\tilde{D}'\tilde{D}]^{-1} E[\tilde{D}'\tilde{Y}]}$$

$$\epsilon_1 = \tilde{D} = D - W'\gamma_{DW} \quad \cdots (1)$$

$\epsilon_2 = \tilde{y} = y - w'\gamma_{yw} \ldots (2)$

Reg $\underline{\epsilon_2}$ on $\underline{\epsilon_1} \longrightarrow \boxed{\hat{\beta}_1}$

(*)

$$y = D'\beta_1 + w'\beta_2 + \epsilon$$

+ $y \sim w \Rightarrow$ $y = w'\gamma_{yw} + \epsilon_2$.

$\boxed{\gamma_{yw} = (w'w)^{-1} w'y}$

+ $D \sim w \rightarrow D = w'\gamma_{Dw} + \epsilon_2$

$\boxed{\gamma_{Dw} = (w'w)^{-1} w'D}$

$\boxed{\hat{\epsilon}_1 = y - w'\hat{\gamma}_{yw} \ldots (1)}$

$\boxed{\hat{\epsilon}_2 = D - w'\hat{\gamma}_{Dw} \quad (2)}$

reg $\hat{\epsilon}_1 \sim \hat{\epsilon}_2$

$$\boxed{\hat{\epsilon}_1 = \hat{\epsilon}_2' \gamma_{\epsilon_1 \epsilon_2} + \epsilon_3}$$

$$\gamma_{\epsilon_1 \epsilon_2} = (\hat{\epsilon}_2' \hat{\epsilon}_2)^{-1} (\hat{\epsilon}_2' \hat{\epsilon}_1)$$

$$\hookrightarrow \quad \hat{\beta}_1 \xrightarrow[\not p]{n \to \infty} \beta_1$$