Assignment 2:

1. **Raindrops are falling at an average rate of 20 drops per square inch per minute. What would be a reasonable distribution to use for the number of raindrops hitting a particular region measuring 5 inches^2 in t minutes? Why? Using your chosen distribution, compute the probability that the region has no rain drops in a given 3 second time interval. A reasonable choice of distribution is P**

Solution: A reasonable choice of distribution is Pois(λt), where $\lambda = 20 \cdot 5 = 100$ (the average number of raindrops per minute hitting the region). Assuming this distribution,

$$P(\text{no raindrops in } 1/20 \text{ of a minute}) = e^{-100/20}(100/20)^0/0! = e^{-5} \approx 0.0067.$$

2. **Let X be a random day of the week, coded so that Monday is 1, Tuesday is 2, etc. (so X takes values 1, 2..., 7, with equal probabilities). Let Y be the next day after X (again represented as an integer between 1 and 7). Do X and Y have the same distribution? What is P(X)**

Solution: Yes, X and Y have the same distribution, since Y is also equally likely to represent any day of the week. However, X is likely to be less than Y. Specifically,

$$P(X < Y) = P(X \neq 7) = \frac{6}{7}.$$

In general, if Z and W are *independent* r.v.s with the same distribution, then $P(Z < W) = P(W < Z)$ by symmetry. Here though, X and Y are *dependent*, and we have $P(X < Y) = 6/7, P(X = Y) = 0, P(Y < X) = 1/7$.