

Assignment 4:

1. Is it possible that an event is independent of itself? If so, when?

Solution: Let A be an event. If A is independent of itself, then $P(A) = P(A \cap A) = P(A)^2$, so $P(A)$ is 0 or 1. So this is only possible in the extreme cases that the event has probability 0 or 1.

2. Is it always true that if A and B are independent events, then A^c and B^c are independent events? Show that it is, or give a counterexample.

Proof. Let A and B be independent. Then

$$P(B^c|A) = 1 - P(B|A) = 1 - P(B) = P(B^c),$$

so A and B^c are independent. Swapping the roles of A and B , we have that A^c and B are independent. Using the fact that A, B independent implies A, B^c independent, with A^c playing the role of A , we also have that A^c and B^c are independent. ■