Design and Analysis of Algorithms Assignment - 6

Department of Information Technology

Indian Institute of Information Technology - Allahabad, India

Gitika Yadav
IIT2019219

Divyatez Singh
IIT2019220

Divyansh Rai
IIT2019221

Abstract:This paper introduces an algorithm that calculates the minimum distance between any pair location(or cells) in a given 2D matrix(with all 0's and 1's, 0 denoting blocked cells) .

Index Terms: 2D-Arrays, Queue, Recursion

INTRODUCTION

Breadth-first search (BFS) is an algorithm for traversing or searching graphs or trees , by exploring all of the neighbor nodes at the present depth prior to moving on to the nodes at the next depth level.

Breadth First Search Breadth first search is a general technique of traversing a graph. Breadth first search may use more memory but will always find the shortest path first. It is a path finding algorithm that is capable of always finding the solution if one exists.

1. **Data Structure:** In this search a queue data structure is used and it is level by level traversal.

2. **Traversal:** Breadth first search expands nodes in order of their distance from the root.

3. **Path:** Each node in the search tree is expanded in a breadth wise at each level.

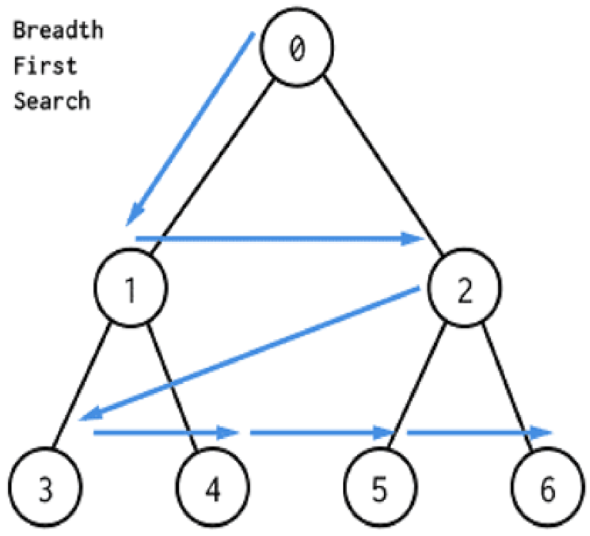


Figure 1: Divide and Conquer

Advantages of Breadth-first search The Breadth-first search technique generally has following mentioned advantages over Brute Force Algorithms.

Time Efficiency Used to find the shortest path between vertices as traverses by level in shortest time.

Space Efficiency: As no need to mention the algorithm, but still, the space complexity also reduces up to a much extent due to the nature of searching level by level

This report further contains:

- Algorithm Designs
- Algorithm Analysis
- Experimental Study and Profiling
- Conclusion
- References
- Appendix

ALGORITHM DESIGN

A general problem based on the breath-first search in which, we level order traversal nodes , initiating by pushing into queue and then make decisions while traveling front of queue.Following structures have been taken into account while making the code for the report.

Algorithmic Steps: For the given problem, we are given the dimension of the given matrix, then the matrix and then then for a given pair location(of cells), we need to find the shortest path from source point to destination point.

The approach we discussed is based on Breadth First Search Algorithm for finding shortest path between two given cells, generally has the steps defined in following algorithmic procedure, which have been implemented while solving the problem:

1. Input dimension of given matrix.
2. Store then the matrix and source cell's and destination cell's location.
3. Store each cell as a node with their row, column values and distance from the source cell.
4. Start BFS with the source cell.
5. Make a visited array with all having "false" values.
6. Keep updating distance from source value in each move.
7. Return distance when destination is met, else return -1 (no path exists in between source and destination).

The above returned cost is the sum of distance from source to destination.

```
Int:
procedure main
        n ->row
        m ->column
        a[n][m]->matrix with input
        si,sj ->source cell location
    di,dj ->destination cell location

    if(source cell or destination cell equals 0):
        print( Path  not possible )

    else:
        Initialize queue (of pair of int) q
        Initialize vector vis[n][m] with all
        values initially false

        Initialize cost -> 0
        set vis[si][sj] -> true
        push ({si,sj}) in q

        while(q.size()>0):
                cost++
                p -> q.size

                while(p--):
                        top -> q.front()
                q.pop()
                x->top.first
                    y->top.second



            if(x>0):
                    if(valid left cell):
                            Left node visited
                            Queue push left node
            if(y>0):
                    if(valid above cell ):
                            Above node visited
                            Queue push above node
            if(x<n-1):
                    if(valid right cell ):
                            Right node visited
                            Queue push right cell
            if(x>0):
                    if(valid below cell ):
                            Below node visited
                            Queue push below cell

            if(destination cell is visited):
                    print(cost)
                    break
            End of while

            if(destination cell is visited):
                    break
        End of while
        if(destination still not visited):
            print( Not  Found )

End of procedure
```

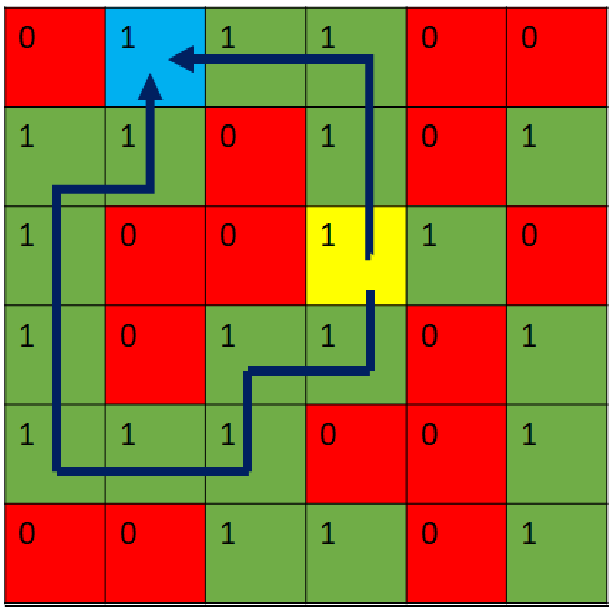


Figure 2: BFS Traversal

ALGORITHM ANALYSIS

APRIORI ANALYSIS : This is the analysis performed prior to running in a stage where the function is defined using a theoretical model. Therefore, complexity is determined by just examining the algorithm rather than running it on a particular system with a different memory,processor, and compiler. So,as we discussed under the heading complexity analysis we arrived at the conclusion that the best time complexity is O(1) and the space complexity O(n*m).

Time complexity Derivation: Let Time complexity of above algorithm be T(n) and it can be expressed as follows:
TIME COMPLEXITY DERIVATION:

T(n) = V (O(1) + O(Edge from vertex) + O(1))

As each vertex can travel in 4 directions:
T(n) = V + 4V + V
T(n) = 2V + E(total number of edges in graph)
T(n) = V + E

Scanning for all adjacent vertices takes O(E) time, since sum of lengths of adjacency lists is E.
Thus on combining we get the overall time complexity as:T(n) = O(V+E)

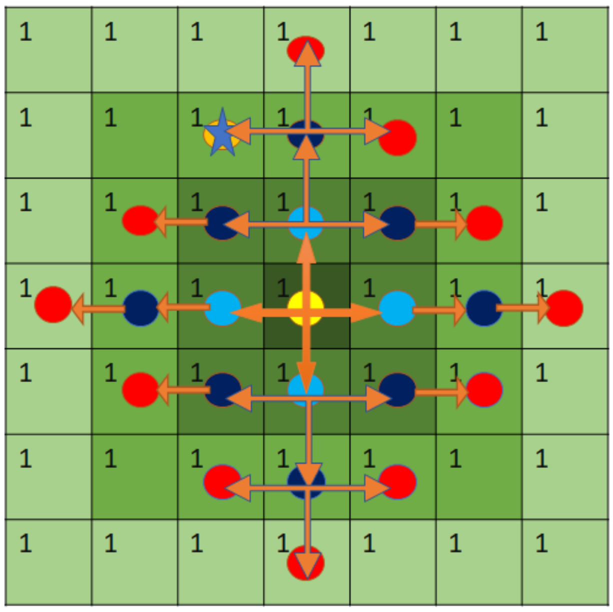
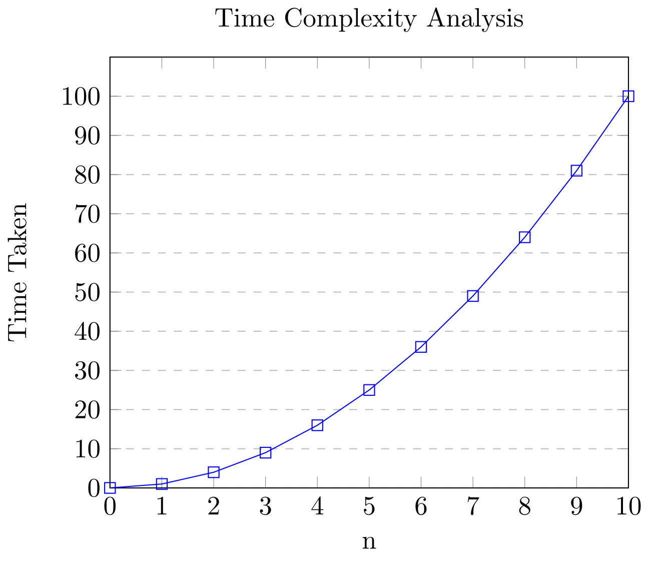


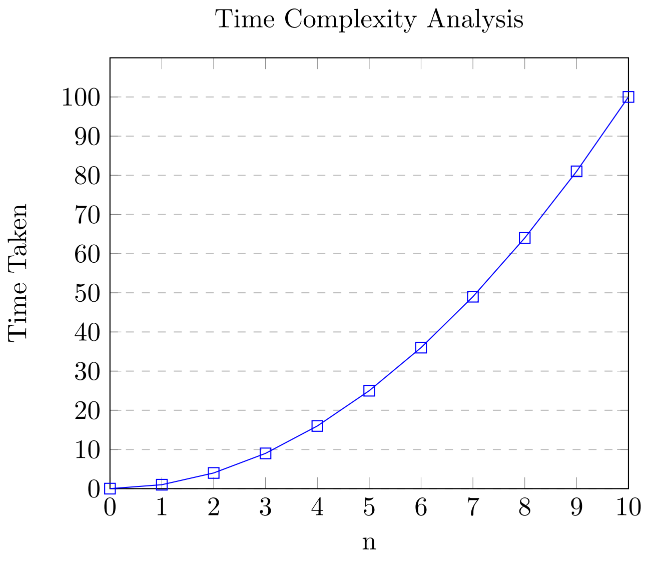
Figure 2: Time Complexity Derivation



Time Analysis: Following is the graph representing the time complexity of the algorithm.

By the experimental analysis, we found that in case of optimized approach, on increasing the number of numbers the graph is strictly increasing with a bend (or concavity) towards horizontal axis. Thus the overall time increases with an increase in size.

Space Analysis: Following is the graph representing the space complexity of the algorithm.



By the experimental analysis, we found that in case of optimized approach, on increasing the number of numbers the graph is strictly increasing. Thus the overall space increases with an increase in size.

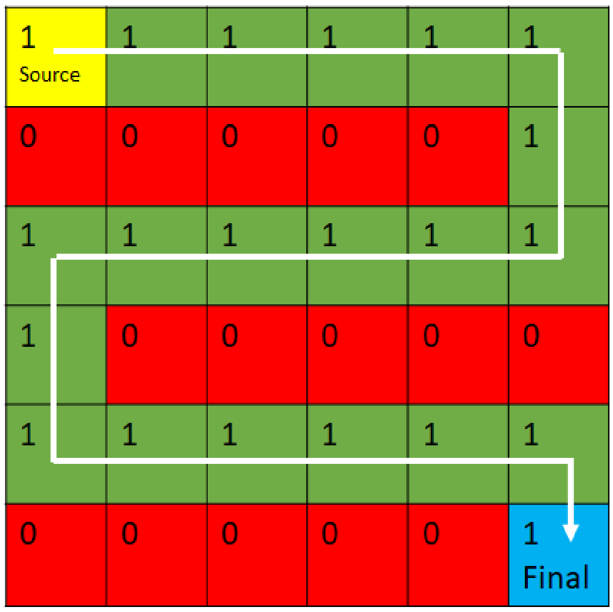


Figure 2: Worst Case Time Complexity

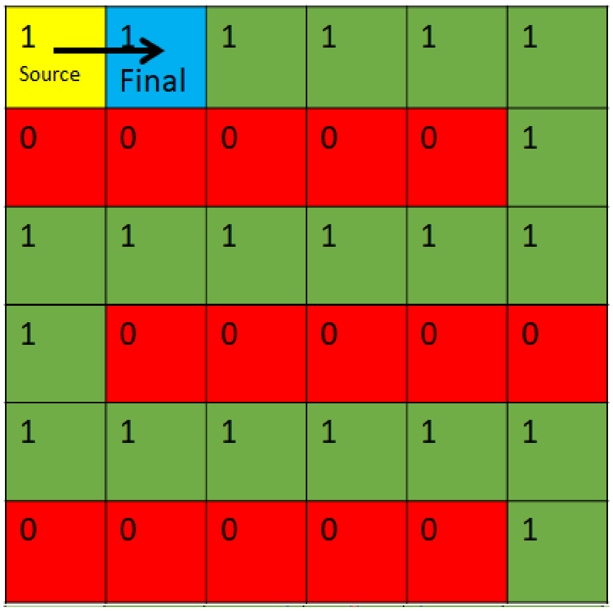


Figure 2: Best Case Time Complexity

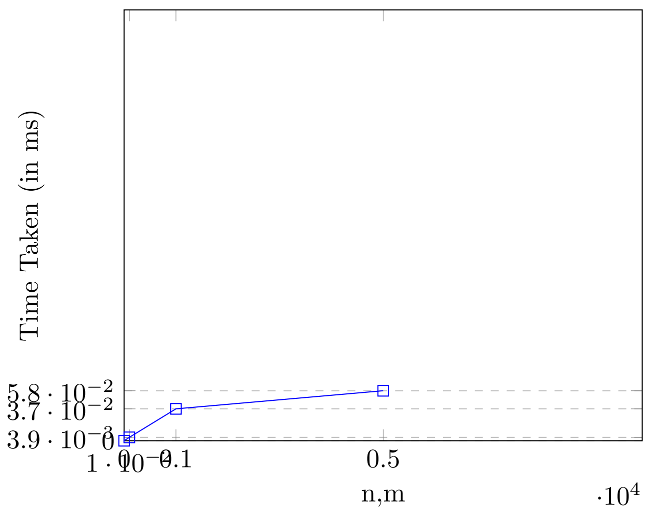
APOSTERIORI ANALISIS: Aposteriori analysis of an algorithm means we per- form analysis of an algorithm only after running it on a system. It directly depends on the system and changes from system to system. So for the a aposteriori analysis of the algorithm,we have run our code on the compiler and get values of the time.

Experimental Analysis

In the following table some cases are plotted for the algorithm on our local machine,

n,m	Time Taken (in ms)
100	0.0039
1000	0.037
5000	0.058

Time Complexity Analysis



CONCLUSION

So, with the above mentioned algorithms and their profiling, we come to the conclusion that this minimum distance between any pair location(or cells) in a given 2D matrix(with all 0's and 1's, 0 denoting blocked cells) is achieving its best time complexity of O(V+E) and space complexity of O(V*V).
Also, when we are dealing with large numbers such as the one occupying 64-bits (long long integers), the complexity is same. So we can take the modulus of distance with 109+7 in order to make sure that it gets fit in the available data types(long long integer types more specifically).

REFERENCES

1. Introduction to Algorithms by Cormen,Charles, Rivest and Stein:
https://web.ist.utl.pt/ fabio.ferreira/material/asa

2. Breadth First Search or BFS for a Graph:
https://www.geeksforgeeks.org/breadth-first-search-or-bfs-for-a-graph/

3. Intro to BFS:
https://cp-algorithms.com/graph/breadth-first-search.html

APPENDIX

To run the code, follow the following procedure:

1. Download the code(or project zip file) from the github repository.
2. Extract the zip file downloaded above.
3. Open the code with any IDE like Sublime Text, VS Code, Atom or some online compilers like GDB.
4. Run the code following the proper running commands(vary from IDE to IDE)
   (a) **For VS Code:** Press Function+F6 key and provide the input on the terminal.
   (b) **For Sublime Text:** Click on the Run button and provide the input.

Code for Implementation is:

```
#include<bits/stdc++.h>
using namespace std;


int main(){
  int n  ,m; cin>>n>>m;
  int a[n][m];
   for(int i=0;i<n;i++){
        for(int j=0j<m;j++){
      cin>>a[i][j];
    }
   }

 int si,sj,di,dj;
  cin>>si>>sj>>di>>dj;
  if(a[si][sj]==0||a[di][dj]==0)
  {
      cout<<"ERROR"<<endl;
  }


  queue<pair<int,int>>q;
  vector<vector<bool>>
  vis(n,vector<bool>(m,false));
  q.push({si,sj});
  vis[si][sj]=true;
  int cost=0;

  while(q.size()>0)
  {
    Cost++;
    int p=q.size();
    while(p--)
    {
      pair<int,int> top=q.front();
      q.pop();
      int x = top.first;
      int y = top.second;
      if(x>0)
      {
        if(vis[x-1][y]==false&&a[x-1][y]==1)
        {
          vis[x-1][y]=true;
          q.push({x-1,y});
        }
      }
      if(y>0)
      {
        if(vis[x][y-1]==false&&a[x][y-1]==1)
        {
          vis[x][y-1]=true;
          q.push({x,y-1});
        }
      }
      if(x<n-1)
      {
        if(vis[x+1][y]==false&&a[x+1][y]==1)
        {
          vis[x+1][y]=true;
          q.push({x+1,y});
        }
      }
      if(y<m-1)
      {
         if(vis[x][y+1]==false&&a[x][y+1]==1)
         {
           vis[x][y+1]=true;
          q.push({x,y+1});
         }
      }
      if(vis[di][dj]==true)
      {
        cout<<" Cost is "<<cost<<endl;
        break;
      }

    }

    if(vis[di][dj]==true)
    {
      break;
    }
  }
  if(vis[di][dj]==false)
  {
    cout<<"Not Found"<<endl;
  }
}
return 0;
}
```