Lab 4

Objectives:

The objective of this lab is to understand and implement M/M/1 queueing models using C programming to analyze different queuing scenarios. The lab focuses on simulating real-world applications of queuing theory, including a bank service system and a football stadium ticket counter. By implementing these simulations, students will compute key performance metrics such as the probability of no waiting time, expected number of customers in the system, and total time spent in the queue. Additionally, the lab includes writing a program to calculate important measures of an M/M/1 queue, such as system utilization, average queue length, and waiting time, for given values of arrival rate (λ) and service rate (μ). Through this lab, students will gain practical insights into the efficiency of single-server queues and how queuing models apply to real-life service systems.

1. **Customers arrive in a bank according to a Poisson's process with mean inter arrival time of 10 minutes. Customers spend an average of 5 minutes on the single available counter, and leave. Write a program in C to find:**

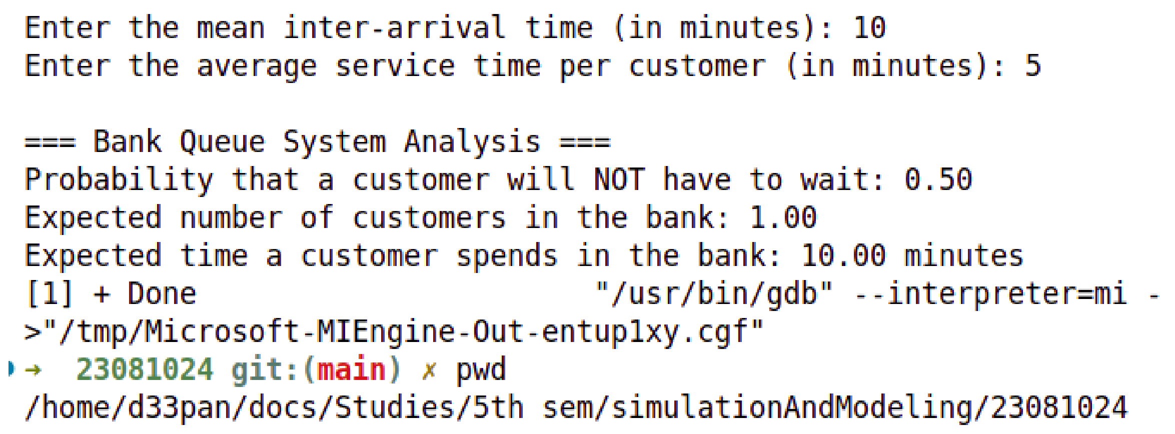
a) **Probability that a customer will not have to wait at the counter.**
b) **Expected number of customers in the bank.**
c) **Time a customer expects to spend in the bank.**

Source Code:

```
#include <stdio.h>

int main() {

    double arrival_time, service_time, lambda, mu, rho, P0, L, W;

    printf("Enter the mean inter-arrival time (in minutes): ");

    scanf("%lf", &arrival_time);

    printf("Enter the average service time per customer (in minutes): ");

    scanf("%lf", &service_time);

    lambda = 1.0 / arrival_time;  // Arrival rate (customers per minute)

    mu = 1.0 / service_time;      // Service rate (customers per minute)

    // Check if system is stable (rho < 1)

    if (lambda >= mu) {

      printf("\nSystem is unstable! Arrival rate must be less than service rate.\n");

        return 1;

    }

    // Compute metrics

    rho = lambda / mu;               // Utilization factor

    P0 = 1.0 - rho;                  // Probability of no waiting

   L = lambda / (mu - lambda);      // Expected number of customers in the bank

   W = 1.0 / (mu - lambda);         // Expected time a customer spends in the bank
```

```
    // Print results

    printf("\n=== Bank Queue System Analysis ===\n");

    printf("Probability that a customer will NOT have to wait: %.2lf\n", P0);

    printf("Expected number of customers in the bank: %.2lf\n", L);

    printf("Expected time a customer spends in the bank: %.2lf minutes\n", W);

    return 0;

}
```

Output:



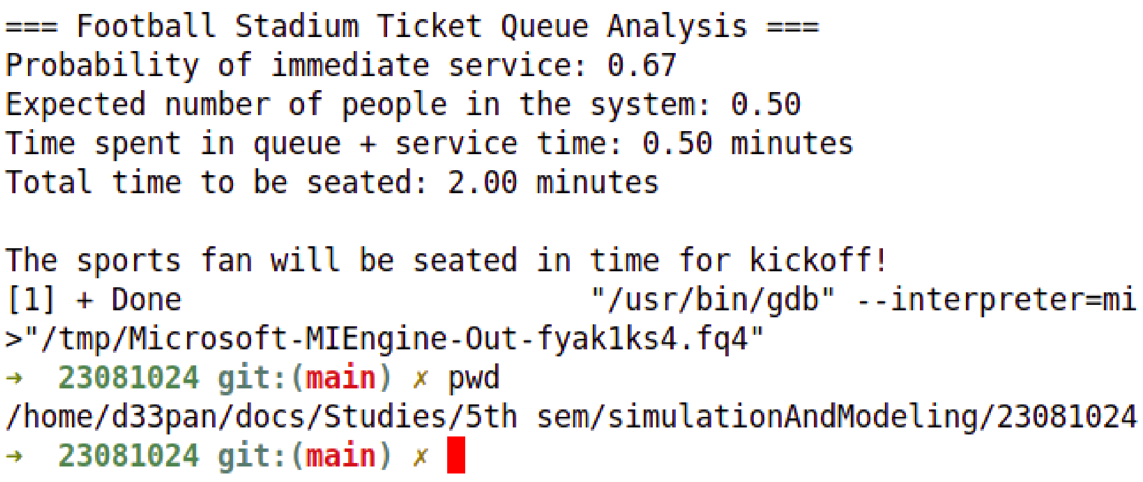
```
Enter the mean inter-arrival time (in minutes): 10
Enter the average service time per customer (in minutes): 5

=== Bank Queue System Analysis ===
Probability that a customer will NOT have to wait: 0.50
Expected number of customers in the bank: 1.00
Expected time a customer spends in the bank: 10.00 minutes
[1] + Done                       "/usr/bin/gdb" --interpreter=mi -
>"/tmp/Microsoft-MIEngine-Out-entup1xy.cgf"
→  23081024 git:(main) ✗ pwd
/home/d33pan/docs/Studies/5th sem/simulationAndModeling/23081024
```

2. **At the ticket counter of football stadium, people come in queue and purchase tickets. Arrival rate of customers is 1/min. It takes at the average 20 seconds to purchase the ticket. WAP in C to calculate total time spent by a sports fan to be seated in his seat, if it takes 1.5 minutes to reach the correct seat after purchasing the ticket. If a fan comes exactly before 2 minutes before the game starts, can sports fan expect to be seated for the kick-off?**

Source Code:

```
#include <stdio.h>

int main() {

    // Given data

    double lambda = 1.0;      // Arrival rate (customers per minute)

    double mu = 3.0;          // Service rate (customers per minute)
    double seat_time = 1.5;   // Time to reach the seat after purchase
    // Utilization factor (traffic intensity)
    double rho = lambda / mu;
    // Expected number of customers in the system (L)
    double L = lambda / (mu - lambda);
    // Expected time in the system (waiting + service time)
    double W = 1.0 / (mu - lambda);
    // Total time spent before being seated
    double total_time = W + seat_time;
    // Print results
    printf("=== Football Stadium Ticket Queue Analysis ===\n");
    printf("Probability of immediate service: %.2lf\n", 1 - rho);
    printf("Expected number of people in the system: %.2lf\n", L);
    printf("Time spent in queue + service time: %.2lf minutes\n", W);
    printf("Total time to be seated: %.2lf minutes\n", total_time);
    // Check if the fan will be seated before kickoff
    if (total_time <= 2.0)
        printf("\nThe sports fan will be seated in time for kickoff!\n");
    else
        printf("\nThe sports fan may miss the kickoff.\n");
    return 0;
}
```

Output:



```
=== Football Stadium Ticket Queue Analysis ===
Probability of immediate service: 0.67
Expected number of people in the system: 0.50
Time spent in queue + service time: 0.50 minutes
Total time to be seated: 2.00 minutes

The sports fan will be seated in time for kickoff!
[1] + Done                       "/usr/bin/gdb" --interpreter=mi
>"/tmp/Microsoft-MIEngine-Out-fyak1ks4.fq4"
➜  23081024 git:(main) ✗ pwd
/home/d33pan/docs/Studies/5th sem/simulationAndModeling/23081024
➜  23081024 git:(main) ✗
```

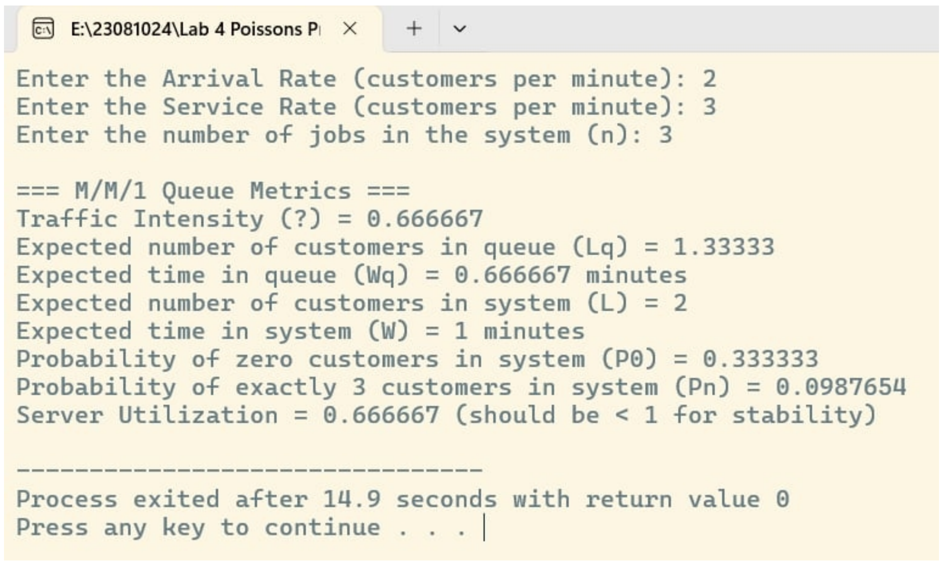
3. **Write a program to calculate measures of a M/M/1 Queue for a given value of Arrival Rate and Service Rate.**

Source Code:

```
#include <iostream>

#include <cmath>

using namespace std;

void mm1_queue(double lambda, double mu, int n) {

   // Check system stability (lambda must be less than mu)

   if (lambda >= mu) {

      cout << "Error: Arrival rate (lambda) must be less than service rate (mu) for
a stable system.\n";

       return;

   }

   double rho = lambda / mu;             // Server utilization (Traffic Intensity)

  double Lq = pow(rho, 2) / (1 - rho);     // Average number of customers in the
queue

  double Wq = Lq / lambda;              // Average time a customer spends in the
queue
```

```
    double L = lambda / (mu - lambda);        // Average number of customers in
the system
    double W = 1 / (mu - lambda);          // Average time a customer spends in the
system
    double P0 = 1 - rho;                   // Probability of zero customers in the system
    double Pn = P0 * pow(rho, n);            // Probability of exactly 'n' customers in
the system
    // Display results
    cout << "\n=== M/M/1 Queue Metrics ===\n";
    cout << "Traffic Intensity (?) = " << rho << endl;
    cout << "Expected number of customers in queue (Lq) = " << Lq << endl;
    cout << "Expected time in queue (Wq) = " << Wq << " minutes\n";
    cout << "Expected number of customers in system (L) = " << L << endl;
    cout << "Expected time in system (W) = " << W << " minutes\n";
    cout << "Probability of zero customers in system (P0) = " << P0 << endl;
    cout << "Probability of exactly " << n << " customers in system (Pn) = " << Pn
<< endl;
    cout << "Server Utilization = " << rho << " (should be < 1 for stability)\n";
}
int main() {
    double lambda, mu;
    int n;
    // Get user inputs
    cout << "Enter the Arrival Rate (customers per minute): ";
    cin >> lambda;
    cout << "Enter the Service Rate (customers per minute): ";
    cin >> mu;
    cout << "Enter the number of jobs in the system (n): ";
    cin >> n;
    // Call function to calculate M/M/1 queue metrics
    mm1_queue(lambda, mu, n);
    return 0;
}
```

Output:



```
Enter the Arrival Rate (customers per minute): 2
Enter the Service Rate (customers per minute): 3
Enter the number of jobs in the system (n): 3

=== M/M/1 Queue Metrics ===
Traffic Intensity (?) = 0.666667
Expected number of customers in queue (Lq) = 1.33333
Expected time in queue (Wq) = 0.666667 minutes
Expected number of customers in system (L) = 2
Expected time in system (W) = 1 minutes
Probability of zero customers in system (P0) = 0.333333
Probability of exactly 3 customers in system (Pn) = 0.0987654
Server Utilization = 0.666667 (should be < 1 for stability)

--------------------------------
Process exited after 14.9 seconds with return value 0
Press any key to continue . . .
```

Conclusion:

In conclusion, this lab provided a practical understanding of M/M/1 queueing models and their real-world applications through C programming. By simulating a bank service system and a football stadium ticket counter, we successfully analyzed key performance metrics such as the probability of a customer not waiting, the expected number of customers in the system, and the total time spent in the queue. Additionally, implementing a general M/M/1 queue program allowed us to compute essential parameters like system utilization, queue length, and waiting time for given arrival and service rates. These experiments reinforced the significance of queueing theory in optimizing service efficiency and minimizing wait times in various domains, such as banking, ticketing, and customer service. Overall, this lab enhanced our understanding of queueing models and their practical implementation in computational simulations.